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VARIOGRAM-BASED FRACTAL ANALYSIS OF SLOPE MORPHOMETRY ON MERCURY

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Abstract

This study presents a quantitative assessment of the fractal characteristics of the spatial distribution of surface slopes within Mercury using a variogram-based fractal dimension (FD) approach applied to the global MESSENGER DEM. The obtained results reveal a clear fractal pattern of Mercury's slope distribution, with mean FD values of 2.127 in the Northern Hemisphere and 2.107 in the Southern Hemisphere, supported by very high coefficients of determination ($R^2 > 0.99$). A statistically consistent, though moderate, hemispherical asymmetry is observed, with higher fractal dimensions and greater spatial heterogeneity in the Northern Hemisphere, particularly in regions dominated by large impact basins and limited volcanic smoothing. In contrast, the Southern Hemisphere exhibits more homogeneous fractal properties, consistent with spatial differences in surface roughness and morphological complexity. These findings provide new insights into the geological evolution of Mercury's hemispheres and highlight the applicability of variogram-based fractal analysis as a methodological tool for comparative studies of planetary topography.

Introduction

Prior to the Mariner 10 spacecraft mission, the scientific community had very limited knowledge of the topographical features of Mercury, the planet closest to the Sun. In 1975, [1] produced the first geologic map of Mercury's surface, identifying various geological features. Mercury's surface is densely covered by impact craters. More than 30,000 craters with diameters exceeding 5 km have been catalogued from spacecraft imagery. The largest of these structures is the Caloris basin, approximately 3.9 billion years old and about 1,550 km in diameter. Owing to Mercury's proximity to the Sun, impactors typically strike the planet at significantly higher velocities than those affecting other terrestrial planets. These high-impact velocities enhance ejecta production and contribute to the formation of an unusually large number of secondary craters across Mercury's surface [2]. In addition to impact craters, Mercury's surface comprises a variety of other geological features, including heavily cratered plains, smooth plains, as well as hilly and mountainous regions (Fig. 1).

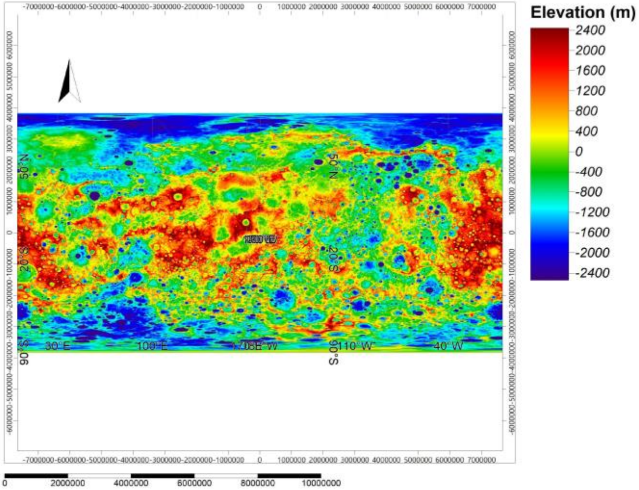

Fig. 1. Digital Elevation Model (DEM) of Mercury showing the main characteristics of its terrain. The surface is dominated by impact craters and elevated regions. Notably, the polar areas are largely characterised by negative elevations, whereas the equatorial regions predominantly exhibit positive elevations.

Subsequent missions, particularly MESSENGER [3], have significantly advanced our understanding of Mercury's surface. The global topography of Mercury varies from approximately – 5365 m to 4457 m (Fig. 2). About 60% of the surface corresponds to negative elevations, while positive elevations account for roughly 40%. In the northern hemisphere, negative landforms dominate, whereas in the southern hemisphere, positive elevations are more prevalent.

In recent decades, the continuous acquisition of new data on planetary bodies within the Solar System has facilitated the development and application of innovative analytical and interpretative methods. Among them, fractal analysis occupies an important place. Fractal geometry provides a scientific framework for describing complex shapes and surfaces [4], making it a suitable methodological tool for investigating planetary topography.

