

CORIMIA IONIZATION MODEL – 2+1 INTERVAL APPROXIMATION FOR COSMIC RAY NUCLEI ($Z > 1$)

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Abstract

This paper considers the ionization model CORIMIA (Cosmic Ray Ionization Model for Ionosphere and Atmosphere) in its two-interval 2+1 approximation with charge-decreasing interval of the ionization loss function. Increasing the number of approximation intervals is the next step in improving the model. The final stage will be the introduction of a 5-interval approximation of the Bohr-Bethe-Bloch formula with charge-decreasing interval. This version includes the introduction of integration limits in an arbitrary energy interval.

Introduction. Particles with Charge $Z > 1$

In [1], the case of cosmic ray protons $Z = 1$, which constitute the majority of their composition [2–4], was considered (see Table 1, below). Particles with charge $Z > 1$ can be divided into several groups according to their nuclei. This classification is possible because experimental studies (Dorman, 2004) [5]; (Toptigin, 1985) [6] have shown that the fluxes of cosmic rays composed of nuclei with similar charges are approximately equal. Table 1 presents the fluxes of the different nuclear groups in the composition of cosmic rays. Experimental results cited by Dorman [5] and Toptigin (in parentheses) [6] are shown.

Table 1: Groups of Nuclei in Cosmic Rays – The first column of the table presents the different nuclear groups found in cosmic rays. The second column shows the nuclear charge Z of the groups, and the third column provides experimental data on the flux ϕ ($\text{m}^{-2} \cdot \text{ster}^{-1} \cdot \text{s}^{-1}$) for energies $E > 2.5$ GeV/nuc. Data without brackets are from Dorman (2004), and those in brackets are from Toptigin (1985).

Table 1.

Nuclear group from CR	Charge Z	Flux ϕ ($\text{m}^{-2} \cdot \text{ster}^{-1} \cdot \text{s}^{-1}$) for energies $E > 2.5$ GeV/nuc
Protons p	1	1300 (1300)
Helium nuclei α	2	88 (94)
Light nuclei L	3–5	1.9 (2)
Medium nuclei M	6–9	5.6 (6.7)
Heavy nuclei H	10–19	2.5 (2)
Very heavy nuclei VH	≥ 20	0.7 (0.3)

Model approximation

In describing the model for calculating the rate of ion pair production due to the passage of charged cosmic ray particles with $Z > 1$, a double integral approximation of the ionization loss function is used. Equation (2) [1] applies in this case. However, besides the two characteristic energy limits of the integral, there is another interval related to the charge reduction of the ionizing particle: $[E_a; E_b][E_{-a}; E_{-b}]$, where the upper bound E_a is defined as [7, 8]:

$$(1) \quad E_a = 0.15 \times Z^2.$$

Here, E_a is the initial energy at which the particle starts to reduce its charge. E_{-b} is the lower bound of the charge reduction interval, when the particle has a charge $Z = 1$ and is described by:

$$(2) \quad E_b = 0.15.$$

From equations (1) and (2), the effective charge of the incoming particle is given by:

$$(3) \quad Z^* = \left(\frac{E}{0.15} \right)^2, \quad 0.15 \leq E \leq E_a.$$

Substituting this expression for the effective charge Z^* into the ionization loss function (equation (2)) [1], we obtain:

$$(4) \quad \frac{1}{\rho} \frac{dE}{dh} = 242 \times Z^2 \times E^{-3/4} = 242 \frac{E}{0.15} E^{-3/4} = 1613.3 E^{1/4}, \quad 0.15 \leq E \leq E_a.$$

The final form of equation (2) [1] becomes:

$$(5) \quad -\frac{1}{\rho} \frac{dE}{dh} = \begin{cases} 1613.3 E^{1/4}, & 0.15 \leq E \leq E_a, & \text{Interval 1}^* \\ 242 Z^2 E^{-3/4}, & E_a \leq E \leq 600 \text{ MeV}, & \text{Interval 1} \\ 2 Z^2, & 600 < E \leq 5 \times 10^6 \text{ MeV}, & \text{Interval 2.} \end{cases}$$

where Interval 1* represents the region of charge reduction of an incoming charged particle in the Earth's atmosphere. The reason for this charge reduction is detailed in Dorman (2004) [5] and is related to the process of electron attachment.

