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FRACTAL APPROACH TO SPACE STRUCTURES, OBJECTS AND DYNAMIC PHENOMENA

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Abstract

This paper presents the fractal theory and the definition of a fractal. It dwells on the basic features of abstract and natural fractals, as well as on the mathematical instruments used to describe these sophisticated, complex and nonlinear shapes and phenomena in the Universe. The study examines some of the earliest attempts to apply the fractal theory in astrophysics and particularly the processes related to gravitational singularities, the shape of spiral galaxies and the dynamic organization and structure of the Universe.

1. Introduction

In his visionary work the American scientist, Benoit Mandelbrot, opened new vistas in mathematics and all sections of physics by discovering the fractal set of geometric shapes. In the preface to the second revised edition of his book, titled *The Fractal Geometry of Nature*[1], he pointed out that he was in search for a new natural geometry, which would not leave out the huge diversity of "irregular" shapes to be seen everywhere in the physical reality, but would rather describe and include them in mathematical studies. This had to be done, because "Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line." He considered his mathematical research both a philosophic and aesthetic drive to a more complex and profound understanding of the forms and processes in reality. The fractal theory has been widely used in the pursuit to attain a new understanding of the complex nonlinear and dynamic processes in astrophysics, geophysics, and even in the sciences examining the human body, brain and consciousness. Since mathematic studies in fractal geometry and the practical application of these studies in specific sciences started only after 1975, there are still many areas, as well as space phenomena and processes, which can benefit substantially from the application of fractal analysis.

2. Purpose

The present paper is an attempt to present the fractal theory and some of the early experiments to apply it to astrophysics. The main point is to approach the widely spread nonlinear dynamic processes in the Universe with the latest mathematical descriptive tools and furthermore to perform a structural analysis of the existing shapes and objects in space.

3. Results and Discussion

3.1. Fractal – definition and characteristics

If we are to use one of the most general definitions, *fractal* is an object which is *self-similar* in one way or another[2]. From a strict mathematical point of view this self-similarity is relevant only to the structure or shape of the object, however, from a more abstract and philosophical point of view a fractal can be considered in a more universal aspect – every time there is a complex *self-similarity, manifested in a single hierarchically structured whole*. Mathematically the fractal refers to equations related to *interation, recursion* or any form of *feedback*.[3] Mathematics and informatics define iteration as algorithmic repetition of any process or function, and recursion – as a particular type of functions referring to themselves or put in other words each member of a recursive function is derived from previous members of the same function. In turn the self-similarity of fractals can be realized by an iterative, recursive or stochastic mathematical method. Fractals of exact self-similarity are most often realized by an iterative method; the quasi-self-similar fractals are usually realized by recursive functions, while the accidental fractals exhibit numeric or statistical characteristics persistent in certain proportions. From a mathematical point of view fractals have the following characteristics[4]:

- The whole projects and manifests itself in itself – each part of the whole is similar to the whole, whereas the self-similarity can be exact, approximate or stochastic;

- Abstract fractals organized mathematically have a fine pattern structure in random small scale, and naturally generated fractals have a self-similar structure and properties in broad but still limited scale;
- Rather high irregularity, difficult to describe using the standard Euclid's geometry language;
- Non-whole (Hausdorff) dimensions which as a rule are bigger than the topological dimensions;
- Simple and recursive definition;
- The so called natural dynamic fractals exhibit a characteristic typical for the complex dynamic systems – self-organisation and natural emergence, which occurs because of the very essential properties of the fractals, and also because of the fact that a form of cyclic causality generates fractality.

