Bulgarian Academy of Sciences. Space Research and Technology Institute.
Aerospace Research in Bulgaria. 37, 2025, Sofia

CORIMIA IONIZATION MODEL – A TWO-INTERVAL APPROXIMATION OF IONIZATION LOSS FUNCTION

Peter Velinov, Simeon Asenovski

Space Research and Technology Institute – Bulgarian Academy of Sciences
e-mail: PeterIYVelinov@gmail.com, asenovski@gmail.com

Keywords: *Cosmic rays (CRs), Ionization, Ionosphere, Atmosphere*

Abstract

This article examines the ionization model CORIMIA (COsmic Ray Ionization Model for Ionosphere and Atmosphere) in its two-interval approximation of the ionization loss function.

Increasing the number of approximation intervals is the next step in improving the model. The last stage was the introduction of a 5-interval approximation of the Bohr-Bethe-Bloch formula with an intermediate interval. This version includes the introduction of integration limits in an arbitrary energy interval.

Introduction

For a better description and approximation of the experimental data, a discretization of the Bohr-Bethe-Bloch function is required, the function must be divided into separate energy intervals [1–6]. We will start with the case of approximation for two characteristic energy intervals of the ionization loss function (dE/dh) for: protons - p, α (alpha) particles (which represent helium cores), light cores (L) - $3 \leq Z \leq 5$, medium cores (M) - $6 \leq Z \leq 9$, heavy cores (H) - $10 \leq Z \leq 19$ and very heavy cores (VH) - $Z \leq 20$. The calculations were made for a stream of cosmic rays entering the Earth's atmosphere vertically (the angle of the cosmic ray velocity vector and the normal to the Earth's surface is zero degrees).

Introducing an intermediate interval in the two-interval approximation of dE/dh significantly increases the accuracy of numerical calculations of q(h) [1, 2]. Increasing the number of approximation intervals is the next step in improving the model.

The last stage was the introduction of a 5-interval approximation of the formula of Bohr-Bethe-Bloch with an intermediate interval [7-9]. This version includes the introduction of integration limits in a random energy interval.

Model approximation

The simplest approximation of the Bohr-Bethe-Bloch formula is as follows [1, 2]:

(1) $$-\frac{1}{\rho}\frac{dE}{dh} = \begin{cases} 242E^{-3/4} & 0.15 \le E \le 600 \text{ MeV} \quad \text{Интервал 1} \\ 2 & 600 < E \le 5 \times 10^6 \text{ MeV} \quad \text{Интервал 2,} \end{cases}$$

where E (MeV) is the kinetic energy of the incoming charged particle, h (g.cm^(-2)) is the thickness of the substance (in this case the Earth's atmosphere), ρ is the density of the substance.

Figure 1 shows a diagram of the two-interval approximation of the ionization loss function for ionizing particles with charge Z = 1.

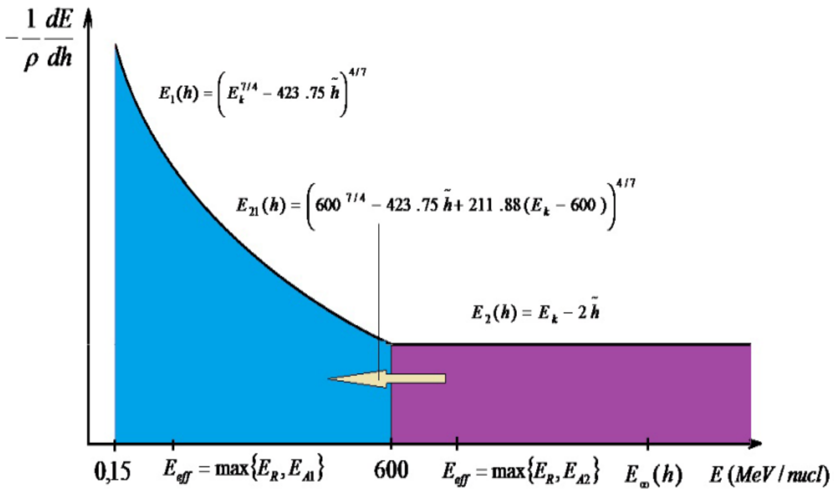

Fig. 1. Two-interval approximation of the ionization loss function for protons (Z = 1). The figure presents the energy loss laws for the individual intervals (E1(h) and E2(h)) as well as the energy loss law at the boundary between the two intervals E21(h).

