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BLACK HOLE THERMODYNAMICS FOR PRODUCING HORIZON OSCILLATIONS

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Abstract

We present a scenario for addressing the information loss problem in black holes which preserves the unitarity of the S-matrix. Our approach is based on quantizing the curvature singularity using classical non-commutative geometry. Also we take advantage of the developments in unification of quantum mechanics and gravity to provide a two-dimensional holographic description of the event horizon. Hence we discretize the holographic screen (horizon) into individual pixels, Planck area in size, each carrying a degree of freedom. We provide a generic phenomenon which arises naturally from black hole perturbation theory to produce a remnant-free final stage evolution and omit the necessity of introducing an additional membrane, as it has been proposed by Susskind. A stronger version of the Page's argument regarding the no-cloning theorem is considered.

Introduction

The 'black hole information loss' paradox conjectured by Hawking in 1974 [1] states that during the black hole formation and evaporation process the quantum-mechanical unitarity is violated. Thus a pure state, given by the density matrix

(1) $$\rho = |\psi><\psi|$$

evolves into a mixed state described by the density matrix

(2) $$\rho = \sum_{n=1}^{N} \rho_n |\psi_n><\psi_n|$$

When a black hole is formed in a semiclassical background geometry the strong gravitational dynamics act on the quantum vacuum, creating high frequency outgoing modes, which are radiated away to infinity, and the black hole evaporates. The model suggests the emitted Hawking particles carry large amount of entropy S, given by

(3) $$S=M^2/M^2_p$$

where M^2_p is the Planck mass, have black body thermal spectrum of temperature $T_H = \kappa/2\pi$ and carry next-to-nothing information about the initial quantum state of the matter which undergoes gravitational collapse, hence $\Delta I = -\Delta S$. The thermal nature of the radiated Hawking quanta implies it does not depend on the internal microstates but rather on the geometry outside the black hole, thus its mass, M_{BH}. The vicinity of the black hole is just a part of the whole quantum system which means some of the correlations remain in the interior region and causality limits the accessibility for an outside observer. In this view it is impossible, even in principle, to reconstruct the initial state.

Any solution to the information paradox must address two important results which follow from the loss of unitarity, namely the pure-to-mixed state evolution and the time-irreversibility (T-asymmetry) of the map between the past and future null infinity regions, denoted as $\mathcal{J}^-$ and $\mathcal{J}^+$, respectively. The fundamental nature of the paradox pushes us in a direction towards abandoning a physical principle that up until now we have believed to understand. Recent developments in AdS/CFT correspondence [2–4] and holography [5–7] strongly advocate the fundamentally non-local universe view. The extreme conditions created by the strong gravitational dynamics in the process of black hole formation/evaporation further bolster the hypothesis. In our efforts to preserve the unitary evolution of the S-matrix, the overwhelming opinion is that in high energy regimes ($\geq M_P$) certain non-local features occur. Thus in super-Planckian physics we should observe violations of the locality/causality which govern quantum field theory

(4) $$[\varphi(x)\,,\ \varphi(y)] = 0$$

where $(x - y)^2 > 0$.

We present a scenario for preserving unitarity without involving exotic physics. The model we propose eliminates the infinities which arise from the singularity region, thus allowing us to establish a complete T-symmetry between $\mathcal{J}^-$and $\mathcal{J}^+$. Furthermore, we put forward a mechanism which naturally follows from black hole thermodynamics and causes instability in the end-stage evolution of the system, hence leading to a complete remnant-free evaporation.

The article is organized as follows. In ‘Redefining the singularity’ section we develop our model by describing the requirements for preserving unitarily. We redefine the problematic singularity region by quantizing it with the use of non-commutative geometry. In ‘Holographic pixilation and horizon oscillations’ section we apply the holographic principle to the event horizon and define the notion of "pixel". Applying holography (pixelating) to the horizon allows us to present a natural oscillating conjecture that we believe emerges as an effect of matter perturbations. The proposed conjecture could account for the membrane in complementarity [8] and leads to a late-time explosion due to certain instabilities which arise from black hole thermodynamics. In ‘Black hole evaporation and information storing’ section we put forward a method for storing the information regarding an infalling matter onto the vacuum in the vicinity of the hole and establish symmetry between the past and future null infinities. In the 'Conclusions' we summarize the work.