3.2. Mathematical definition of fractal structures

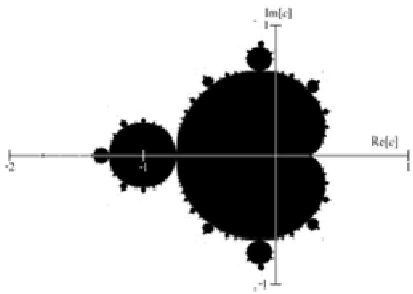

Fig. 1. Graphic image of the Mandelbrot Set [5]

If one examines *the Mandelbrot set* with the purpose to clarify the mathematical aspect of the fractal concept, one will arrive at the conclusion that this set is identified as a fractal determined by a set of points C in the complex plane characterized by the following iterative sequence:

$Z_0=0; \quad Z_{n+1}=Z_n^2 + c$

so that this sequence does not converge to infinity. The several opening iterative lines look like this:

if $c = X + i.Y$, where X and Y are real numbers, and i is the imaginary number $\sqrt{-1}$ and $Z_0 = 0$, then:

$Z_1 = Z_0^2 + c = X + i.Y$

$Z_2 = Z_1^2 + c = (X + i.Y)^2 + X + i.Y = X^2 - Y^2 + X + (2.X.Y + Y).\ i$

$Z_3 = Z_2^2 + c = \ldots$

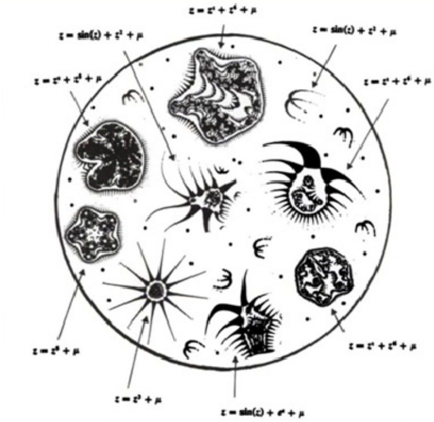

Fig. 2. Biomorphic fractal patterns of a quadratic functional definition different from the classic one[6]

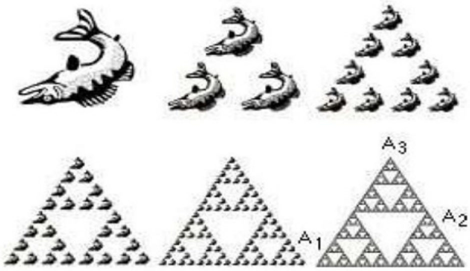

Fig. 3. Serpinski's triangle constructed by multiple reduction of image prototype[7]

Graphically this set is displayed by a complex coordinate system and by taking into account whether a certain point in the coordinate system belongs to the Mandelbrot set or not (*Fig. 1*). The point is that various functions can be placed in an iterative sequence and their *Mandelbrot sets* can be obtained. Put in other words different functional models can be placed in the specific mathematical feedback (the iterative sequence) and this will construct different fractals. *Fig.2* shows fractal forms quite similar to the forms of living organisms or to forms existing in the Universe. The particular functional equations of the displayed fractals are as follows: $z=z^3+\mu$; $z=z^5+\mu$; $z=z^3+z^5+\mu$; $z=\sin(z)+z^2+\mu$; $z= z^3+z^6+\mu$, etc., where μ is constant.

There are other types of fractals not related to complex numbers. One of the commonly quoted classic examples of a non-complex, iterative fractal is Serpinski's Triangle S (ref. *Fig.3*). It is worth noting that when constructing this fractal, during the first recursive steps, it has a rather rough and formless structure, but the progressing recursion results in a gradual perfection of each detail of the fractal. Thus original rough properties are shaped by recursive construction, and are transformed into fine patterns. One can trace the mathematical formalization behind this special property of fractals. The S set has three components each of which is a reduced image of S. Each of these components has three sub-components thus resulting in a nine-fold reduced image on the next level and this goes on infinitely. A simple observation of this iterative fractal leads to the functional correlation: $S = f_1(S) \cup f_2(S) \cup f_3(S)$, where f_1, f_2, f_3 are reductions of the S space by a factor of ½ , and the three main components of S are as follows: $f_1(S)$, $f_2(S)$, $f_3(S)$. This definition is applicable to any set of objects. If the collection of prototype cards is $F = (f_1, f_2, f_3)$, referred to as *Iterative functional system*, then in respect of each set $F(T) = f_1(T) \cup f_2(T) \cup f_3(T)$, it can be evidenced that S is of core significance: by starting with a certain compact set T_0 and $K \geq 1$, we can describe recursively $T_\kappa = F(T_{\kappa-1})$; then $T_\kappa \rightarrow S$ with $K \rightarrow \infty$, in other words T_κ tends to *Serpinski's triangle S* in Hausdorff dimensions with K tending to infinity and at that independently from the type of the initial set T_0 (ref. *Fig.3*, where *Serpinski's triangle* S is realized, made of a set of fishes). Therefore, S is referred to as a *fractal set attractor* of the iterative-functional system, described by the equation: $S = f_1(S) \cup f_2(S) \cup f_3(S)$, and the initially rough form, from which the fractal structure gradually moves away systematically by each new step of the iteration, is referred to as a *fractal set repeller* [8].

