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PHASED SATELLITE IMPACT WITH IDEAL AND OTHER LEAD ANGLES

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Abstract

Threats posed by counterspace capabilities are directed against space systems, their supporting ground infrastructure, and data links between space systems and ground infrastructure. Space countermeasures include direct attack and co-orbital anti-satellite systems, cyber attacks, electronic warfare, and directed energy. This is a condition, however, of the impossibility of stationary dominance and control, which exacerbates the need to develop opportunities to influence the enemy's satellites, while at the same time guaranteeing the sustainable operation of one's own space capabilities. This can be accomplished by ground or space-based means. Since outer space is a vacuum and is incommensurable with the Earth's atmosphere, the change in the trajectory of satellites in space is fundamentally different from the change in the trajectory of combat aircraft. The movement of satellites in a vacuum along a certain orbit, due to the distortion of space by the planet Earth, leads to movement at a much higher speed than all aircraft flying in the Earth's atmosphere, and the movement in the orbit does not require thrust compared to airplanes. When it is necessary, however, to change the satellite's trajectory, energy is required to be expended by changing the satellite's velocity ΔV. *This is usually achieved by burning chemical fuels or expelling accelerated gases through a propulsion system.*

Introduction

In 2022, the EU identified space as a strategic area, which was the basis for the EU Space Strategy for Security and Defence. The strategy outlines the counter capabilities and the main threats in space and their ground infrastructure. The High Representative will produce a classified annual analysis of space threats at the EU level using Member States' intelligence. According to the adopted strategy, the space domain comprises all elements that are relevant to the functioning of space systems and the provision of space services, including the physical space environment, the various orbits and spacecraft, their ground and launch infrastructure, radio frequency links, user terminals, the information associated with and delivered by these space assets, the associated cyber environment and the core industrial space sector. The threats posed by counter-space capabilities are directed against space systems, the ground infrastructure that supports them, and the data links between space systems and ground infrastructure. Counter-space capabilities include direct attack and coordinated anti-satellite systems, cyber-attacks, electronic warfare, and directed energy. These can disrupt, damage, or destroy with reversible or irreversible effects. The space sector and its supply chains are also vulnerable to interference from competitors. To this it is necessary to add the requirements for fourth and a half generation fighters - "stealth", improved tactical avionics, thrust vector control, active phased array radar (air targets about 200 km), integrated into a network-centric battlespace, where fighter jets have a much greater range to perform multi-purpose missions.

For the Air Force to be able to use the full capabilities of a platform of this generation, appropriate specialists in: Linux, Solaris and Windows Server network operating systems; Relational database management systems - Oracle, MySQL, SQLite and Integrated Database Management Language – SQL; Network communication devices - Hub, switch, Router, Firewall device; Computer networks – models, standards and protocols; Principles of operation of software security in computer networks; Communication protocols for exchange of radar, weather and pre-flight information - Eurocontrol ASTERIX, ICAO, OLDI; Standardized database for aeronautical information - (Digital Aeronautical Flight Information File (DAFIF); Protected communication protocols providing C2 systems - Tactical Data Links (TDL) - Link16, Link22, etc.

When the countries that purchased the F-16 Block 70 platform are expected to acquire the capabilities to fully use its combat capabilities, it is expected that the sixth generation of combat aircraft platforms will enter service in developed countries, to which requirements such as: cyber warfare; space war; possibility to perform manned and unmanned missions; integration of the platform with fleets of drones and satellites; ground sensors in a high-traffic network environment to provide a complete data-to-solution capability. Advanced digital capabilities - including high-capacity networks, artificial intelligence elements, data fusion, and battlefield command, control, and communications (C3) capabilities.

These requirements lead to the need for an integral consideration of the air and space domain of the combat space, and from there to the direct involvement of satellites as part of the combat platforms operating on the ground, in the air, in the sea, and in the information space [4-5].

This suggests that new specializations such as cyber and electronic warfare, which can be conducted effectively under appropriate command and control systems, will dominate in the future. For this reason, command and control systems need to be established as a separate specialization in higher military schools. Adequate logistics must be added to them.