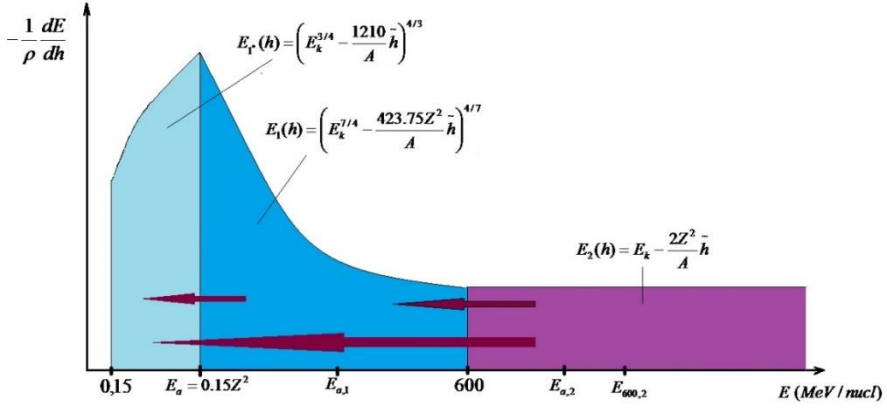


Fig. 1. Double-integral (2+1) approximation of the ionization loss function for charged cosmic ray particles with charge $Z > 1$

The figure shows the segmentation of energy losses into different intervals $E1^*(h)$, $E1(h)$, $E2(h)$, as well as the segment boundaries at the respective energies when the particle crosses between intervals: $E11(h)$, $E21(h)$, $E31(h)$.

The first step in model development is to determine the energy losses functions for the different intervals in equation (5). As in the case of particles with charge $Z = 1$, we will use the quantity of transferred matter $h\sim$.

The amount of transferred matter $h\sim$ in Interval 1* can be expressed as [7]:

$$(6) \quad \tilde{h} = \tilde{h}_{1^*} = \frac{1}{1613.3} \int_{E(h)}^{E_k} \frac{AdE}{E^{1/4}} = \frac{A}{1613.3} \frac{4}{3} \left(E_k^{3/4} - E^{3/4}(h) \right),$$

where A is the atomic mass of the particle with charge Z . From equation* (6), the energy losses law in Interval 1* can be written as:

$$(7) \quad E_{1^*}(h) = \left(E_k^{3/4} - \frac{1210}{A} \tilde{h} \right)^{4/3}.$$

For the transferred matter in Interval 1, we have:

$$(8) \quad \tilde{h} = \tilde{h}_1 = \frac{1}{242} \int_{E(h)}^{E_k} \frac{AdE}{Z^2 E^{-3/4}} = \frac{A}{Z^2 242} \frac{4}{7} \left(E_k^{7/4} - E_1^{7/4}(h) \right).$$

From expression (8), we can express the energy loss law in Interval 1 as:

$$(9) \quad E_1(h) = \left(E_k^{7/4} - \frac{423.75Z^2}{A} \tilde{h} \right)^{4/7}.$$

In a similar way, we can derive the energy losses law in the final interval – Interval 2. The amount of transferred matter $h_{\sim}$ is given by:

$$(10) \quad \tilde{h} = \tilde{h}_2 = \frac{A}{2Z^2} \int_{E(h)}^{E_k} dE = \frac{A}{2Z^2} (E_k - E_2(h)).$$

The energy losses law in Interval 2 is:

$$(11) \quad E_2(h) = E_k - \frac{2Z^2}{A} \tilde{h}.$$

The next important step is to derive analytical expressions for the atmospheric cut-off thresholds $E_{Aj}E_{\{Aj\}}E_{Aj}$ in the different intervals. The transferred matter for Interval 1* can be expressed as:

$$(12) \quad \tilde{h} = \tilde{h}_{1^*} = \frac{A}{1613.3} \int_{E(h)=0.15}^{E_{A1^*}(h)} \frac{dE}{E^{1/4}} = \frac{A}{1613.3} \frac{4}{3} \left(E_{A1^*}^{3/4}(h) - 0.15^{3/4} \right).$$