Redefining the singularity

In the present Section we consider non-commutative geometry as a method for avoiding the formation of a singularity, in a spherically symmetric solutions, to the Einstein vacuum equations. The simplest solution of such type is the Schwarzschild metric

(4) $$ds^2 = -(1-2M/r)dt^2 + (1-2M/r)^{-1}\,dr^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2)$$

In the particular metric the event horizon is located at $r = 2M$ and the singularity is at $r = 0$. We suppose the Schwarzschild black hole settles rapidly to a quasi-stationary state, entirely parameterized by mass, angular momentum and charge, given by M, J, and Q, respectively. The quasistationarity is because of the monotonic thermal emission due to the strong gravity effects on the quantum fields.

The current definition of a singularity as infinitely dense point-like "object" leads to certain major problems such as temperature divergence at the black hole endpoint (big curvature), $R \to \infty$ as $r \to 0$ and complete breakdown of general relativity at short distance scales. It further leads to geodesic incompleteness in the interior region as $r \to 0$, and is thus responsible for the information loss problem. It has been argued that non-commutative (NC) geometry can cure all of the above problems by introducing a minimal length scale, hence removing the point-like zero-size "objects", $a(t) = 0$.

We quantize the singularity by first presenting the non-commutative nature of space-time in terms of the commutator

(5) $$[x^\mu, x^\nu] = i\Theta^{\mu\nu}$$

where $\Theta^{\mu\nu}$ is the anti-symmetric matrix (non-commutative operator), with dimensionality of $(lenght)^2$, and places the minimal length scale of $\sqrt{\Theta} \sim 10^{-17}cm$. Here x^μ and x^ν are coordinate operators where $\mu,\ \nu \to 1, 2, 3$. We believe non-commutativity is an integral part of the manifold. NC geometry removes the point-like objects and favors the smeared over particular radius ones. The notion comes from its possible interpretation as a gravitational analog of the uncertainty principle. When we replace the delta-function by a Gaussian distribution the matter source is given by

(6) $$\rho(r) = \frac{M}{(4\pi\Theta)^{3/2}} exp(-r^2/4\Theta)$$

in $D = 4$ dimensions, where M is the mass of the source which is no longer concentrated in a region of zero size $a(t) = 0$, but is rather smeared across a region the size of the minimum length scale, $\sqrt{\Theta}$. Once we have introduced the limit $\sqrt{\Theta}$, and have established that the notion of point-like objects is no longer relevant in that context, we argue the infinities associated with singularity disappear. Thus as we consider the singularity as a gravitational source, it could be represented as

(7) $$M(r) = 4\pi \int_o^r \rho(x) x^2 dx$$

where $\rho > 0$. Therefore we obtain finiteness of mass given by

(8) $$m = 4\pi \int_0^\infty \rho(r) r^2 dr$$

Because of the finiteness of the parameters, any density source will effectively vanish as $r \to \infty$. Indeed we find that by using non-commutative geometry we can redefine the otherwise problematic physical singularity. Therefore we have got rid of the infinities and get a nice constant curvature which at $r = 0$ is described by the *Ricci* scalar

(9) $$R(0) = \left(\frac{4M}{\sqrt{\pi}\Theta^{3/2}} \right)$$

Once we apply NC quantization to the singularity region we cure the temperature divergence at the black hole endpoint, and thus assign a finite value to the curvature scalar R, as $r \to 0$. Detailed calculations have been carried out in [9, 10].

The framework we have provided deals with the pathological problems by eliminating the infinities and the curvature divergence. However, the effects of non-commutativity on large scales are negligible. It has been argued the given model substitutes the singularity region with the so-called *De Sitter* core which is characterized by finite-value parameters and exhibits repulsive gravity features. Therefore, the strong energy condition $(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}) X^\mu X^\nu \geq 0$, is violated in this newly defined singularity region. Violations of the strong energy condition are present in many models and should not be taken as a surprise. We believe every potential solution to the information loss problem should address the problematic matter by either singularity quantization or different approaches.