The satellite race also includes the development of measures to destroy or counteract the space capabilities of opposing countries. The main categories of these weapons can be anti-satellite strikes.

Anti-satellite strike

When defining the angle of the satellite relative to the observer, it is not advisable to ignore that the planet Earth is a flattened spheroid (Fig. 1).

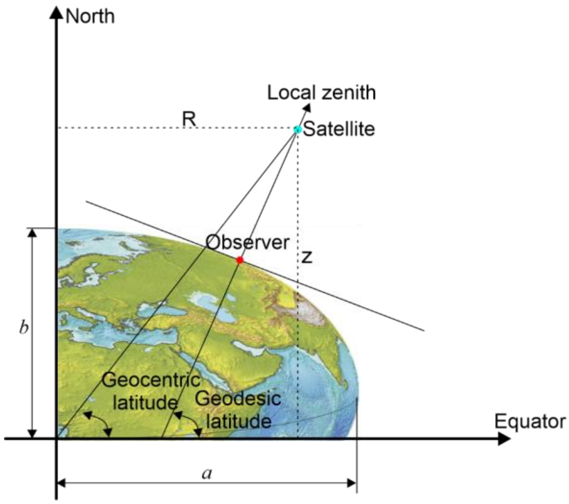

Fig. 1. Cross section of a flattened spheroid

Local zenith - this is the direction from a point on the Earth's surface perpendicular to (at right angles to) the local horizon (a plane that is tangent to the Earth's surface from the observer's position). While in the case of a sphere, this direction points to the center of the Earth; however, in the case of a flattened spheroid, this is not the case except at the equator and poles.

The rotation ellipsoid best fits the surface of the flattened spheroid. It can be defined either by the major semi-axis (a) and the minor semi-axis (b), or by the magnitude of the major semi-axis and the contraction (sparsity).

The reference spheroid WGS-84 (World Geodetic System, 1984) used in the GPS system will be used to solve the problem.

In WGS-84, the equatorial radius of the Earth (major semi-axis) a= 6378137 m, the polar radius of the Earth (minor semi-axis) b is related to the equatorial radius by the flattening f:

(1) $$b = a(1 - f)$$

The flattening f in WGS-84 is only 1/ 298.257223563, i.e., the difference with an ideal sphere is very small. Using this value, the Earth's polar radius would be 6356752.31 km - only 21384.68 meters difference from the equatorial radius.

Taking this into account, we assume that the geodesic latitude φ is the angle between the local zenith direction and the Earth's equatorial plane. The angle locked between the center of the Earth and the equatorial plane is called the geocentric latitude (from the observer's position) [2].

This makes it possible to determine geocentric latitude from geodetic latitude:

(2)
$$\tan(\textit{geocentric latitude}) = \frac{b^2}{a^2}\tan(\textit{geodesic latitude}) =$$
$$= (1-f)^2 \tan(\textit{geodesic latitude}).$$

The pointing error produced by assuming a spherical Earth when tracking the satellite with high-gain antennas requires a relatively narrow beamwidth, which can result in loss of communication.

The satellite sub-point is the point on the Earth's surface that lies directly beneath the satellite. For the case of a spherical Earth, this point is the intersection of the line from the centre of the Earth to the satellite and the Earth's surface.

(3)
$$\mathrm{H_c} = \sqrt{\mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2} - \mathrm{R_{Earth}}$$

As can be seen in Figure 1, the calculation for the flattened Earth is a bit more complicated. In this case, the satellite subpoint is the point on the Earth's surface where the satellite would appear at the zenith [1].