For particles with charge Z>1, the ionization losses function takes the following form:

(2) $$-\frac{1}{\rho}\frac{dE}{dh} = \begin{cases} 242Z^2E^{-3/4} & 0.15 \le E \le 600 \text{ MeV} \quad \text{Интервал 1} \\ 2Z^2 & 600 < E \le 5 \times 10^6 \text{ MeV} \quad \text{Интервал 2,} \end{cases}$$

(3) $$\tilde{h} = \int \rho(h)dh,$$

(4) $$\tilde{h} = L_j(E_k) - L_j\left(E_{k_{min}}\right) = \int_{E_{k_{min}}}^{E_k} \frac{dE_k}{\frac{1}{\rho}\frac{dE_k}{dh}},$$

(5) $$\tilde{h} = \frac{1}{242}\int_{E(h)}^{E} \frac{dE}{E^{-3/4}} = \left(\frac{4}{7\times 242}\right)\left(E^{7/4} - E(h)^{7/4}\right) = \frac{1}{423.75}\left(E^{7/4} - E(h)^{7/4}\right).$$

Model equations

The law of energy losses in the first interval takes the form:

(6) $$E_1(h) = \left(E_k^{7/4} - 423.75\tilde{h}\right)^{4/7},$$

where $E_1(h)$ is the kinetic energy of the incoming proton at altitude h for the first energy interval, and E_k is the initial kinetic energy. In the case of the two-interval approximation, it is assumed that when the energy of the incoming particle reaches E(h)=0.15 MeV, it is absorbed in the atmosphere.

From the above equation, we can easily calculate the amount of matter $\tilde{h}$ passed by a particle with initial energy E=600 MeV:

(7) $$\tilde{h} = \frac{1}{423.75}\left(600^{7/4} - 0.15^{7/4}\right) = 171.65 \text{ g.cm}^{-2}.$$

Depending on the specific atmospheric model (CIRA, US Standard Atmosphere, etc.) the calculated value of the atmospheric thickness in equation (7) corresponds to 12 - 12.5 km - altitude.

In a similar way, we can find the energy loss law of the second characteristic energy interval. For the amount of substance passed $\tilde{h}$ we have:

(8) $$\tilde{h} = \frac{1}{2}\int_{E(h)}^{E} dE = \frac{1}{2}\left(E - E(h)\right).$$

From this equation, we easily obtain the law for energy losses in the second interval:

(9) $$E_2(h) = E_k - 2\tilde{h}.$$

The next step required for the development of the model is the determination of the atmospheric cutoff thresholds E_{Aj}, (j is the number of the corresponding interval) for the different characteristic energy intervals. The atmospheric cutoff threshold E_A (MeV) represents the minimum energy that a particle must have in order to enter the atmosphere without being "reflected". To determine E_A, we again use the amount of matter passed $\tilde{h}$:

(10) $$\tilde{h} = \tilde{h}_1 = \frac{1}{242}\int_{E(h)=0.15}^{E_{A1}(h)} \frac{dE}{E^{-3/4}} = \frac{1}{423.75}\left(E_{A1}(h)^{7/4} - 0.15^{7/4}\right).$$

From equation (9), we can express the atmospheric cutoff threshold for the first interval:

(11) $$E_{A1}(h) = \left(423.75\tilde{h} + 0.15^{7/4}\right)^{4/7} \leq 600.$$

For the second interval, we have:

(12) $$\tilde{h} = \tilde{h}_1 + \tilde{h}_2 = \frac{1}{242}\int_{0.15}^{600} \frac{dE}{E^{-3/4}} + \frac{1}{2}\int_{600}^{E_{A2}(h)} dE = \frac{1}{423.75}\left(600^{\frac{7}{4}} - 0.15^{\frac{7}{4}}\right) + \frac{1}{2}\left(E_{A2}(h) - 600\right)$$

From the expression for the amount of substance passed (12), we can express the atmospheric cutoff threshold for the second interval:

(13) $$E_{A2}(h) = \frac{127200 - 600^{7/4} + 0.15^{7/4} + 423.75\tilde{h}}{211.88} \approx 257 + 2\tilde{h} \geq 600.$$

In addition to E_A, the geomagnetic cutoff threshold E_R is also of essential importance. The effective cutoff threshold is determined by the expression:

(14) $$E_{eff} = max\{E_R, E_A\} = max\{\left(\sqrt{R^2 + 0.88} - 0.938\right), E_A\},$$

where R is the geomagnetic hardness of the geomagnetic field. After introducing the effective geomagnetic cutoff threshold E_{eff}, the first energy interval of equations (1,2) is in the range: [E_{eff}÷600 MeV] (for the case when E_{eff}>0.15).