Holographic pixilation and horizon oscillations

In this section we apply the holographic principle [5, 6] to quantize the event horizon into pixels. Pixelizing the horizon is akin to placing a Planck grid onto it, where each cell (pixel) is of size $l_P{}^2$, where $l_P{}^2 = 2.59 * 10^{-66} cm^2$. We argue the holographic interpretation of the horizon combined with the redefined notion of singularity (see preceding section) give a generic method for producing Planckian-amplitude horizon oscillations which substitute the physical membrane in the complementarity conjecture [8]. Moreover, in the context of perturbation theory the proposed horizon oscillations lead to a remnant-free explosion of the hole due to thermodynamic instabilities in the final stage of the evaporation.

By performing a coordinate transformation we can map 3D space into 2D

(10) $$x^{\mu} = \begin{pmatrix} x^i \\ x^j \\ x^k \end{pmatrix} \rightarrow x^{\mu} = \binom{x^i}{x^j}$$

By that simple operation we project 3D bulk physics to a 2D holographic screen at a rate of bit per $l_P{}^2$. We define a *bit* as a binary degree of freedom or simply a two-state system which describes a classical unit of information. In the current framework, each pixel can host a single bit of information. The two states, that each pixel could be in, represented in terms of vectors have the form $\binom{1}{0}$ and $\binom{0}{1}$ for excited (on) and vacuum state (off), respectively. A one-pixel state is given by

(11) $$|\Psi> = (\tfrac{1}{2})(|on> + |off>)$$

Hence a pixel on a holographic screen can be either thermally excited or in a vacuum state. Each Planck cell is limited by the uncertainty principle and the cosmic censorship conjecture by the M_P. Therefore a surface of area A, consisting of N number of pixels can be interpreted as N-$state$ system. Thus a black hole with surface area A will be described as N-$state$ system which obeys the *Bekenstein-Hawking* area/entropy law in Planck units, $S = A/4l_p{}^2$, where $l_P{}^2 = G$. Therefore the number of different states the whole system can be in N, is given by the logarithm of the dimensions of the Hilbert space $\mathcal{H}$

(12) $$N = ln\, dim(\mathcal{H})$$

In a way, N is the minimum required number of bits to describe a given system. The horizon-related entropy S, measures the lack of information about the collapsed matter as far as an observer in the $r > 2M$ region is concerned. The fate of an infalling matter as it crosses the event horizon and approaches the singularity (dS core) is examined in Fig. 1.

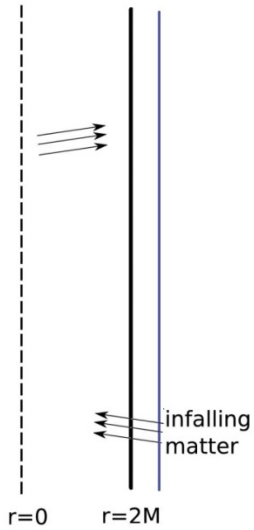

Fig. 1. Black hole in Eddington-Finkelstein coordinates. The dashed line (r = 0) is the singularity. The bold black vertical line denoted as r = 2M is the event horizon. The blue line indicates the position of the horizon as it oscillates. The amplitude is Planckian, thus the horizon oscillates a Planck length in outward direction from r = 2M.

We make a simple assumption which can be interpreted as a stronger version of Page's argument. The conjecture provides an even tighter security for the no-cloning theorem. In fact, one does not have to wait for half of the black hole's mass to have evaporated in order to be able to retrieve a single bit. Suppose we use classical black hole mechanics to describe the perturbations

(13) $$dM = \frac{\kappa}{8\pi}\,()dA + \Omega dJ$$

where κ is the surface gravity, Ω is the angular velocity and J is the angular momentum. We may define the no-back reaction conjecture as lack of perturbations on the metric due to quantum fields and fluctuations. As matter crosses the $r = 2M$ region (horizon) there will be no perturbations to the background metric. Such will be observed only after the infalling matter reaches $r = 0$, gets thermalized, and then gets embedded onto the holographic screen. Information no longer gets destroyed as it reaches the singularity due to its redefined nature. As we have argued in previous section the strong energy condition can be violated in the presence of significant gravity dynamics. Violation of the condition leads to repulsive gravity behavior by the gravitational field. Therefore, as infalling matter hits the singularity it gets thermalized (vacuumized) and reflected back towards the horizon where it will be embedded. The time for the thermalized matter to get embedded onto the horizon is of order the scrambling time t_s, where $t_s = R\,ln((R/l_P)/(/))$. Therefore in no scenario can the linearity of quantum mechanics be violated. An integral part of the black hole formation/evaporation process is the transformation of fine-grained degrees of freedom into coarse-grained degrees of freedom and vice versa. Moreover as the vacuumized (high energetic) matter gets embedded onto the holographic screen, it takes it out of its vacuum state, and thus causes the Planckian oscillations. The frequency of the oscillations obeys the simple relation