To calculate the altitude of the satellite above the footpoint, the following formula is used:

(4)
$$\mathrm{H_s} = \frac{\mathrm{R}}{\cos(\text{geodesic latitude})} - \frac{a}{\sqrt{1-(2f-f^2).\sin^2(\textit{geocentric latitude})}}.$$

Constant input parameters required

- gravitational constant of the planet Earth - μ=398600.436233 [km^3/s^2];
- gravitational constant of the moon - mμ=4902.800076 [km^3/s^2];
- gravitational constant of the sun - sμ=132712440040.944 [km^3/s^2];
- angular rotational velocity of the Earth in inertial space - 7.2921150 ± 0.0000001)$\times 10^{-5}$ [rad/s];
- Earth's radius at the equator - $R_{Equator}$=6378.1363 km;
- flattening factor of planet Earth - f=1/ 298.257223563;
- Earth's gravitational flattening factor - j_2=0.01108263;

- astronomical unit (the distance from the Earth to the Sun) - 14597870.691 km.

Ideal lead angle

To perform a kinetic impact, one satellite must intercept and rendezvous with the second satellite at the same time.

Coplanar collision

The main parameter for making this transfer is time. In which the engines will be activated. It is necessary to calculate in advance how long it will take the interceptor to reach the collision point. Knowing the speeds of the interceptor and the target.

In order for the intercept to take place, the interceptor needs to overtake the target by the so-called lead angle – α_l (the angular velocity of the target at the time of the interceptor's transfer) when the Hohmann transfer starts. This overtaking angle, shown in Fig. 2, represents [3].

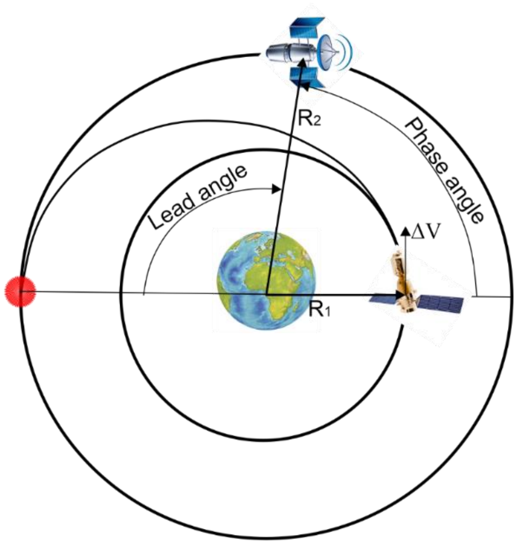

Fig. 2. ΔV of the randezvous

(5) $$\alpha_1 = \pi\left[1-\left(\frac{1+\frac{R_i}{R_t}}{2\frac{R_i}{R_t}}\right)^{\frac{3}{2}}\right].$$

where: R_i – radius of the interceptor's circular orbit;
R_t - radius of the target circular orbit.

In order to calculate the exact moment for making a transfer, it is necessary to know the phase angle (it is locked between the radius of the interceptor vector and the vector of the radius of the target in the direction of the interceptor's movement).

(6) $$\varphi = \pi - \alpha_l,$$

where: α_l - lead angle.

Normalization

Normalization allows for the removal of all free parameters from the equations so that the calculations made can be scaled in any system, depending only on the ground and satellite properties.

For a coplanar Hohmann transfer, three radii need to be normalized [3]:

(7) $$R_{n1} = \sqrt{\frac{2R_t}{R_i + R_t}} = \sqrt{\frac{2(R_{Earth}+H_t)}{(R_{Earth}+H_i)+(R_{Earth}+H_t)}} = \sqrt{\frac{2(R_{Earth}+H_t)}{2R_{Earth}+H_i+H_t}};$$

(8) $$R_{n1} = \sqrt{\frac{R_i}{R_t}} = \frac{R_{Earth}+H_i}{R_{Earth}+H_t};$$

(9) $$R_{n3} = \sqrt{\frac{2R_i}{R_i + R_t}} = \sqrt{\frac{2(R_{Earth}+H_i)}{(R_{Earth}+H_i)+(R_{Earth}+H_t)}} = \sqrt{\frac{2(R_{Earth}+H_i)}{2R_{Earth}+H_i+H_t}};$$

where:

R_i - geocentric radius of the initial circular orbit;

R_t - geocentric radius of the target circular orbit;

H_i - altitude of initial orbit;

H_t - target orbit altitude.