Regarding Mercury, the application of fractal analysis remains limited. Only a few studies addressing this topic can be found in the literature. [5] applied fractal methods to analyse asteroid craters on the planet. [6] explored spatial variations of Mercury's terrain and gravity field based on fractal dimension. In a conference paper, [7] demonstrated that Mercury's topography exhibits scaling fractal properties, transitioning from monofractal behaviour at scales below 10 km to a multifractal regime at larger scales.

This scarcity of such studies highlights one of the main objectives of the present work. Unlike previous studies focusing on elevation and gravity field, the present work applies variogram-based fractal analysis to slope gradients, which represent first-order derivatives of topography. By applying the variogram-based fractal dimension approach, the current study aims to comprehensively characterise the spatial patterns of slope distribution across Mercury’s surface, providing new insights into its geological evolution.

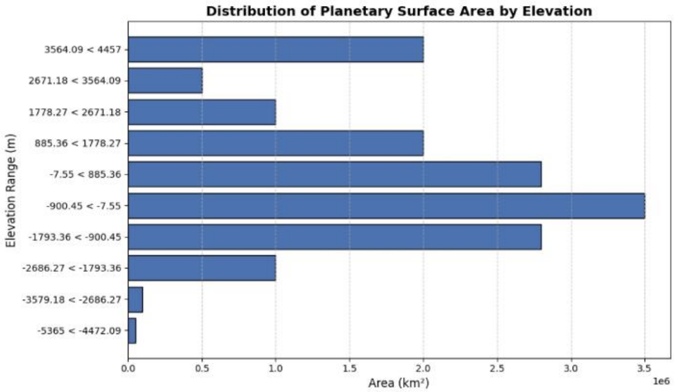

Fig. 2. The bimodal distribution of Mercury’s topographic elevations. Negative elevations are more extensive than positive ones, reflecting the dominant influence of impact events that have shaped the planet’s present-day surface morphology. The highest peaks are another characteristic dominating the positive elevations.

Methods and Data

Preprocessing, Noise Reduction, and Slope Calculation of the Mercury DEM

The global Mercury Digital Elevation Model (DEM), derived from MESSENGER/USGS data at 665 m spatial resolution, was pre-processed to mitigate high-frequency orbital noise and striping artefacts prior to morphometric analyses. The original DEM values were provided with a scale factor of 0.5; all raster values were multiplied by this factor to convert digital numbers into metric elevations (meters). This step ensured that subsequent slope calculations were physically meaningful.

To suppress high-frequency noise while preserving primary geological features, the scaled DEM was subjected to Gaussian smoothing with a standard deviation (σ) of 1 pixel (~ 665 m). This approach attenuates small-scale striping and random measurement noise without significantly altering surface morphology. The Gaussian-filtered DEM retained the original georeferencing and projection and served as the input for slope computation and subsequent variogram-based fractal analysis. This preprocessing strategy ensures that the derived slope values reflect genuine geomorphic variability rather than instrumental or orbital noise.

Slope (Table 1) was calculated using the Terrain Analysis → Morphometry → Slope module in SAGA GIS, which implements the algorithm of [8, 9] for estimating slope derivatives. The method employs a 3 × 3 moving window to approximate the partial derivatives of elevation in the x- and y-directions (dz/dx and dz/dy) and computes slope as the magnitude of the local gradient vector. The slope angle theta (θ) is calculated as:

(1) $$\text{theta} = \arctan(\text{sqrt}((dz/dx)^2 + (dz/dy)^2))$$

where dz/dx and dz/dy represent the first derivatives of elevation along the x and y directions, respectively. In this study, slope values are expressed in degrees.

Although the classical variogram-based formulation of FD is defined for topographic surfaces, its application to slope fields is justified here as a measure of spatial roughness of gradient magnitudes rather than true geometric fractal dimension.