The atmospheric cut-off for Interval 1* is then defined by:

$$(13) \quad E_{A1^*}(h) = \left(0.15^{3/4} + \frac{1210}{A} \tilde{h} \right)^{4/3} \leq E_a.$$

Similarly, the remaining two cut-off thresholds can be expressed. The transferred matter in Interval 1 is:

$$(14) \quad \tilde{h} = \tilde{h}_{1^*} + \tilde{h}_1 = \frac{A}{1613.3} \int_{0.15}^{E_a} \frac{dE}{E^{1/4}} + \frac{A}{242Z^2} \int_{E_a}^{E_{A1}} \frac{dE}{E^{-3/4}} = \frac{A}{1210} \left(E_a^{3/4} - 0.15^{3/4} \right) + \frac{A}{423.75Z^2} \left(E_{A1}^{7/4} - E_a^{7/4} \right).$$

From equation (14), the cut-off threshold for Interval 1 can be expressed as:

$$(15) \quad E_{A1}(h) = \left(423.74 \frac{Z^2}{A} \tilde{h} - 0.35Z^2 \left(E_a^{3/4} - 0.15^{3/4} \right) + E_a^{7/4} \right)^{4/7}.$$

The transferred matter for the final Interval 2 is the sum of the transferred matter across all intervals, and is given by:

$$(16) \quad \tilde{h} = \tilde{h}_{1^*} + \tilde{h}_1 + \tilde{h}_2 = \frac{A}{1613.3} \int_{0.15}^{E_a} \frac{dE}{E^{1/4}} + \frac{A}{242Z^2} \int_{E_{A2}(h)}^{600} \frac{dE}{E^{-3/4}} + \frac{A}{2Z^2} \int_{600}^{E_{A2}(h)} dE = \frac{A}{1210} \left(E_a^{3/4} - 0.15^{3/4} \right) + \frac{A}{423.75Z^2} \left(600^{7/4} - E_a^{7/4} \right) + \frac{A}{2Z^2} (E_{A2}(h) - 600).$$

The final expression for the atmospheric cut-off for Interval 2 is:

$$(17) \quad E_{A2}(h) = 2 \frac{Z^2}{A} \tilde{h} - 0.002Z^2 \left(E_a^{\frac{3}{4}} - 0.15^{\frac{3}{4}} \right) - 0.005 \left(600^{\frac{7}{4}} - E_a^{\frac{7}{4}} \right) + 600$$

The effective reflection threshold is determined taking into account the increase in the geomagnetic cut-off ER and atmospheric cut-off EA .

Upon entering the atmosphere, a cosmic ray particle with charge Z begins to lose energy according to the energy loss laws for the different intervals in equation (5). In other words, its kinetic energy E_k changes based on the loss mechanisms and charge reduction processes, depending on the interaction it undergoes.

When the particle crosses a boundary between energy intervals, it is necessary to derive specific energy loss laws for each case. Using double integral approximations of the Bohr-Bethe-Bloch ionization loss function, and accounting for the charge reduction interval, the following boundary conditions are possible [7–9]:

- $E_{1*1}(h)$ – when the particle crosses the boundary between Interval 1 and Interval 1* (at energy E_a);
- $E_{21}(h)$ – when the particle crosses the boundary between Interval 2 and Interval 1 (at 600 MeV);
- $E_{31}(h)$ – when the particle crosses both the boundary between Interval 2 and Interval 1 (600 MeV), and the boundary between Interval 1 and Interval 1* (at E_a).

To, derive the energy loss laws at the interval boundaries the transferred matter h is used.

