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FLIGHT DYNAMICS MODEL OF A LARGE TRANSPORT AIRCRAFT

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Abstract

In the presented paper, linearized and nonlinear mathematical models of three-dimensional motion of Douglas DC-8-63 transport aircraft are considered. A source code was developed in the GNU Octave environment by means of which serial computations were made for impulse deviations of elevator and ailerons. The results obtained from both the linearized and the non-linear model are shown graphically, compared, and commented.

Introduction

Airplane flight dynamics refers to carrying out a study of how airplanes move and maneuver in three-dimensional space, [1]. It involves understanding the forces and moments that act on an aircraft during flight, as well as the aerodynamic principles that govern the behavior of an aircraft in the air. It is all about trading off stability against control. By and large, stability refers to the tendency of an aircraft to return to a steady state after a disturbance ceases to act. Control refers to the pilot’s ability to maneuver the aircraft and maintain control. The aircraft design process comes with a trade-off between stability and control qualities. Overall, airplane flight dynamics is critical for ensuring the safety and efficiency of air travel.

Previous research on the topic includes D. Scholtz’s noteworthy monography, [2] who investigated the flight dynamics of the Douglas DC-3 by means of both linearized and non-linear models. In addition, McLean [3] and Nelson [4] have developed linearized and non-linear models, including short numerical examples involving generic aircraft.

The objectives of the presented paper are, firstly, to carry out numerical experiments with linearized and non-linear models of aircraft motion and, secondly, to compare the obtained results. Decoupled aircraft responses (longitudinal and lateral) are considered separately after applying a pulse control input by elevator and ailerons. Inertial and aerodynamic coupling effects are not taken into consideration whatsoever.

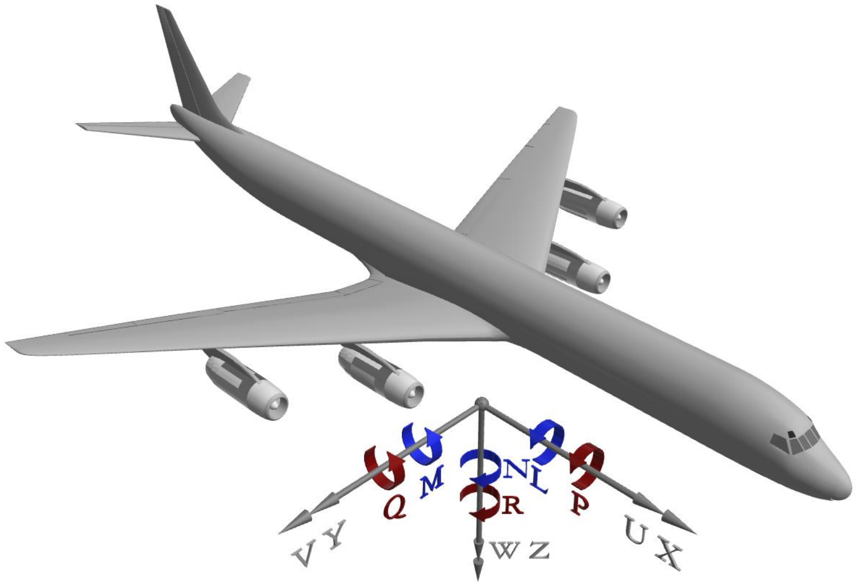

Fig. 1. Adopted body reference frame

Governing equations

Governing equations have been derived from theorems of linear and angular momentum conservation as follows:

$$\frac{d\mathbf{v}}{dt} = \frac{\mathbf{F}}{m} - \boldsymbol{\omega} \times \mathbf{v}$$
$$(1) \qquad \frac{d\boldsymbol{\omega}}{dt} = I^{-1} \cdot \left(\mathbf{M} - \boldsymbol{\omega} \times I\boldsymbol{\omega}\right)$$

where $\mathbf{F}$ = [X, Y, Z]^T, $\mathbf{M}$ = [L, M, N]^T are vectors of externally applied forces/moments, $\mathbf{v}$ = [U, V, W]^T, $\boldsymbol{\omega}$ = [P, Q, R]^T stand for linear/angular velocity vectors, Fig. 1, m is rigid body mass (constant), I is inertia tensor describing mass distribution within the rigid body:

$$(2) \qquad I = \begin{Vmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{Vmatrix}$$

The body reference frame is solely considered. Positive signs of moments and angular velocities, forces, and linear velocities are shown in Fig. 1.