Table 1. Global slope distribution of Mercury's surface. The results indicate that nearly 97% of the total surface area has slope values lower than 1.5°.

Class	Slope Interval (Degrees)	Distribution (%)
1	0° < 0.72°	57.06%
2	0.72° < 1.44°	39.88%
3	1.44° < 2.16°	2.03%
4	2.16° < 2.88°	1.09%
5	2.88° < 3.60°	0.63%
6	3.60° < 4.32°	0.40%
7	4.32° < 5.04°	0.28%
8	5.04° < 5.76°	0.20%
9	5.76° < 6.48°	0.14%
10	> 6.48°	0.29%

Fractal Analysis of Mercury's Topography Using a Variogram-Based Method

To investigate the fractal nature of Mercury's surface, a local fractal analysis was conducted on the global Mercury DEM at 665 m spatial resolution. Given the large size of the dataset (11520 × 23040 pixels), the DEM was divided into two hemispherical tiles (Northern and Southern ones) to reduce computational load and facilitate memory-efficient processing within the Python environment of QGIS.

Local fractal dimensions were computed using a variogram approach [10, 11] implemented in a Python script executed within the QGIS environment. A moving window of 64 × 64 pixels with a 32-pixel step was applied to each tile. Within each window, the semi-variance was estimated for a limited number of lag distances (up to five pixels), using horizontal and vertical offsets only, rather than a fully isotropic variogram:

(1) $$\text{gamma}(h) = 1/2 * \text{mean}\,[(z(x) - z(x+h))^2]$$

where z(x) represents the elevation at position x, and h is the lag distance.

The slope of the log-log relationship between semi-variance and lag distance was then used to derive the local fractal dimension:

(2) $$FD = 3 - (m / 2), \quad 2 <= FD <= 3$$

where m denotes the regression slope. The coefficient of determination (R^2) from the linear regression in log-log space was also recorded for each window as a measure of fit quality.

The computed FD and R^2 values were assigned to the corresponding pixels within each moving window and written to separate GeoTIFF files. Prior to analysis, elevation values were normalised to zero mean and unit variance to ensure numerical stability and comparability across windows. Each hemispherical tile was processed independently, after which the outputs were merged to obtain global maps of local fractal dimensions and R^2. This tiling strategy ensured memory-efficient processing without compromising the spatial consistency of the DEM and, at the same time, constituted a prerequisite for a comparative analysis between the two planetary hemispheres.

Data and software

The analysis of Mercury's topography was performed using digital elevation data from the Mercury Global Digital Elevation Model (DEM) v2 [12], derived from observations by the MESSENGER spacecraft mission [3]. The DEM provides global coverage of Mercury's surface at a spatial resolution of 665 × 665 m per pixel, representing topographic variations across the entire planet. Elevation values are expressed in meters and were scaled according to the original data specifications.

Raster data processing and analysis were conducted using open-source geographic information systems (GIS). The primary platforms employed in this study were SAGA-GIS (System for Automated Geoscientific Analyses; [13]) and QGIS. This software environment enabled the extraction of slope values, the implementation of variogram-based fractal dimension analysis, and the spatial visualisation necessary to support the objectives of the study.

Results and Discussion

The results of this research are quantitatively summarised in Table 2 and graphically illustrated in Fig. 3 and Fig. 4. The interpretations are presented below.

Table 2. Fractal Analysis of Slope in the Mercurian Hemispheres

	Northern hemisphere	**Southern hemisphere**
FD min	2.005	2.000
FD max	2.773	2.876
FD mean	2.127	2.107
FD StdDev	0.0639	0.0527
R^2 min	0.805	0.589
R^2 max	0.999	0.999
R^2 mean	0.997	0.998
R^2 StdDev	0.0034	0.0071

The results presented in Table 2 clearly demonstrate that Mercury's slope distribution exhibits a pronounced fractal behaviour. The mean fractal dimension (FD) is 2.127 for the Northern Hemisphere (NH) and 2.107 for the Southern Hemisphere (SH). The high coefficients of determination further confirm the variogram-based fractal model, with mean R^2 values of 0.997 for the NH and 0.998 for the SH.