If the energy of the incoming particle crosses the boundary E_a , we have:

$$(18) \quad \tilde{h} = \tilde{h}_{1*} + \tilde{h}_1 = \frac{A}{1613,3} \int_{E_{11*}(h)}^{E_a} \frac{dE}{E^{1/4}} + \frac{A}{242Z^2} \int_{E_a}^{E_k} \frac{dE}{E^{-3/4}} = \frac{A}{1210} \left(E_a^{3/4} - E_{11*}^{3/4} \right) + \frac{A}{423,75Z^2} \left(E_k^{7/4} - E_a^{7/4} \right).$$

The energy loss law in this case is:

$$(19) \quad E_{11*}(h) = \left(\frac{2,85}{2Z^2} - \left(E_k^{7/4} - E_a^{7/4} \right) - \frac{1210}{A} \tilde{h} + E_a^{3/4} \right)$$

When the incoming particle's energy crosses the boundary at 600 MeV, the transferred matter h can be expressed as:

$$(20) \quad \tilde{h} = \tilde{h}_1 + \tilde{h}_2 = \frac{A}{242Z^2} \int_{E_{21}(h)}^{600} \frac{dE}{E^{-3/4}} + \frac{A}{2Z^2} \int_{600}^{E_k} dE = \frac{A}{423,75Z^2} \left(600^{7/4} - E_{21}^{7/4} \right) + \frac{A}{2Z^2} (E_k - 600).$$

From equation (20), we can derive the energy loss law when the incoming particle's energy crosses the 600 MeV boundary:

$$(21) \quad E_{21}(h) = \left(211,88(E_k - 600) - \frac{423,75Z^2}{A} \tilde{h} + 600^{7/4} \right)^{4/7}$$

If the kinetic energy E_k of the incoming particle crosses both boundaries – at 600 MeV and at $E_a = 0.15 \times Z^2$ – then the total transferred matter is:

$$(22) \quad \tilde{h} = \tilde{h}_{1^*} + \tilde{h}_1 + \tilde{h}_2 = \frac{A}{1613,3} \int_{E_{21^*}(h)}^{E_a} \frac{dE}{E^{\frac{1}{4}}} + \frac{A}{242Z^2} \int_{E_a}^{600} \frac{dE}{E^{-\frac{3}{4}}} + \frac{A}{2Z^2} \int_{600}^{E_k} dE = \frac{A}{1210} (E_a^{3/4} - E_{21^*}^{3/4}) + \frac{A}{423,75Z^2} (600^{7/4} - E_a^{7/4}) + \frac{A}{2Z^2} (E_k - 600).$$

The final energy loss law when both thresholds (600 MeV and E_a) are crossed becomes:

$$(23) \quad E_{21^*}(h) = \left(E_a^{\frac{3}{4}} + \frac{2,85}{2Z^2} \left(600^{\frac{7}{4}} - E_a^{\frac{7}{4}} \right) + \frac{605}{Z^2} (E_k - 600) - \frac{1210}{A} \tilde{h} \right)^{4/3}$$

Based on the derived energy loss laws for each interval (equations (7, 9, 11) 4.33, 4.35, 4.37) and for cases when boundaries between intervals are crossed (equations (19, 21, 23) 4.45, 4.47, 4.49), we can now express the ionization loss function from equation (5, 4.31) as follows:

For Interval 1*:

$$(24) \quad \frac{1}{\rho} \left(\frac{dE}{dh} \right)_{1^*} = 1613,3 E_{1^*}^{1/4} = 1613,3 \left(E_k^{3/4} - \frac{1210}{A} \tilde{h} \right)^{1/3}$$

$$(25) \quad \frac{1}{\rho} \left(\frac{dE}{dh} \right)_1 = 242Z^2 E_1^{-3/4} = 242Z^2 \left(E_k^{7/4} - \frac{423,75Z^2}{A} \tilde{h} \right)^{-3/7}$$