For the sake of clear and unambiguous visualization, we shall write down the following expression:

$$FR(patern/function) \xrightarrow{\text{iteration}} Attractor$$

The presented notation should be interpreted as follows:

Fractality with a certain pattern and/or with a particular mathematically definable function, after a certain number of iterations, transforms into its attractor.

If the *Mandelbrot set* is examined as attractor and it is denoted as M, then its complex fractality may be displayed in the following way:

$$FR(Z_{n+1}=Z_n^2 + c) \rightsquigarrow M.$$

And if *Serpinski's triangle* is examined under this scheme, as a typical non-complex fractal, with its S, the result will be as follows:

$$FR(\ f_1(S) \cup f_2(S) \ \cup f_3(S)\ ) \rightsquigarrow S.$$

In a more generic way it can also be expressed as follows:

$$FR(Triplicity\ ) \rightsquigarrow S,$$

because as was already shown each image, characterized by triplicity, can become a pattern of this fractality and according to the above-mention iteration method can be transformed into a fractal attractor – *Serpinski's triangle.*

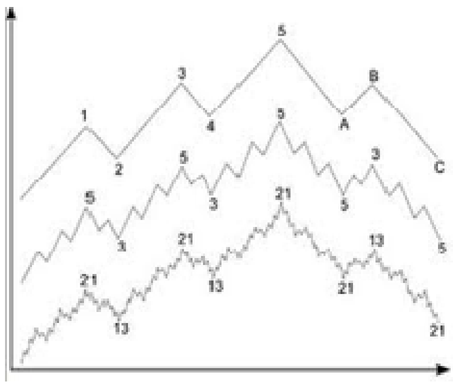

Fig. 4. Fractality of the stock exchange index

The main property of a fractal – its self-similarity, can be used to calculate its dimensions[9]. If we use simple geometric figures and if we create a degree of self-similarity within them, e.g. if a segment is increased twice, we will obtain two copies of the segment; if a square is increased twice, four copies of the first square will be obtained; if a triangle is increased twice again 4 copies of the first triangle will be obtained; if a cube is increased twice 8 copies of the first cube will be obtained. On the basis of these examples the following can be argued: if D is dimensions, K is the rate of increase, and N is the number of the obtained identical figures, then the following correlation is valid: $K^D = N$. Hence D is derived from the following formula: $D = \log N / \log K$. If this formula is applied to *Serpinski's triangle* with the already used random set of fishes, it can be seen that upon the first iteration of self-similarity 3 identical images are obtained ($N = 3$), each of which is 1/2 of the prototype, hence $K = 2$. Then the Hausdorff dimensions of the fractal called *Serpinski's triangle* are: $D = \log 3 / \log 2 = 1.5849625$.