An important step in the development of the model for calculating the rate of electron formation due to the passage of charged cosmic particles through the

Earth's atmosphere is the determination of the law of energy losses at the boundaries between the characteristic energy intervals. This is necessary due to the fact that it is possible for the energy of the incoming particle to pass from one interval to another. In the case of a two-interval approximation, it is necessary to find the law of energy losses for the passage of kinetic energy from Interval 2 to Interval 1. For this purpose, the amount of matter passed h̃ is used:

$$\tilde{h} = \tilde{h}_1 + \tilde{h}_2 = \frac{1}{242}\int_{E_{21}}^{600} \frac{dE}{E^{-\frac{3}{4}}} + \frac{1}{2}\int_{600}^{E_k} dE = \frac{1}{423.75}\left(600^{\frac{7}{4}} - E_{21}^{\frac{7}{4}}\right) + \frac{1}{2}(E_k - 600) \tag{15}$$

where Ek is the initial kinetic energy of the entering particle, such that it passes from Interval 2 to Interval 1. From equation (15), we can express the law of energy losses at 6he boundaries between Interval 1 and Interval 2:

$$423.75\tilde{h} = 600^{7/4} - E_{21}^{7/4} + 211.88(E_k - 600) \Rightarrow$$
$$E_{21} = \left(600^{7/4} - 423.75\tilde{h} + 211.88(E_k - 600)\right)^{4/7}. \tag{16}$$

The initial energy E_∞ (h) of particles that reach kinetic energy E=600 in Interval 2 can be expressed as:

$$E_\infty(h) = 600 + 2\tilde{h}. \tag{17}$$

From the thus obtained laws for energy losses on the two intervals, as well as the law for energy losses on the boundary between Interval 1 and Interval 2, we can respectively express the explicit form of the ionization loss function in the case of a two-interval approximation for a unit charge of the incoming particle (proton):

$$\frac{1}{\rho}\left(\frac{dE}{dh}\right)_1 = 242E(h)^{-\frac{3}{4}} = 242E_1(h)^{-\frac{3}{4}} = 242\left(E_k^{\frac{7}{4}} - 423.75\tilde{h}\right)^{-\frac{3}{7}}. \tag{18}$$

(19) $$\frac{1}{\rho}\left(\frac{dE}{dh}\right)_{21} = 242E(h)^{-\frac{3}{4}} = 242E_{21}(h)^{-\frac{3}{4}} =$$

$$242(600^{\frac{7}{4}} - 423.75\tilde{h} + 211.88(E_k - 600))^{-3/7}$$

(20) $$\frac{1}{\rho}\left(\frac{dE}{dh}\right)_{2} = \frac{1}{\rho}\left(\frac{dE}{dh}E_2(h)\right) = 2.$$

To calculate the rate of formation of free electrons due to the passage of a stream of charged cosmic rays, a three-dimensional model for electron production is used:

(21) $$q(h) = \sum_i q_i(h) = \frac{1}{Q}\sum_i \int_{E_i}^{\infty}\int_{A=0}^{2\pi}\int_{\theta=0}^{\pi/2+\Delta\theta} D_i(E)\left(\frac{dE}{dh}\right)_i \sin\theta\, d\theta\, dA\, dE,$$

where q(h) is the rate of free electron formation, Q=35eV is the energy required to form one electron-ion pair, E_i are the effective cutoff thresholds calculated from equation (14), A is the azimuthal angle, θ is the angle to the vertical, dE/dh are the ionization loss functions for the different intervals calculated from equation (17,18). D_i (E) are the spectra of the incoming cosmic rays (GCR, SCR, and ACR). The spectrum of galactic cosmic rays (GCR) D(E) has the form:

(22) $$D(E) = K(0.939 + E_k)^{-\gamma}\left(1 + \frac{\alpha}{E_k}\right)^{-\beta},$$

where K, α, β, and γ are parameters of the GCR spectrum that must be determined for the different groups of nuclei (p, He, L, M, H, VH). The first term of equation (22) K(0.938+E_k)^(-γ) describes the primary spectrum of the GCR. The second term (1+α/E_k)^(-β) describes the modulation by the solar wind. Depending on the different phases of solar activity (minimum, average, maximum, etc.), the parameters of the second term change.