$$(26) \quad \frac{1}{\rho} \left(\frac{dE}{dh} \right)_2 = \frac{2Z^2}{A}$$

$$(27) \quad \frac{1}{\rho} \left(\frac{dE}{dh} \right)_{11^*} = 1613,3 E_{11^*}^{1/4} = 1613,3 \left(\frac{2,85}{2Z^2} (E_k^{7/4} - E_a^{7/4}) - \frac{1210}{A} \tilde{h} + E_a^{3/4} \right)^{1/3}$$

$$(28) \quad \frac{1}{\rho} \left(\frac{dE}{dh} \right)_{21} = 242Z^2 E_{21}^{-3/4} = 242Z^2 \left(211,88(E_k - 600) - \frac{423,75Z^2}{A} \tilde{h} + 600^{7/4} \right)^{-3/7}$$

$$(29) \quad \frac{1}{\rho} \left(\frac{dE}{dh} \right)_{21^*} = 1613,3 E_{21^*}^{1/4} = 1613,3 \left(\frac{2,85}{2Z^2} (600^{7/4} - E_a^{7/4}) - \frac{605}{Z^2} (E_k - 600) - \frac{1210}{A} \tilde{h} + E_a^{3/4} \right)^{1/3}$$

Based on the previous considerations, we can express the explicit form of the model for calculating the electron production rate $q(h)q(h)q(h)$ – the general formula [4]. When determining the form of the general formula, it is necessary to account for different possible cases depending on the energy of the effective reflection threshold, as well as on the differences in atmospheric cut-off energies for the separate intervals.

Case 1.

The energy of the effective reflection threshold is less than the upper boundary of Interval 1*:

$$0.15 < EA1 \leq E_{eff} \leq Ea < E_{\infty} < 600 \text{ MeV.}$$

In this case, the general formula (4) becomes:

(30)

$$\begin{aligned} q(h) = & \frac{\rho(h)}{Q} \left(\int_{E_{eff}}^{E_a} D(E) \left(\frac{dE}{dh} \right)_{1*} dE_k + \int_{E_a}^{E_{a\infty}} D(E) \left(\frac{dE}{dh} \right)_{11*} dE_k + \int_{E_{a\infty}}^{600} D(E) \left(\frac{dE}{dh} \right)_1 dE_k \right. \\ & \left. + \int_{600}^{E_{600}} D(E) \left(\frac{dE}{dh} \right)_{21} dE_k + \int_{E_{600}}^{\infty} D(E) \left(\frac{dE}{dh} \right)_2 dE_k \right) \\ = & \frac{1613,3\rho(h)}{Q} \int_{E_{eff}}^{E_a} D(E) \left(E_k^{3/4} - \frac{1210}{A} \tilde{h} \right)^{1/3} dE_k \\ & + \frac{1613,3\rho(h)}{Q} \int_{E_a}^{E_{a\infty}} D(E) \left(\frac{2.85}{Z^2} (E_k^{7/4} - E_a^{7/4}) - \frac{1210}{A} \tilde{h} \right. \\ & \left. + E_a^{3/4} \right)^{1/3} dE_k \\ & + \frac{242Z^2\rho(h)}{Q} \int_{E_{a\infty}}^{600} D(E) \left(E_k^{7/4} - \frac{423,75Z^2}{A} \tilde{h} \right)^{-3/7} dE_k \\ & + \frac{242Z^2\rho(h)}{Q} \int_{600}^{E_{600}} D(E) \left(211,88(E_k - 600) - \frac{423,75Z^2}{A} \tilde{h} \right. \\ & \left. + 600^{7/4} \right)^{-3/7} dE_k + \frac{2Z^2\rho(h)}{Q} \int_{E_{600}}^{\infty} D(E) dE_k \end{aligned}$$

Where:

- E_k is the energy at infinity (initial kinetic energy,
- E_a is the energy threshold defined by the energy loss law for Interval 1:

$$(31) \quad E_{a\infty}(h) = \left(E_a^{7/4} + \frac{423,75Z^2}{A} \tilde{h} \right)$$

E_{600} is the energy at infinity at the 600 MeV boundary, which follows from the energy loss law for Interval 2 and is given in the expression [4, 7–11]:

$$(32) \quad E_{600} = 600 + \frac{2Z^2}{A} \tilde{h}$$

Case 2a.