Then again in his subsequent works Mandelbrot goes on to discuss the so called *multifractal*. What is meant is a fractal structure with more than one base pattern. In other words not only singular fractals (fractals with a single pattern) can be observed in reality, but also multifractals – complex fractal structures with more than one functional patterns. An example of such a naturally generated, but already mathematically abstract fractal is the functional trajectory of the stock exchange movements. It has for a long time been established, on the basis of the so called *technical analysis*, that there is a fractal similarity in the charts of stock exchange indexes between the different time intervals which are visualized (*Fig. 4*). The daily chart of the EURUSD index may be constructed by several iterations of the model set out by the chart of the same index, but visualized subsequently on a 4-hour or 15-minute chart. In this case it can be noticed that different patterns evolve: W-image, N-image, И-image or V-image and so on. Indeed, these are the so called different patterns of the multifractal, developing in the form of the particular stock exchange index. According to the above mentioned more general notation any particular multifractal may be expressed in the following way:

mFR(W-патерн, N-патерн, И-патерн, V-патерн...) $\rightsquigarrow$ EURUSD, Which is to say that the EURUSD is a *multifractal* (this is denoted by *m*), obtained by iterating the following patterns: W-pattern, N-pattern, И-pattern, V-pattern and so on.

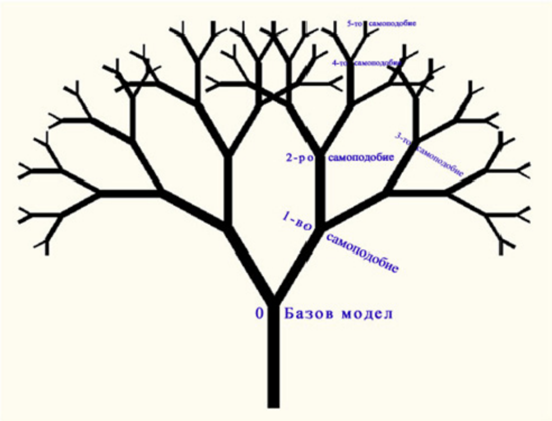

Fig. 5. Degree of self-similarity of natural fractals

According to the author of this paper, *when analyzing the naturally originating fractals, one specific new value may also be introduced – degree of self-similarity*. *Fig.5* shows the typical form of the natural fractals of trees. It reveals how the basic pattern – *fork (Y)* is realized in this particular tree-like structure by *degree of self-similarity 5*. This whole-number positive variable in fact indicates how many hierarchical levels there are in a particular natural fractal and also how many degrees of recursive self-reference were realized through the basic pattern in order to achieve the structure of the particular object. If a certain natural fractal structure is multifractal, it will exhibit several degrees of self-similarity, each one manifested at different hierarchical depth. If in this particular case of a non-complex fractal we denote its attractor by Y, the result will be the following:

$$FR(f_1(Y) \cup f_2(Y)) \rightsquigarrow Y,$$

or put in a more general way, but anyway expressed adequately, indicating the existence of a *natural fractal of a specific degree of self-similarity* 5 we will obtain:

$$nFR(\mathrm{fork}_5) \rightsquigarrow Y.$$

Here the sign *n* must be understood exactly in this way: *there exists a natural fractal*, and *index* 5 of the *fork* pattern stands for *the degree of self-similarity* of the particular fractal in *Fig.5*. The potential application of the fractal analysis and descriptive method to astrophysical objects and to the objects distribution structure in the Universe will be demonstrated on the basis of fairly easy to understand mathematical definitions.

3.3. Fractality of the natural forms in the Universe related to waves, convection and turbulence. Elaboration of the fractal analysis