Using the calculations made so far, as well as the considerations presented, we can express the explicit form of the model for calculating the rate of free electron formation due to the passage of cosmic rays (equation (21) in the case of a two-interval approximation of the ionization loss function. Depending on the size of the effective cutoff threshold (equation (14), there are several cases of representation of the model.

Case 1. The energy of the cutoff threshold is less than the upper limit of the first interval: 0.15<Eeff<600. In such a case, equation (21) takes the form:

(23)

$$q(h) = \frac{\rho(h)}{Q}\left(\int_{E_{eff}}^{600} D(E)\left(\frac{dE}{dh}\right)_1 dE_k + \int_{600}^{E_\infty(h)} D(E)\left(\frac{dE}{dh}\right)_{21} dE_k + \int_{E_\infty(h)}^{\infty} D(E)\left(\frac{dE}{dh}\right)_2 dE_k\right)$$

$$= \frac{242\rho(h)}{Q}\int_{E_{eff}}^{600} D(E)(E_k^{\frac{7}{4}} - 423.75\tilde{h})^{-3/7} dE_k + \int_{600}^{E_\infty(h)} D(E)(600^{\frac{7}{4}} - 423.75\tilde{h} + 211.88(E_k - 600))^{-3/7} dE_k + \frac{2\rho(h)}{Q}\int_{E_\infty(h)}^{\infty} D(E) dE_k$$

Case 2. The energy at the cutoff threshold is greater than the upper limit of the first interval, but is less than the initial energy E∞ (h) of particles that reach kinetic energy E=600 in Interval 2: 600<Eeff<E∞ (h). Thus, for equation (21), we have:

$$q(h) = \frac{\rho(h)}{Q}\left(\int_{E_{eff}}^{E_\infty(h)} D(E)\left(\frac{dE}{dh}\right)_{21} dE_k + \int_{E_\infty(h)}^{\infty} D(E)\left(\frac{dE}{dh}\right)_2 dE_k\right) = \int_{E_{eff}}^{E_\infty(h)} D(E)(600^{\frac{7}{4}} - 423.75\tilde{h} + 211.88(E_k - 600))^{-3/7} dE_k + \frac{2\rho(h)}{Q}\int_{E_\infty(h)}^{\infty} D(E) dE_k \qquad (24)$$

Case 3. The energy at the cutoff threshold is greater than the initial energy E∞ (h) of particles that reach kinetic energy E=600 in Interval 2: E∞ (h)<Eeff<∞. Equation (21) takes the form:

$$q(h) = \frac{\rho(h)}{Q}\int_{E_{eff}}^{\infty} D(E)\left(\frac{dE}{dh}\right)_2 dE_k = \frac{2\rho(h)}{Q}\int_{E_{eff}}^{\infty} D(E) dE_k. \qquad (25)$$

The three cases introduced complete the model description for calculating electron production when protons enter the Earth's atmosphere.

The three cases introduced here complete the description of the model for calculating electron production upon proton (Z = 1) entry into the Earth's atmosphere.

Discussion

The results obtained have been applied to the calculation of electrical parameters in the atmosphere [10], in the propagation of radio waves [1, 2, 11], in various geophysical events [12–16], in the calculation of ionization profiles of other planets and satellites of planets [17, 18]. There are possible connections between ionization by cosmic rays in the atmosphere and some climatic parameters [15, 19, 20].

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ЙОНИЗАЦИОННИЯТ МОДЕЛ CORSIMA – ДВУ-ИНТЕРВАЛНА АПРОКСИМАЦИЯ НА ФУНКЦИЯТА НА ЙОНИЗАЦИОННИТЕ ЗАГУБИ

П. И. Й. Велинов, С. Асеновски

Резюме

В настоящата статия е разгледан йонизационният модел CORIMIA (COsmic Ray Ionization Model for Ionosphere and Atmosphere) в неговата дву-интервална апроксимация на функцията на йонизационните загуби.

Увеличаването на броя на интервалите на апроксимация е следваща стъпка при усъвършенстването на модела. Последният етап бе въвеждането на 5-интервална апроксимация на формулата на Бор-Бете-Блох с междинен интервал. Тази версия включва въвеждане на граници на интегриране в произволен енергетичен интервал.