The energy of the effective reflection threshold is greater than the upper boundary of Interval 1*, but less than the 600 MeV boundary. The energy at infinity (initial energy) is less than 600 MeV:

$$E_a < E_{eff} < E_{\infty} < 600 < E_{600} \text{ MeV.}$$

Then the general equation [4] takes the form:

$$\begin{aligned}
 (33) \quad q(h) &= \frac{\rho(h)}{Q} \left(\int_{E_{eff}}^{E_{a\infty}} D(E) \left(\frac{dE}{dh} \right)_{11^*} dE_k + \int_{E_{a\infty}}^{600} D(E) \left(\frac{dE}{dh} \right)_1 dE_k + \right. \\
 &+ \int_{600}^{E_{600}} D(E) \left(\frac{dE}{dh} \right)_{21} dE_k + \int_{E_{600}}^{\infty} D(E) \left(\frac{dE}{dh} \right)_2 dE_k = \\
 &= \frac{1613.3\rho(h)}{Q} \int_{E_{eff}}^{E_{a\infty}} D(E) \left(\frac{2.85}{Z^2} (E_k^{7/4} - E_a^{7/4}) - \frac{1210}{A} \tilde{h} + E_a^{3/4} \right)^{1/3} dE_k + \\
 &+ \frac{242Z^2\rho(h)}{Q} \int_{E_{a\infty}}^{600} D(E) \left(E_k^{7/4} - \frac{423.75Z^2}{A} \tilde{h} \right)^{-3/7} dE_k + \\
 &+ \frac{242Z^2\rho(h)}{Q} \int_{600}^{E_{600}} D(E) \left(211.88(E_k - 600) - \frac{423.75Z^2}{A} \tilde{h} + 600^{7/4} \right)^{-3/7} dE_k + \\
 &+ \frac{2Z^2\rho(h)}{Q} \int_{E_{600}}^{\infty} D(E) dE_k.
 \end{aligned}$$

Case 2b.

The energy of the effective reflection threshold is greater than the upper boundary of Interval 1*, but less than the 600 MeV boundary. The energy at infinity (initial energy) is greater than 600 MeV:

$$E_a < E_{eff} < 600 < E_{a\infty} < E_{600} \text{ MeV.}$$

Then the general equation [4] becomes:

$$\begin{aligned}
 (34) \quad q(h) &= \frac{\rho(h)}{Q} \left(\int_{E_{eff}}^{600} D(E) \left(\frac{dE}{dh} \right)_1 dE_k + \int_{600}^{E_{a\infty}^*} D(E) \left(\frac{dE}{dh} \right)_{21^*} dE_k \right. \\
 &+ \int_{E_{a\infty}^*}^{E_{600}} D(E) \left(\frac{dE}{dh} \right)_{21} dE_k + \int_{E_{600}}^{\infty} D(E) \left(\frac{dE}{dh} \right)_2 dE_k = \\
 &= \frac{1613.3\rho(h)}{Q} \int_{E_{eff}}^{600} D(E) \left(E_k^{7/4} - \frac{423.75Z^2}{A} \tilde{h} \right)^{1/3} dE_k + \\
 &+ \frac{1613.3\rho(h)}{Q} \int_{600}^{E_{a\infty}^*} D(E) \left(\frac{2.85}{Z^2} (600^{7/4} - E_a^{7/4}) - \right. \\
 &- \frac{605}{Z^2} (E_k - 600) - \frac{1210}{A} \tilde{h} + E_a^{3/4} \left. \right)^{1/3} dE_k + \\
 &+ \frac{242Z^2\rho(h)}{Q} \int_{E_{a\infty}^*}^{E_{600}} D(E) \left(211.88(E_k - 600) - \frac{423.75Z^2}{A} \tilde{h} + 600^{7/4} \right)^{-3/7} dE_k + \\
 &+ \frac{2Z^2\rho(h)}{Q} \int_{E_{600}}^{\infty} D(E) dE_k
 \end{aligned}$$

$E_{a\infty}^*$ is expressed by the energy loss law of a particle crossing the boundaries at 600 MeV and E_a , and is given by [7–9]:

$$(35) \quad E_{a\infty}^* = \frac{1}{212} \left(127200 - 600^{7/4} + E_a^{7/4} + 423.75\tilde{h} \right).$$

Case 3.