Linearized model

Linearized mathematical models of the nonlinear system (1) presented here have been borrowed from textbooks [3], [4], and a monograph [2]. System response is resolved by function `[y,t,x] = lsim(sys,u,t)` available in the Control package in GNU Octave.

Longitudinal dynamics (state space)

$$\begin{aligned} &\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \\ &\mathbf{x} = \|u \quad w \quad q \quad \theta\|^T \quad \mathbf{u} = \|\delta_E \quad \delta_F\|^T \end{aligned} \tag{3}$$

$$A = \begin{Vmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & U_0 & 0 \\ \tilde{M}_u & \tilde{M}_w & \tilde{M}_q & 0 \\ 0 & 0 & 1 & 0 \end{Vmatrix} \quad B = \begin{Vmatrix} X_{\delta E} & X_{\delta F} \\ Z_{\delta E} & Z_{\delta F} \\ \tilde{M}_{\delta E} & \tilde{M}_{\delta F} \\ 0 & 0 \end{Vmatrix} \tag{4}$$

$$\begin{aligned} &\tilde{M}_u = M_u + M_{\dot{w}} Z_u \\ &\tilde{M}_w = M_w + M_{\dot{w}} Z_w \\ &\tilde{M}_q = M_q + M_{\dot{w}} U_0 \\ &\tilde{M}_{\delta E} = M_{\delta E} + M_{\dot{w}} Z_{\delta E} \quad \tilde{M}_{\delta F} = 0 \left(\textit{no flaps}\right) \end{aligned} \tag{5}$$

$$\begin{aligned} &\mathbf{y} = C\mathbf{x} + D\mathbf{u} \\ &\mathbf{y} = \|\alpha \quad \gamma \quad a_Z\|^T \quad \mathbf{u} = \|\delta_E \quad \delta_F\|^T \end{aligned} \tag{6}$$

$$\alpha = \frac{w}{U_0} \quad \gamma = \theta - \alpha \quad a_Z = \dot{w} - U_0 q \quad n_Z = \frac{a_Z}{g} \tag{7}$$

$$C=\begin{Vmatrix} 0 & \dfrac{1}{U_0} & 0 & 0 \\ 0 & \dfrac{1}{U_0} & 0 & 1 \\ Z_u & Z_w & 0 & 0 \end{Vmatrix} \quad D=\begin{Vmatrix} 0 & 0 \\ 0 & 0 \\ Z_{\delta E} & Z_{\delta F} \end{Vmatrix} \tag{8}$$

Lateral dynamics (state space)

$$\begin{gathered} \dot{\mathbf{x}}=A\mathbf{x}+B\mathbf{u} \\ \mathbf{x}=\begin{Vmatrix}\beta & p & r & \varphi\end{Vmatrix}^T \quad \mathbf{u}=\begin{Vmatrix}\delta_A & \delta_R\end{Vmatrix}^T \end{gathered} \tag{9}$$

$$A=\begin{Vmatrix} Y_v & 0 & -1 & g/U_0 \\ L'_\beta & L'_p & L'_r & 0 \\ N'_\beta & N'_p & N'_r & 0 \\ 0 & 1 & 0 & 0 \end{Vmatrix} \quad B=\begin{Vmatrix} Y'_{\delta A} & Y'_{\delta R} \\ L'_{\delta A} & L'_{\delta R} \\ N'_{\delta A} & N'_{\delta R} \\ 0 & 0 \end{Vmatrix} \tag{10}$$