The visual analysis of Fig. 3 and Fig. 4 reveals that the highest latitudes, particularly the polar and peripolar regions (> 50° N and S), are characterised by the largest spatial variability in FD values. This pattern indicates a more complex and heterogeneous terrain marked by sharper topographic transitions. Several factors may contribute to this behaviour. On the one hand, Mercury's equatorial regions experience extreme diurnal temperature variations (approximately -170°C to +430°C) and may contribute to differences in slope smoothing. In contrast, temperature contrasts at higher latitudes are less pronounced, favouring the preservation of steeper relief elements.

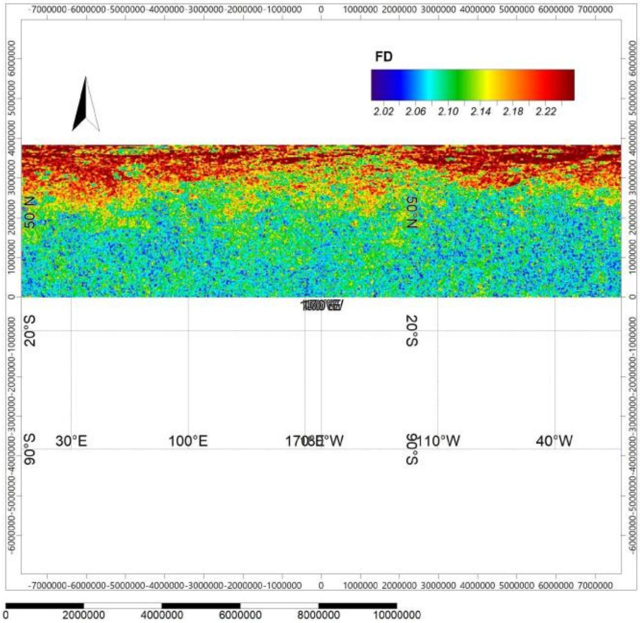

Fig. 3. Spatial distribution of slope fractal dimension (FD) within Mercury's Northern Hemisphere (NH)

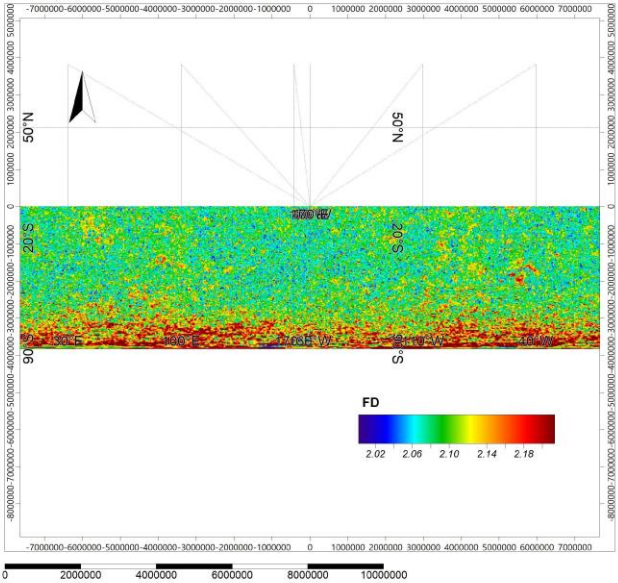

Fig. 4. Spatial distribution of slope fractal dimension (FD) within Mercury's Southern Hemisphere (SH)

Additional explanations are rooted in Mercury's geological evolution. As shown in Fig. 3, the highest FD values in the Northern Hemisphere are concentrated around deep impact structures in the polar regions. In the Southern Hemisphere, at comparable latitudes, FD variations are generally weaker and spatially more restricted. This hemispherical asymmetry likely reflects differences in volcanic resurfacing intensity. During Mercury's early history, extensive volcanic activity in equatorial regions resulted in partial infilling and flattening of pre-existing impact craters. Consequently, although large craters are present, their present-day depths are reduced. The Caloris Basin represents a prominent example, having likely been significantly deeper prior to volcanic resurfacing.