This shows that it is necessary to calculate the optimal angle of observation (α_{ol}) for the meeting of the two satellites:

(10) $$\alpha_{\text{ol}} = \pi \left[1 - \left(\frac{1 + \frac{R_i}{R_t}}{2 \frac{R_i}{R_t}} \right)^{\frac{3}{2}} \right].$$

(11) $$\Delta\alpha = \alpha_l - \alpha_{ol}.$$

Waiting time:

(12) $$T_{waiting} = \frac{\Delta\alpha_l . SP}{2\pi}.$$

Modified equinoctial orbital elements

The modified equinoctial orbital elements (p, f, g, h, k, L) are suitable for analysis and optimization of trajectories. They apply to circular as well as elliptical and hyperbolic orbits and show no features for zero eccentricity and orbital inclinations equal to 0 and 90 degrees. However, two of the components are singular for an orbital inclination of 180 degrees (Fig. 3).

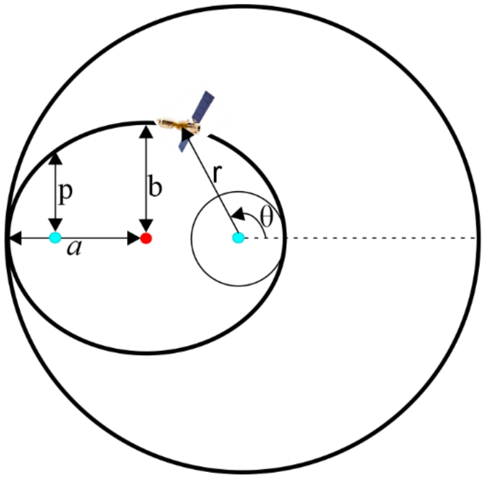

Fig. 3. Semi-latus rectum

Relationship between classical orbital and modified equinoctial elements [3]:

(13) $$a = \frac{p}{1 - f^2 - g^2};$$

(14) $$e = \sqrt{f^2 + g^2};$$

(15) $$i = 2\tan^{-1}\sqrt{h^2 + k^2};$$

(16) $$\omega = \tan^{-1}\left(\frac{g}{f}\right) - \tan^{-1}\left(\frac{k}{h}\right);$$

(17) $$\Omega = \tan^{-1}(k, h);$$

(18) $$\theta = L - (\Omega + \omega) = L - \tan^{-1}\left(\frac{g}{f}\right);$$

(19) $$u = \omega + \theta = \tan^{-1}(h\sin L - k\cos L, h\cos L + k\sin L).$$

where: a - semi-major axis; i - orbital inclination; e - orbital eccentricity; ω - perigee argument; Ω - ascending node length; L - true latitude (along with declination and ascending node, true longitude indicates the exact direction from the ground that the satellite will be at a given time); ν - true anomaly; f, g - the x- and y-axis components of the eccentricity vector in an orbital coordinate system; h, k - the x- and y-axis components of the nodal vector in an orbital coordinate system; $p = \frac{b^2}{a} = a\left(1 - e^2\right)$ - semi-latus rectum (the length of the chord through one of the foci, perpendicular to the major axis).

Transforming the Keplerian elements (a, e, i, ω, Ω, ν) into a state vector (x, y, z, vx, vy, vz) (where: v-velocity of the satellite)

A radius vector is a vector defining the position of a point in space relative to some predetermined fixed point called the origin of the coordinate system.

(20) $$R_m = \frac{a\left(1 - e^2\right)}{1 + e\cos(\nu)},$$

where: R_m – magnitude of the radius vector of the satellite, determining its position.

The latitude argument (u) is an angular parameter that defines the position of the satellite moving along a Kepler orbit and represents the angle subtended between the ascending node and the satellite.

Results

Ideal lead angle

Input data: initial altitude – 400 km; true anomaly on the initial orbit – 55^o; final altitude – 4000 km; ideal initial lead angle.