$$\begin{aligned} L'_\beta &= L_\beta+\frac{I_{xz}}{I_x}N_\beta & N'_\beta &= N_\beta+\frac{I_{xz}}{I_x}L_\beta \\ L'_p &= L_p+\frac{I_{xz}}{I_x}N_p & N'_p &= N_p+\frac{I_{xz}}{I_x}L_p \\ L'_r &= L_r+\frac{I_{xz}}{I_x}N_r & N'_r &= N_r+\frac{I_{xz}}{I_x}L_r \\ L'_{\delta A} &= L_{\delta A}+\frac{I_{xz}}{I_x}N_{\delta A} & N'_{\delta A} &= N_{\delta A}+\frac{I_{xz}}{I_x}L_{\delta A} \\ L'_{\delta R} &= L_{\delta R}+\frac{I_{xz}}{I_x}N_{\delta R} & N'_{\delta R} &= N_{\delta R}+\frac{I_{xz}}{I_x}L_{\delta R} \\ Y'_{\delta R} &= \frac{Y_{\delta R}}{U_0} & Y'_{\delta A} &= \frac{Y_{\delta A}}{U_0} \end{aligned} \tag{11}$$

(12) $$\mathbf{y} = C\mathbf{x} + D\mathbf{u}$$
$$\mathbf{y} = \left\| v \quad a_Y \right\|^T \quad \mathbf{u} = \left\| \delta_A \quad \delta_R \right\|^T$$

(13) $$v = \beta U_0 \quad a_Y = \dot{v} - g\varphi + U_0 r \quad n_Y = \frac{a_Y}{g} \quad \psi = \int r dt$$

(14) $$C = \begin{Vmatrix} U_0 & 0 & 0 & 0 \\ Y_v U_0 & 0 & 0 & 0 \end{Vmatrix} \quad D = \begin{Vmatrix} 0 & 0 \\ Y'_{\delta A} U_0 & Y'_{\delta R} U_0 \end{Vmatrix}$$

Non-linear model

Non-linear equations of motion have been derived from system (1) by means of the SymPy package available in GNU Octave and the source code shown in Appendix 1. The equations are subsequently solved for the highest derivative of the state variables by means of the `ode45` function. The function implements an explicit Runge-Kutta (order 4, 5) method with Dormand – Prince adaptive scheme. Given products of inertia *Ixy = Iyz = 0*, the obtained equations are as follows:

(15)
$$\dot{U} = \frac{X}{m} - QW + RV + G_x$$
$$\dot{V} = \frac{Y}{m} - RU + PW + G_y$$
$$\dot{W} = \frac{Z}{m} - PV + QU + G_z$$
$$\dot{P} = \frac{1}{I_{xx}I_{zz} - I_{xz}^2}\begin{bmatrix} I_{xz}\left(-I_{yy}PQ + N + Q\left(I_{xx}P - I_{xz}R\right)\right) + \dots \\ I_{zz}\left(I_{yy}QR + L + Q\left(I_{xz}P - I_{zz}R\right)\right) \end{bmatrix}$$
$$\dot{Q} = \frac{1}{I_{yy}}\left[M - P\left(I_{xz}P - I_{zz}R\right) - R\left(I_{xx}P - I_{xz}R\right)\right]$$
$$\dot{R} = \frac{1}{I_{xx}I_{zz} - I_{xz}^2}\begin{bmatrix} I_{xx}\left(-I_{yy}PQ + N + Q\left(I_{xx}P - I_{xz}R\right)\right) + \dots \\ I_{xz}\left(I_{yy}QR + L + Q\left(I_{xz}P - I_{zz}R\right)\right) \end{bmatrix}$$

Alternatively, the scalar expansion of system (15) might be found in the textbook [5].