Fig. 6. Naturally generated fractal patterns

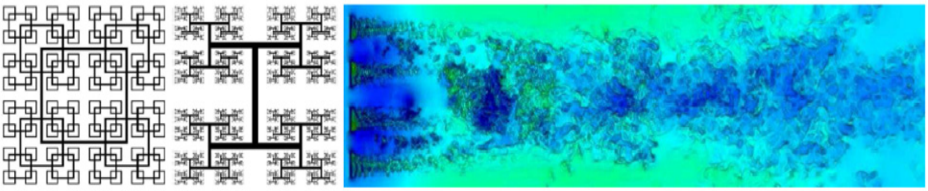

Fig. 7. Fractals and turbulence

The wave process was contemplated in the environment of its natural emergence and its self-similarity was noticed as early as ancient times (ref. *Fig. 6*). The big ocean waves are made of smaller waves, which in turn are further furrowed in waves. The same process is to be observed in the dessert too, where the gigantic sand dunes have a wave-like structure in several levels. If the convection flows of matter are observed for instance in the clouds of the Earth or Jupiter or on the surface of the Sun, fractal patterns are easy to discern in their intensive turbulence. One of the important new ideas in the fluids science is the fact that turbulence is modelled through fractal patterns (*Fig. 7*).[10] Each natural movement of fluid in space or earth environment has, generally expressed, two states: laminar flow and turbulent flow. Under certain conditions the laminar flow having parallel current lines naturally transforms into a turbulent flow. Turbulence structures are similar at various space levels of observation and here again what can be seen is a dynamic fractality. Turbulence causes the imaging of vortex cores varying in scale which represent the basic dissipative mechanism[11] for the transmission and dissipation of energy in the flows. The big vortex cores start the fractal expansion process and at the same time they dissipate energy, causing turbulence of a lower magnitude, and it is exactly in this way that fractality is realized at smaller hierarchical levels in the whole of the fractal. This process evolves structurally and after a certain "life"-time and stability it starts gradually to subside (if no more energy enters the system) by the process of dissipating energy to the intermolecular level.

However, if energy keeps entering the system where the turbulence occurred, fractality is stabilized and this is easy to observe as a stable turbulence picture. The following notation can be produced:

$$FR(\text{Vortex core}) \rightsquigarrow \text{Turbulence} \rightarrow \text{Energy dissipation}$$

According to the author of this study in this way a conclusion can be reached that matter, manifested in different physical states, is entirely penetrated by fractality, i.e. actually everywhere in the Universe matter is organized on the basis of self-similarity generated by cyclic causality. Using the above example, taken from nature, demonstrating by turbulence not only single cyclic vortexes, but also a structural texture of fractally recurrent vortex cycles, which transmit and dissipate energy, the scope of the causal analysis of the examined processes can be significantly expanded.

The causal fabric of the Universe is not composed only of simple, complex and cyclic causal links, but also of vortex causal processes which are essentially fractals. In other words the cyclic causal link as one singular closed cycle transforms into a nucleus and pattern of a causal vortex-fractal, which means that the following notation can be formulated:

$$FR(\text{Cyclic causality}) \rightsquigarrow \text{Causal fractal} \rightarrow \text{Energy dissipation.}$$

Moreover, the presented example of the turbulence process evidences that the emergence and transformation of the dynamic fractality in the Universe passes through four stages:

(i) *emergence* – natural switching from non-fractal (i.e. non-self-similar structure) into fractal structure or put in other words the beginning of the self-organisation of the fractal process, at which there is already one degree of self-similarity;

(ii) *evolution* – unfolding of the fractality pattern to ever smaller hierarchical levels in the fractal totality or put in other words, fractal „growth” during which the degree of self-similarity increases;

(iii) *subsiding* – discontinuation of the realization of the fractal pattern to lower levels in the self-similar hierarchy and gradual fading away of entire hierarchical branches of the fractal, where the degree of self-similarity starts decreasing;

(iv) *liquidation* – the fading away of different hierarchical levels of the fractal reaches a critical phase which causes the self-similarity in a given structure to disappear, i.e. the degree of self-similarity is equal to zero.

It is important to note that while there is a strong and constant causal impulse bringing in a steady inflow of energy, the fractal is evolving and trying to attain its attractor – its proportional and hierarchically self-similar manifested form and structure, which is simultaneously a balanced and steady energy state of the system. Conversely, it has to be emphasized that the above described four stages in the evolution of the dynamic fractal should not be examined only in single-model fractal structures. These are to be seen in the so called *multifractals*, whereas with them it is possible one pattern to be at the emergence and evolution stage in the overall structure of the multifractal, and another pattern to be at the subsiding and liquidation stage, at that the dynamics of the first pattern may and may not be contingent on the dynamics of the second one.