Output data (fig. 4, 5):

- ideal initial lead angle - 44.735801^o;
- trajectory times - total mission time - 3953.396757 s;
- first impulse - interceptor true anomaly – 55^o; target true anomaly - 99.735801^o; true anomaly of transfer orbit – 0^o;
- second impulse - interceptor true anomaly - 311.269235^o; target true anomaly – 235^o; true anomaly of transfer orbit – 180^o;
- $\Delta Vx_{firstimpulse}$=-627.702647 m/s; $\Delta Vy_{firstimpulse}$= 439.522125 m/s; $\Delta Vz_{firstimpulse}$= 0 m/s; $\Delta V_{firstimpulse}$= 766.283441 m/s;
- $\Delta Vx_{secondimpulse}$=563.951296 m/s; $\Delta Vy_{secondimpulse}$=-394.882949 m/s; $\Delta Vz_{secondimpulse}$= 0 m/s; $\Delta V_{secondimpulse}$= 688.457412 m/s;
- ΔV_{total}= 1454.740853 m/s.

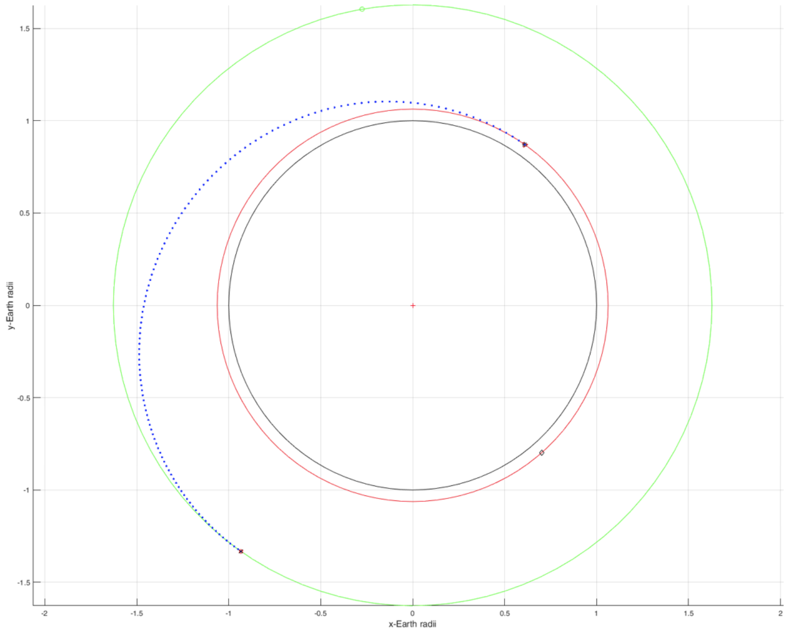

Fig. 4. Anti-satellite phasing strike with ideal angle

Designations in Fig. 5 and Fig. 8.

Small red triangle - initial location of interceptor; small green circle - initial location of target; the two black asterisks – locations of the interceptor and target (location of the two impulses); black circle – spherical Earth; red circle – initial orbit; green circle – final orbit.; small triangle mangenta - location of target at initial impulse.

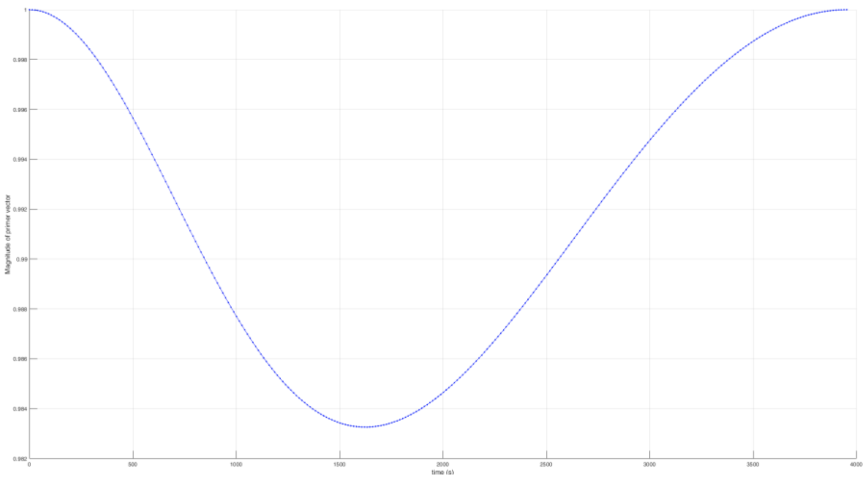

Fig. 5. Magnitude of primer vector for ideal lead angle

Other lead angle

Input data: initial altitude – 400 km; true anomaly on the initial orbit – 55^{o}; final altitude – 4000 km; other initial lead angle.