Gravity force vector is to be added to the aforementioned system according to

$$\text{(16)}\qquad \begin{aligned} G_x &= -g\sin\theta \\ G_y &= g\cos\theta\sin\varphi \\ G_z &= g\left(\cos\theta\cos\varphi - 1\right) \end{aligned}$$

where θ is aircraft pitch, φ is roll angle, and g = *9.80665* m/s^2 is standard gravity net acceleration. This, in turn, introduces the following additional relations as far as aircraft attitude is concerned:

$$\text{(17)}\qquad \begin{aligned} \dot{\theta} &= Q\cos\varphi - R\sin\varphi \\ \dot{\varphi} &= P + R\tan\theta\cos\varphi + Q\tan\theta\sin\varphi \end{aligned}$$

Note, a singularity would arise should pitch angle $\theta = \pm \pi/2$ rad. A workaround would be to propagate quaternions from angular rates P, Q, R instead of Euler angles according to a recipe published in link [6].

Externally applied forces **F** and moments **M** are further calculated, taking into account dimensional stability and control derivatives. This approach is commonly known as Taylor series approximation. It is a representation of a function as an infinite sum of terms that are calculated from values of the function's derivatives at a single point, a.k.a. equilibrium condition.

$$\text{(18)}\qquad \begin{aligned} \frac{X}{m} &= X_u\left(U_0 - U\right) + X_w W + X_{\delta E}\delta E + X_{\delta F}\delta F \\ \frac{Y}{m} &= Y_v V + Y_{\delta A}\delta A + Y_{\delta R}\delta R \\ \frac{Z}{m} &= Z_u\left(U_0 - U\right) + Z_w W + Z_{\delta E}\delta E + Z_{\delta F}\delta F \\ L &= I_x\left(L_v V + L_p P + L_r R + L_{\delta A}\delta A + L_{\delta R}\delta R\right) \\ M &= I_y\left(M_u U + M_w W + M_{\dot{w}}\dot{W} + M_q Q + M_{\delta E}\delta E + M_{\delta F}\delta F\right) \\ N &= I_z\left(N_v V + N_p P + N_r R + N_{\delta A}\delta A + N_{\delta R}\delta R\right) \end{aligned}$$

By using the first few terms of the Taylor series, a polynomial approximation could be created that is accurate near the equilibrium point. Values of the first partial derivatives, [7] at certain flight conditions are given in Appendix 2. Conversion to SI units has been carried out in advance. In equations above, subscripts denote the following deflections, in the current study case, in radians: δE elevator, δA aileron, δR rudder, δF flaps. Negative elevator trailing edge up gives positive nose-up pitch response. Negative right aileron up and left aileron down gives positive right wing down roll response, [8]. A shorthand notation is used here, for example, $Xu = \partial X/\partial U$ and so on.

Results

Initial conditions U0 = 74.2 m/s, mass m = 86104 kg, Mach number M = 0.219, product of inertia Ixz = 37962.9 kg.m^2, moments of inertia Ixx = 4189477.460 kg.m^2, Iyy = 3986104.768 kg.m^2, Izz = 7565464.152 kg.m^2 as well as values of dimensional stability and control derivatives are published in Appendix 2 (imperial units). These values, available in paper [7], apply to Douglas DC-8-63 aircraft. The following results were obtained for longitudinal and lateral aircraft responses after applying a pulse of elevator/aileron deflection of +0.02 rad, respectively, within 10 seconds. Overlaid charts in Figs. 2 and 3 show results obtained by means of both linearized and non-linear models. Both figures depict state vectors of longitudinal (3) and lateral (9) motions. In the chart caption, a ternary operator is used to describe the input pattern. In Fig. 4, the airplane's response to the same input is depicted in three-dimensional space.