3.4. Application of the fractal approach in astrophysics

3.4.1. Holography, structural fractality and the holographic principle in astrophysics

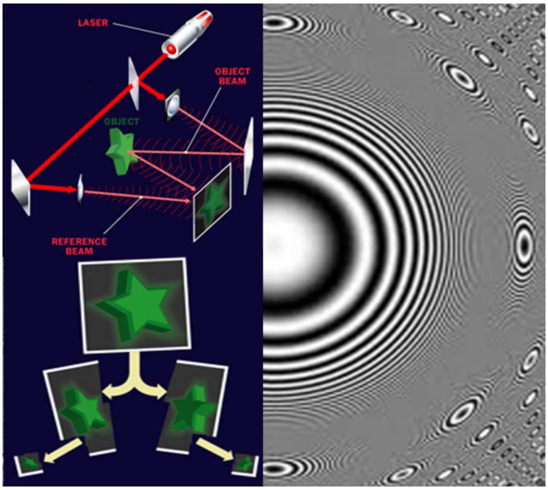

Fig. 8. Hologram flow chart. Retaining the model of the whole upon breaking a holo-plate[12]

In case a demonstration is needed on how fractal structures evolve naturally as a result of a certain cyclic-causal process, then the holographic phenomenon seems to be the best option. When a particular wave phenomenon is included in a cyclic causal link to itself, e.g. a ray of coherent light, it is split into a primary ray and a referent ray, and while the two rays are interacting between each other interference occurs that can cause a fractalised record of the environment where the interference took place. In the holo-process the referent ray interferes with the primary ray and the latter has been dissipated repeatedly under different angles by a particular object. The interesting point with this instance of wave cyclic causality is that fractality can be detected not only in the fixed interference picture itself, usually recorded on a holographic plate, but also in the very particle structure of this picture – no matter how many pieces the holographic plate is broken into, it retains the model and information recorded in the whole plate anyway (ref. *Fig.8*). Thus a kind of fractality is reached, which is not based on the geometry of forms, but on the very internal structure of the object. If the plate is exposed to a suitable laser beam, a 3D holo-image will emerge out of it because the interaction between the laser beam and the fractally structured plate will create a cascade of a huge number of light refractions, as a result of which a 3D image appears, which was recorded initially on one 2D object – the holo-plate. Thus it becomes clear that the holographic setup can be used to record and present the entire information concerning the shape of a n-dimensional object on (n-1)-dimensional plate. This in contemporary theoretical physics and astrophysics appears to be the so called *holographic principle*, which was first formulated by Hooft and Susskind[13,14]. This principle was inspired by the black holes thermodynamics and the insight with them is that the information contents of all objects that have fallen into a particular black hole may be entirely located in the surface fluctuations of the gravitational singularity event horizon. Hence the theory assumes that the entire Universe can be seen as a 2D information structure "displayed" on the cosmologic horizon in a way that the three dimensions that we observe are only an effective description of the macro level. Susskind applies the holographic principle and the fractal ideas to substantiate the so called *Principle for information conservation in the Universe* – a concept derived from astrophysics which is of crucial significance in modern science.

3.4.2. Fractal approach to gravitational singularity

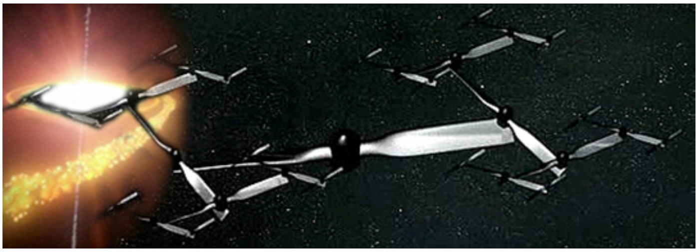

Fig. 9. A still from the film „Through the Wormhole. The Riddle of Black Holes"- 16.06.2010, visualising Susskind's ideas

In his book *The Black Hole War*[15], as well as in his interview presented by Morgan Freeman in the *Through the Wormhole. The Riddle of Black Holes* series, Susskind demonstrated the application of the fractal approach to the gravitational singularities. In scientific circles *The Black Hole War* is understood to stand for the long lasting scientific dispute with Hawking where Susskind presents witty thought experiments and mathematical constructs in order to show that no information about the cause-and-effect processes is lost at the physical level even at the black hole boundaries. After a series of twists and turns Susskind nonetheless “won” the war and Hawking accepted his arguments. By a thought experiment examining the different points of view of observers moving in reference to a black hole, Susskind demonstrated the fractal structure of matter, and its transformation into an image on the holographic plate covering the entire surface of a given black hole. In order to visualize his idea he described a dynamic fractal with a moving propeller of an airplane falling into a black hole (ref. *Fig.9*).