Output data (fig. 6, 7):

- initial lead angle – 55^{o};
- ideal initial lead angle - 44.735801^{o};
- trajectory times - total mission time - 4288.741885 s;
- first impulse - interceptor true anomaly – 76.737924^{o}; target true anomaly - 121.473726^{o}; true anomaly of transfer orbit – 0^{o};
- second impulse - interceptor true anomaly - 333.007159^{o}; target true anomaly – 256.737924^{o}; true anomaly of transfer orbit – 180^{o};
- $\Delta Vx_{firstimpulse}$=-745.847375 m/s; $\Delta Vy_{firstimpulse}$= 175.789664 m/s; $\Delta Vz_{firstimpulse}$= 0 m/s; $\Delta V_{firstimpulse}$= 766.283441 m/s;
- $\Delta Vx_{secondimpulse}$=670.096893 m/s; $\Delta Vy_{secondimpulse}$=-157.935942 m/s; $\Delta Vz_{secondimpulse}$= 0 m/s; $\Delta V_{secondimpulse}$= 688.457412 m/s;
- ΔV_{total}= 1454.740853 m/s.

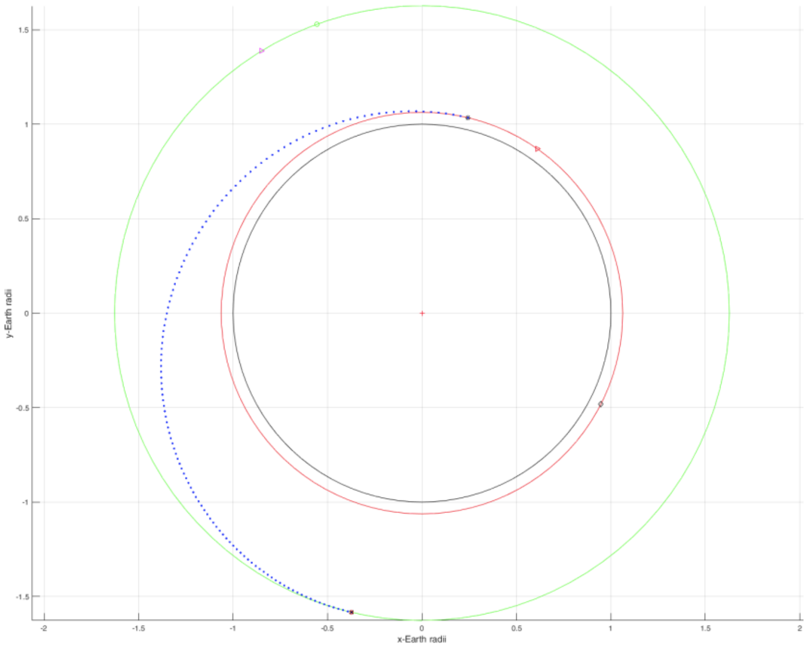

Fig. 6. Anti-satellite phasing strike with other lead angle

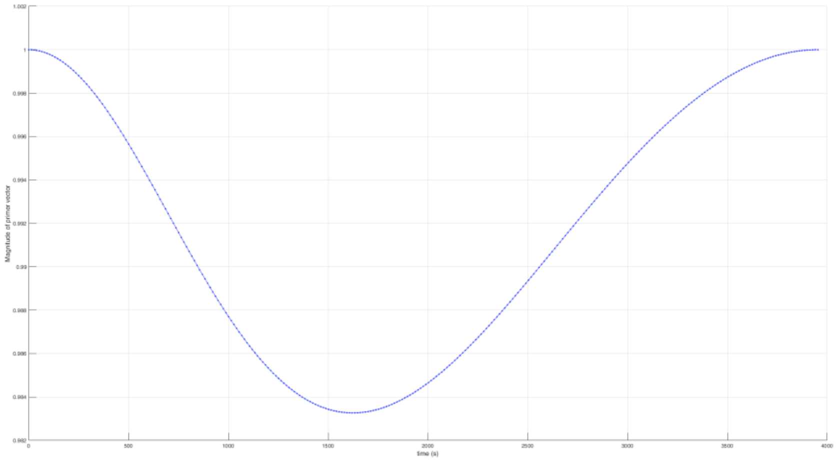

Fig. 7. Magnitude of primer vector for another lead angle

Conclusions

The satellite race involves the development of measures to destroy or counter the space capabilities of opposing nations. The main categories of these weapons can be anti-satellite weapons and directed energy weapons. Providing an ideal lead vector will preserve the total velocity expenditure of the two pulses, but reduce the time to achieve collision with the target. This is an important feature for interceptors because, in order to be able to strike other space objects, they must have a large ΔV budget to be able to deliver a kinetic strike or get close enough to otherwise affect them. Satellites in orbit constantly change their place in the orbit and in relation to other space objects, and in relation to the Earth, occupying a specific place can only be carried out in geosynchronous and geostationary orbits. In all other orbits, they will constantly change their position and will be at odds with the paradigm of dominance in a certain area. In return, the satellites provide exceptional opportunities for situational awareness of other types and branches of troops.

References

1. Lambeth, B., Next steps in the military uses of space, United States Air Force, 2003.