$$(14) \qquad \omega = \sqrt{\frac{-T_{\mu\nu}}{M_{BH}}}$$

where $T_{\mu\nu}$ is the energy/momentum tensor (Hawking radiation) and M is the mass of the black hole. The minus sign indicates the black hole adiabatically loses mass. Small value of $T_{\mu\nu}$ would imply the hole is massive. This follows from the straightforward relation $T = 1/M$. Thus, a massive black hole would oscillate with lower frequency. Another way of stating the above conjecture would be

$$(15) \qquad \frac{d^2A(t)}{dt^2} = -T_{\mu\nu}$$

The oscillations do not lead to violations of the equivalence principle because of their Planckian amplitude and relatively low frequency throughtout the black hole evolution. They can only become noticible at the final stage of the evaporation process when the whole system becomes thermodynamically unstable. An observer close to the horizon will measure classical Unruh vacuum with no deviations. In fact, until the black hole reaches its lower mass bound the stress tensor will not diverge. However, as $T \rightarrow M_P$ the black hole explodes due to the present instabilities. Hence no timeslice with drama for the observer can be constructed.

We argue that the physical membrane in black hole complementarity can be substituded by the natural oscillations of the horizon. When a black hole is described by a distant observer the entropy of the hole S_{BH}, is viewed to arise from the fine-grained degrees of freedom on the stretched horizon. As it has been shown in [8] the stretched horizon is located a l_p away from the event horizon in outward direction. Therefore, each point from the event horizon is projected (corresponds) onto the membrane. Thus, the entropy of the global horizon equals the entropy of the stretched horizon, $S_{horizon} = S_{stretched} = A/4l_P{}^2$ which obeys the Bekenstein/Hawking entropy bound. Since there is equality between the degrees of freedom of the global horizon and the stretched horizon, we argue the oscillations conjecture omits the need for a membrane. Hence an observer at infinity can falsely interpret the horizon oscillations as a physical membrane located just outside the $r = 2M$ region.

We further suggest the generic oscillations can easily lead to a complete evaporation. As the black hole loses mass, in accordance with (14), the frequency of the oscillations ω and thus its temperature T, will monotonically increase. As we have already argued the proposed scenario does not lead to violations of the equivalence principle. However, as T becomes of order M_P, the whole system becomes thermodynamically unstable and the black hole explodes leaving no remnant behind. This view was first suggested by Hawking concerning primordial black holes of approximate mass $\sim 10^{15} g$ produced by quantum fluctuations in the early Universe [11]. The basic calculations behind particle production in curved spacetime are also provided. The proposed final-stage explosion releases in an instant the remaining of the information. It should be noted there is nothing that prevents complete remnant-free evaporation from occuring. The contemporary models fail to address the question of how to evaporate the singularity without presenting remnants which are equally troubling. The proposed remnants, stable or quasi-stable, must be able to carry infinitely many states so they can reproduce the initial quantum state of the matter that has collapsed to form a black hole. The amount of entropy the remnants should be able to carry is of order $M^2/M_P{}^2$ in Planck units, where M is the mass of the collapsing matter. In the proposed picture, however, since we have given a mechanism for quantizing the singularity region (removing the infinities), we have cured the pathological problems, hence there is nothing, in principle, to prevent the $r = 0$ region from evaporating.

Black hole evaporation and information storing

In the present Section we examine black hole evaporation and discuss a possible way for storing the information of an infalling matter.

The evaporation process is due to the strong gravitational dynamics acting on the quantum fields which thus leads to amplification of the energy density of the quantum fluctuations. Hence outgoing modes are radiated to infinity. The calculations suggest the emitted Hawking particles are purely thermal with black body thermal spectrum of $T = (\kappa/2\pi)$, which implies they do not carry information about the initial quantum state of the infalling matter [12]. The thermal radiation depends solely on the fixed background metric of the hole. It is in no way correlated with the interior microstates. As matter, in a certain quantum state $|\psi>$, falls in, the Hilbert space $\mathcal{H}$, partitions into two independent Hilbert spaces for the interior and exterior region of the black hole, given respectively by $\mathcal{H}_{in}$ and $\mathcal{H}_{out}$, which are uncorrelated.