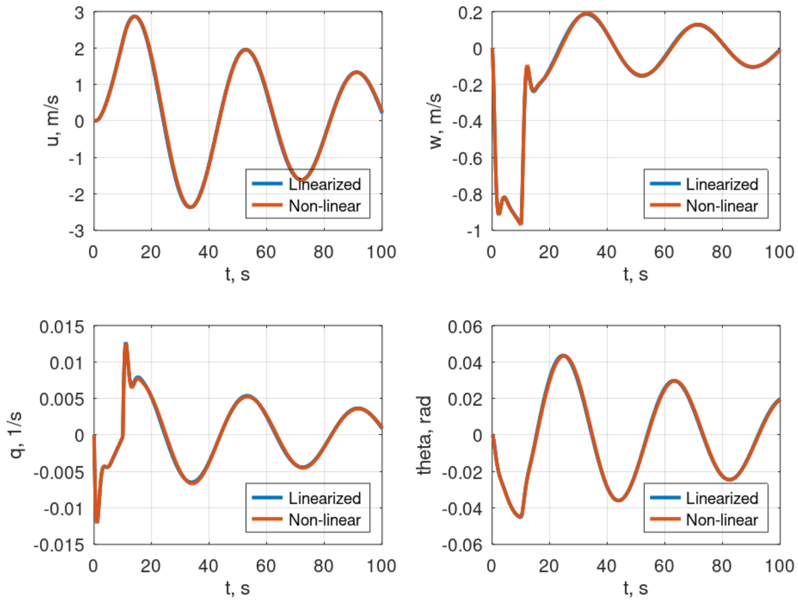

Fig. 2. Longitudinal motion, elevator δ_E = (t<10) ? 0.02 : 0 rad

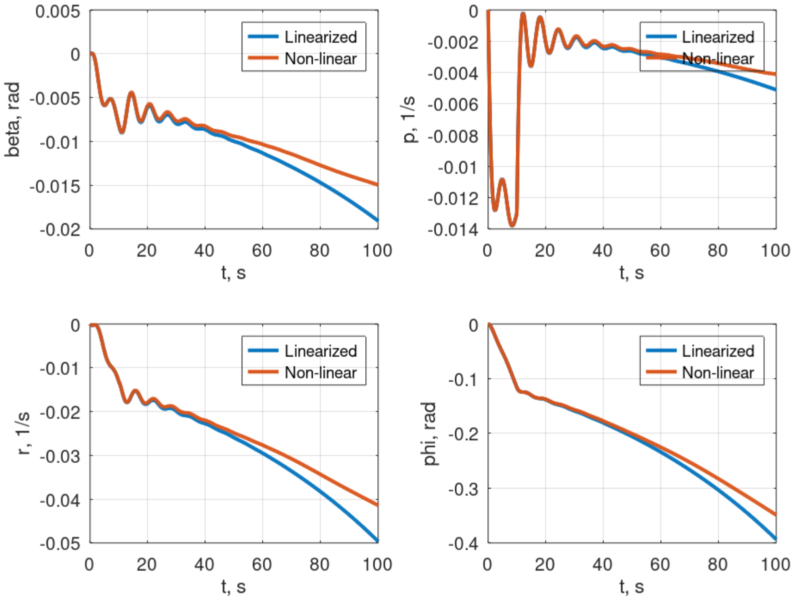

Fig. 3. Lateral motion, aileron δ_A = (t<10) ? 0.02 : 0 rad

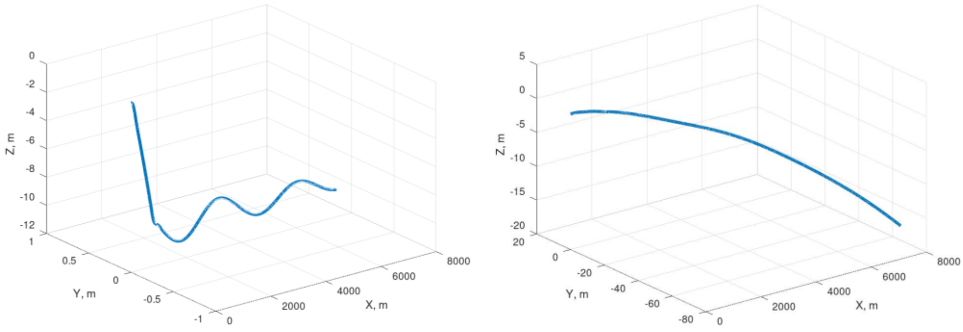

Fig. 4. Airplane response, longitudinal (left) and lateral motion (both non-linear)

Conclusion

In case of longitudinal motion, no significant difference between linearized and non-linear models is observable. With regard to lateral motion, however, an approximation error is being accumulated. This might be accounted for by the poor accuracy of the proposed linearized model of aircraft lateral motion.