2. Wright, D., Grego L., The Physics of Space Security, American Academy of Arts and Sciences, 2005.
3. Yuan, J., Cheng Y., Orbital Dynamics and Mission Trajectory Desig, Science Press, Beijing, China, 2019.
4. Zagorski, N., Analysis of the military application of unmanned aircraft and main direction for their development, Aerospace Research in Bulgaria, Volume 33/2021, p.p. 237-250, ISSN 1313-0927 (print), DOI: https://doi.org/10.3897/arb.v33.e17.
5. Zagorski, N., Aerospace systems “aerial launch” at the modern stage, Aerospace Research in Bulgaria, 34, 34, Bulgarian Academy of Sciences. Space Research and Technology Institute, , p.p. 80-94, Volume 34/2022, ISSN:p-ISSN 1313 - 0927, DOI:https://doi.org/10.3897/arb.v34.e07.

ФАЗИРАН УДАР НА САТЕЛИТ С ИДЕАЛЕН И ДРУГ ЪГЪЛ НА ИЗПРЕВАРВАНЕ

А. Маринов

Резюме

Заплахите, породени от противокосмическите способности, са насочени срещу космическите системи, поддържащата ги наземна инфраструктура и връзките за данни между космическите системи и наземната инфраструктура. Космическите противодействия включват директна атака и коорбитални анти-сателитни системи, кибератаки, електронна война и насочена енергия. Това е условие обаче за невъзможност за стационарно господство и контрол, което изостря необходимостта от разработване на възможности за въздействие върху спътниците на противника, като в същото

време се гарантира устойчива работа на собствените космически възможности. Това може да бъде постигнато чрез наземни или космически средства. Тъй като космическото пространство е вакуум и е несъизмеримо със земната атмосфера, изменението на траекторията на спътниците в пространството е коренно различно от изменението на траекторията на бойните самолети. Движението на спътниците във вакуум по определена орбита, следствие изкривяването на пространството от планетата Земя, води до движение с много по-висока скорост от всички въздухоплавателни средства, летящи в земната атмосфера, а движението в орбитата не изисква тяга. Когато е необходимо, обаче да се промени траекторията на движение на сателита, се изисква изразходване на енергия чрез промяна на скоростта на сателита ΔV. В настоящата статия е направено изчисление на анти-сателитен фазиран удар с идеален и неидеален изпреварващ ъгъл на прехващач спрямо цел.