(16) $\quad \mathcal{H} \rightarrow \mathcal{H}_{\text{in}} \otimes \mathcal{H}_{\text{out}}$

Because causality/locality limits the correlations between the independent Hilbert spaces, the outgoing modes are of thermal spectrum.

We have so far only addressed one of the aspects which we believe every potential solution to the information loss problem should tackle, namely the geodesic divergence in the $r < 2M$ region, and have only partly touched on the second issue - establishing T-symmetry between the $\mathcal{J}^-$ and $\mathcal{J}^+$ regions, Fig. 2.

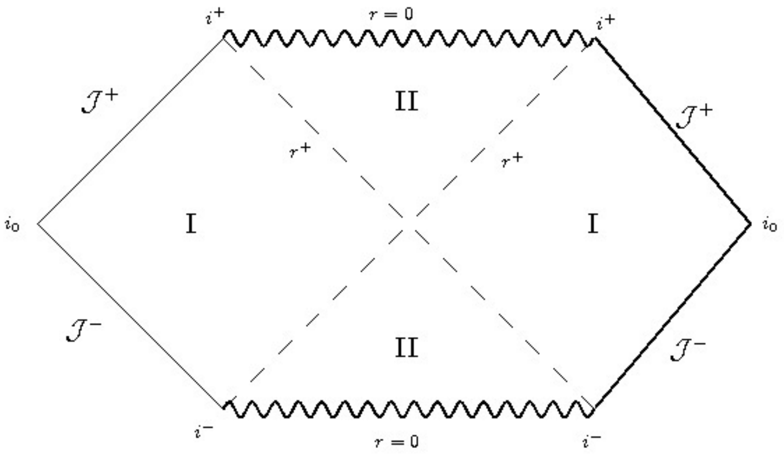

Fig. 2. Penrose diagram for the time-reversibility between $\mathcal{J}^-$ and $\mathcal{J}^+$

As it has been recently argued the uniqueness of the vacuum might be an effective field theory [13, 14]. Thus any non-local fundamental framework reduces to classical local quantum field theory in low-energy regimes. Tracing radiation back from I^+ to the source (horizon) should not result in high-energetic excitations in the near-horizon region. Instead, one finds slight deviations from the unique Minkowski vacuum. Therefore there must be various states, locally indistinguishable, all energetically degenerate. In the provided framework a semiclassical black hole consists of many vacuum states $|\psi_n>$ where $n = 1, 2, 3... A/4$ in Planck units. The number agrees with the Bekenstein-Hawking area/entropy bound. In fact, vacuum degeneracy implies the number of different vacua in the region surrounding the black hole is given by the exponential of the Bekenstein bound

(17) $$|\psi n > = exp[A \ / \ 4l_P^2]$$

where $A = 16\pi M^2 l_P{}^4$. As far as an infalling observer is concerned the vacuum in the vicinity of the horizon is unique. Moreover the equivalence principle (no drama) holds and an accelerated observer should not encounter any deviations from the Unruh vacuum. Furthermore non-uniqueness of the vacuum allows for the possibility of information to be stored on it and later be encoded in the outgoing

Hawking radiation. Hence the degenerate vacuum surrounding the black hole should be able to store information without getting energetically excited.

That being said, we argue that as matter falls inside a black hole it leaves or imprints the information regarding its initial quantum state onto the degenerate vacuum. This should leave the classical perturbations given by Eq. (13) unchanged. Therefore, as the black hole radiates thermal Hawking particles, they "pick up" information from the vacuum. In principle, information about infalling matter could be found in the emitted Hawking radiation. However, description of the exact mechanism by which the quanta "pick up" the information requires a working theory of quantum gravity, and it appears to be beyond our scope as of now.

The proposed mechanism establishes time-symmetric (T-symmetry) map between $\mathcal{I}^-$ and $\mathcal{I}^+$. The two regions become completely identical. All of the information at I^- can be obtained from I^+ and vice versa.