Volcanic processes also affected the polar regions, but to a lesser extent. As a result, a subtle hemispherical dichotomy is observed. The Northern Hemisphere exhibits sharper topographic contrasts and higher FD values compared to the Southern Hemisphere (Fig. 3 and Fig. 4). Craters in the north tend to be steeper, consistent with differences in impact preservation state. In contrast, craters near the south pole generally display gentler slopes, implying earlier cessation of volcanic and tectonic activity.

The results indicate that Mercury's two hemispheres experienced distinct geological histories and fractal characteristics of slope distribution in the Northern Hemisphere relative to the Southern Hemisphere. This provides a comparative framework for assessing hemispherical differences in topography complexity rather than direct constraints on absolute geological age.

Conclusion

This study demonstrates that the slope distribution of Mercury's surface exhibits a well-defined fractal nature, quantified using a variogram-based fractal dimension approach. The obtained fractal dimension values (mean FD = 2.127 for the Northern Hemisphere and 2.107 for the Southern Hemisphere), supported by very high coefficients of determination (mean $R^2 = 0.997$ for the Northern Hemisphere and 0.998 for the Southern Hemisphere), indicate that Mercury's slopes are organised in a scale-invariant manner. This spatial organisation reflects the cumulative influence of long-term geological processes acting on the planet's surface.

Comparative hemispherical analysis reveals a clear, though moderate, asymmetry. The Northern Hemisphere exhibits a slightly higher mean fractal dimension and greater spatial variability, characteristics that are spatially associated with large impact structures and regions that experienced limited volcanic smoothing, particularly at high latitudes. In contrast, the Southern Hemisphere displays more homogeneous fractal properties, suggesting earlier or more extensive surface modification.

The spatial patterns of fractal dimension further indicate that higher FD values correspond to areas with pronounced topographic gradients, such as deep basins and relatively well-preserved cratered terrains, whereas lower FD values dominate smoother plains affected by volcanic resurfacing. This confirms that slope-based fractal metrics are sensitive to variations in morphological complexity and the degree of surface modification.

The results support the interpretation that Mercury's hemispheres underwent distinct geological histories, marked by differences in impact preservation and volcanic resurfacing intensity. The variogram-based fractal analysis of Mercury's slope provides new opportunities for future comparative studies of Mercury in particular and other terrestrial planetary bodies more broadly.

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ФРАКТАЛЕН АНАЛИЗ НА МОРФОМЕТРИЯТА НА НАКЛОНИТЕ НА МЕРКУРИЙ НА БАЗАТА НА ВАРИОГРАМА

Р. Илиев, Б. Рангелов

Резюме

Изследването прилага вариограмен подход за определяне на фракталната димесия (ФД) на разпределението на наклоните на повърхността на Меркурий въз основа на глобалния ЦМР (DEM) от мисията MESSENGER. Резултатите показват ясно фрактално поведение с високи коефициенти на детерминираност ($R^2 > 0.99$). Средните стойности на ФД са 2.127 за Северното полукълбо и 2.107 за Южното, като Северното полукълбо демонстрира по-голяма пространствена хетерогенност и по-висока сложност, особено в райони с големи импактни басейни и ограничено вулканично изглаждане. Южното полукълбо показва по-хомогенни фрактални характеристики, което отразява разлики в интензивността на вулканично преоформяне и заравняване на релефа. Тези умерени, но статистически значими разлики подкрепят идеята за различна геоложка еволюция на двете полукълба на Меркурий и потвърждават полезността на вариограмния фрактален анализ за сравнителни планетарни изследвания.