Significant oscillations are noticeable at the beginning and end of the control input. Presumably, low values of corresponding damping derivatives might be pointed out as a cause.

Disclaimer

The project source code might be downloaded from GitHub, [9]. The code runs on GNU Octave only (no MATLAB whatsoever).

References

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9. https://github.com/samolet4e/Flight-dynamics

Appendix 1. Source code in SymPy for deriving governing equations (15)

```
clear;
pkg load symbolic

syms Ixx Ixz Iyy Izz
I = [[Ixx,0,-Ixz];[0,Iyy,0];[-Ixz,0,Izz]];

syms P Q R L M N
w = [P;Q;R]; Mo = [L;M;N];

wdot = simplify(inv(I)*(Mo - cross(w,I*w)));

syms U V W m X Y Z
v = [U;V;W]; Fo = [X;Y;Z];

vdot = simplify(Fo/m - cross(w,v));
```

Appendix 2. Data for Douglas DC-8-63, [7]

Geometrical and inertial		Longitudinal dimensional derivatives	
H, ft	0	Xu, sec^{-1}	−0.0291
M, –	0.219	Xw, sec^{-1}	0.0629
m, slugs	5,900	Xδe, $(ft/sec^2)/rad$	0
Ix, $slug.ft^2$	3,090,000	Zu, 1/sec	−0.2506
Iy, $slug.ft^2$	2,940,000	$Z\dot{w}$, –	0
Iz, $slug.ft^2$	5,580,000	Zw, 1/sec	−0.6277
Ixz, $slug.ft^2$	28,000	Zδe, $(ft/sec^2)/rad$	-10.19
U0, ft/sec	243.5	Mu, 1/(sec.ft)	-7.7e-06
W0, ft/sec	0	$M\dot{w}$, 1/ft	-1.068e-03
δf, deg	35	Mw, 1/(sec.ft)	-0.0087
xcg/ĉ	0.15	Mq, 1/sec	-0.7924
θ, deg	0	Mδe, $1/sec^2$	-1.35

Lateral dimensional derivatives		Lateral dimensional derivatives	
Nβ, $1/sec^2$	0.763	Yδa, $(ft/sec^2)/rad$	0
Nv = Nβ/U0, 1/(sec.ft)	3.1e−03	Yδr, $(ft/sec^2)/rad$	5.79
Np, 1/sec	−0.1192	Lβ, $1/sec^2$	−1.335
Nr, 1/sec	−0.268	Lv = Lβ/U0, 1/(sec.ft)	-5.48e-03
Nδa, $1/sec^2$	−0.0496	Lp, 1/sec	−0.95
Nδr, $1/sec^2$	−0.39	Lr, 1/sec	0.612
Yv, 1/sec	−0.1113	Lδa, $1/sec^2$	−0.726
Yβ, $(ft/sec^2)/rad$	-27.1	Lδr, $1/sec^2$	-0.1848

МОДЕЛ НА ДИНАМИКАТА НА ПОЛЕТА НА ГОЛЯМ ТРАНСПОРТЕН САМОЛЕТ

К. Методиев

Резюме

В настоящата статия са разгледани линеаризиран и нелинеен математически модели на пространственото движение на транспортен самолет Douglas DC-8-63. Разработен е сорс код в среда GNU Octave, с помощта на който са направени серийни пресмятания за импулсни отклонения на кормилото за височина и елероните. Получените резултати както от линеаризирания, така и чрез нелинейния модел са показани в графичен вид, сверени и коментирани.