Conclusions

We have provided a model which addresses both of the issues that contemporary resolution proposals suffer from. Namely, the geodesic divergence (incompleteness) and T-asymmetry between $\mathcal{I}^-$and $\mathcal{I}^+$. By using classical non-commutative geometry we are able to cure the pathological problems of the singularity region. The framework also redefines the singularity by attributing repulsive gravity features to it. Also we get rid of infinities associated with $r = 0$. In the proposed picture information no longer gets destroyed as it reaches the singularity but rather gets thermalized (vacuumized). By combining this proposal with the holographic description of the horizon we are able to present a phenomenon which, we believe, arise naturally. The conjectured Planckian-amplitude horizon oscillations not only account for the physical membrane proposed in black hole complementarity, thus omitting the necessity of its very existence, but also take care of the late-time evolution of the system by leading to final-stage explosion which does not leave remnants behind. The put-forward vacuum degeneracy surrounding the black hole stores the information about infalling matter. This way the emitted particles are of thermal spectrum and "pick up" the information, hence get certain quantum corrections, if you will. Therefore we make the $\mathcal{I}^-$ and $\mathcal{I}^+$ regions identical (time-symmetric) and preserve the unitarity of the S-matrix.

References

1. Hawking, S.W., Breakdown of Predictability in Gravitational Collapse, Phys. Rev. D 14, 2460 (1976).
2. Maldacena, J. M., The large n limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. 2 (1998) 231–252, hep-th/9711200.

3. Gubser, S.S., I.R. Klebanov, and A.M. Polyakov, Gauge theory correlators from non-critical string theory, Phys. Lett. 1998, B428, 105–114, hep-th/9802109.
4. Witten, E., Anti-de sitter space and holography, Adv. Theor. Math. Phys. 2 (1998) 253–291, hep-th/9802150.
5. Hooft, G. The Holographic Principle, arXiv:hep-th/0003004v2.
6. Bousso, R., The Holographic Principle, Rev. Mod. Phys. 2002, 74, 825–874.
7. Banks, T., Holographic space-time and its phenomenological implications, Int. J. Mod. Phys. 2010, A25, 4875–4887.
8. Susskind, L., Thorlacius L., Uglum J. The stretched horizon and black hole complementarity. Physical Review D: Particles and Fields. 48, 3743–3761. PMID 10016649, DOI: 10.1103/PhysRevD.48.3743.
9. Nicolini, P., Anais Smailagic, Euro Spallucci, Noncommutative geometry inspired Schwarzschild black hole, DOI: 10.1016/j.physletb.2005.11.004.
10. Arraut, I., D. Batic, M. Nowakowski, Maximal extension of the Schwarzschild spacetime inspired by noncommutative geometry, arXiv:1001.2226v1 [gr-qc].
11. Hawking, S.W., Black Hole Explosions? Nature. 1974, 248, 30–31; DOI: 10.1038/248030a0.
12. Hawking, S. W., Particle creation by black holes, Phys. 1975, 43, 199–220.
13. Hossenfelder, S., Comment on the black hole firewall, arXiv:1210.5317v1 [gr-qc].
14. Hawking, S.W., M.J. Perry, A. Strominger, Soft Hair on Black Holes, Phys. Rev. Lett. 116, 231–301.

ТЕРМОДИНАМИКА НА ЧЕРНИ ДУПКИ КАТО ПРИЧИНА ЗА ТРЕПТЕНИЯ НА ХОРИЗОНТА

А. Йосифов, Л. Филипов

Резюме

Представяме модел за решение на информационния парадокс при черни дупки, който запазва унитарността на S-матрицата. Подходът ни се базира на квантизиране на сингулярността, използвайки класичекса некомутативна геометрия. Също така използване напредъка в опитите за унифициране на квантова механика и гравитация, за да представим холографско описание на хоризонта. Тоест, квантизираме холографския екран (хоризонт) на индивидуални „пиксели“, с размер на Планк, всеки от които носи по един бит информация. Предлагаме класически феномен, който произтича от пертурбационна теория за черни дупки и води до безостатъчен финален етап, като също така и премахва необходимостта от представяне на допълнителна физическа мембрана, както бе предложено от Съскинд. Предлага се и по-добра версия на аргумента на Пейдж, отнасяща се до теоремата за клониране на квантова информация.