The energy of the effective reflection threshold is greater than the 600 MeV boundary but less than $E_{a\infty}^*$: $600 < E_{\text{eff}} < E_{a\infty}^* < E_{600}$ MeV. In this case, we have:

$$(36) \quad \begin{aligned} q(h) = & \frac{\rho(h)}{Q} \left(\int_{E_{\text{eff}}}^{E_{a\infty}^*} D(E) \left(\frac{dE}{dh} \right)_{21^*} dE_k + \int_{E_{a\infty}^*}^{E_{600}} D(E) \left(\frac{dE}{dh} \right)_{21} dE_k + \right. \\ & + \int_{E_{600}}^{\infty} D(E) \left(\frac{dE}{dh} \right)_2 dE_k = \\ & = \frac{1613.3 \rho(h)}{Q} \int_{E_{\text{eff}}}^{E_{a\infty}^*} D(E) \left(\frac{2.85}{Z^2} (600^{7/4} - E_a^{7/4}) - \right. \\ & - \frac{605}{Z^2} (E_k - 600) - \frac{1200}{A} \tilde{h} + E_a^{3/4} \Big)^{1/3} dE_k + \\ & + \frac{242Z^2 \rho(h)}{Q} \int_{E_{a\infty}^*}^{E_{600}} D(E) \left(211.88(E_k - 600) - \frac{423.75Z^2}{A} \tilde{h} + 600^{7/4} \right)^{-3/7} dE_k + \\ & + \frac{2Z^2 \rho(h)}{Q} \int_{E_{600}}^{\infty} D(E) dE_k \end{aligned}$$

Case 4.

The energy of the effective reflection threshold is greater than the 600 MeV boundary as well as $E_{a\infty}^*$ and E_{600} : $600 < E_{a\infty}^* < E_{600} < E_{\text{eff}} < \infty$.

The rate of electron production due to the passage of charged cosmic rays is:

$$(37) \quad q(h) = \frac{\rho(h)}{Q} \int_{E_{\text{eff}}}^{\infty} D(E) \left(\frac{dE}{dh} \right)_2 dE_k = \frac{2Z^2 \rho(h)}{Q} \int_{E_{\text{eff}}}^{\infty} D(E) dE_k$$

With these four cases, the model for calculating the rate of free electron production due to the passage of charged cosmic rays is completed, using the double-integral approximation of the ionization loss function based on the Bohr-Bethe-Bloch formula. Fig. 1 illustrates the double-integral approximations of the ionization loss function in the case of ionizing particles with charge $Z > 1$.

Conclusion

The results obtained have been applied to the calculation of ionization profiles in the ionosphere, ionization and electrical parameters in the atmosphere of the Earth, planets and their satellites [12–14], in the propagation of radio waves [4], in various geophysical events [15–21]. There are possible connections between ionization by cosmic rays in the atmosphere and some climatic parameters.

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МОДЕЛЬ ИОНИЗАЦИИ CORIMIA – 2+1 ИНТЕРВАЛЬНОЕ ПРИБЛИЖЕНИЕ ДЛЯ ЯДЕР КОСМИЧЕСКИХ ЛУЧЕЙ ($Z > 1$)

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Аннотация

В данной статье рассматривается модель ионизации CORIMIA (Cosmic Ray Ionization Model for Ionosphere and Atmosphere) в ее двухинтервальном 2+1 приближении с интервалом убывания заряда функции ионизационных потерь. Увеличение числа интервалов аппроксимации является следующим шагом в совершенствовании модели. Заключительным этапом станет введение 5-интервального приближения формулы Бора-Бете-Блоха с интервалом убывания заряда. Эта версия включает введение пределов интегрирования в произвольном интервале энергии.