The conclusion that can be drawn is that a fractal structure, similar in form to the fractality on the holographic plate, appears on the surface of the gravitational singularity; this fractal structure is a record and a dynamic reflection of the entire reality. This situation visualizes through *the holographic principle* how the macro-cosmic structures in the Universe, which have distinct fractal characteristics, touch, determine and interact with the fractal structure of the holographic patterns shaped on the event horizon of the black hole by the specific radiation of Hawking[17,18], emanating from this gravitational singularity. According to theoretic physics virtual particles and anti-particles exist in the texture of the Universe and they are constantly emerging and disappearing. They appear on the basis of a particular emitted quantum of energy and afterwards they re-merge and re-emit the initial quantum of energy. This process is not disrupted in the absence of gravitational singularity, but in the presence of gravitational singularity it is possible for a particle after emerging to sink into the black hole and its antipode to break free from the gravitational vortex. This is exactly what produces Hawking’s radiation, which according to Susskind realizes the information conservation law, also under these extreme conditions.

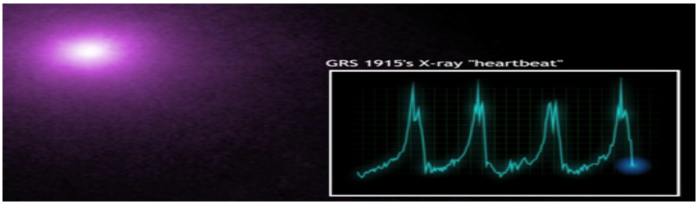

Fig. 10. The "heart pulsation" of a black hole[16]

Nonetheless, fractal processes taking place on the boundary of gravitational singularity may be accessed and perceived in their reality by our senses as well. The peak astrophysics facility – the Chandra observatory – provides opportunities to investigate X-ray radiation in deep space including radiation coming also from systems with probable black holes. Thanks to this facility the American scientists have detected a specific pulsation generated by the self-regulating feedback system of jet and the accretion disk of a black hole about 14 times the Sun's mass, also known by the name of: GRS 1915+105.[19] It turned out that it's not only the supermass black holes in the centre of galaxies that have a self-regulatory cyclic-causal mechanism[20], but black holes of relatively normal size also possess such a mechanism. Exactly this regulatory mechanism is the cause of the "heart pulsation" of GRS 1915+105[21](ref. *Fig.10*). It has a typical fractal form and bears a close resemblance to the proven fractal morphology of the electrocardiogram of the human heart pulsation. This fractality may be represented graphically or in audio form[22], however, it is essentially part of the fractal process taking place on the boundary of gravitational singularity.

3.4.3. Fractal approach to the structure of spiral galaxies

Fig. 11. Fractal spiral structure quite similar to the structure of spiral galaxies.[23]

There is another example of fractal approach to astrophysic structures. This is easy to do by computer simulation of vortex movements and shapes similar to the ones visible for instance in our Galaxy (ref. *Fig. 11).* It is evident that matter in the Universe is distributed fractally: hundreds of milliards of stars, grouped together, form a galaxy, groups of galaxies are attracted to one another and form galaxy clusters, and these merge and form super-clusters.[24] What matters here is the fact that the different or similar levels of these structures, often interact among each other thus realizing different phenomena of a causal-cyclic nature. For example a particular energy radiation of our Galactic centre may totally re-structure the spiral organization of our Galaxy or of any particular star system – this is an interaction between two different hierarchical levels in the fractal structure of the Milky Way. Nonetheless the different star systems influence one another so in some cases this leads also to the birth of new solar systems – e.g. a particular star explodes and compresses a gas cloud up to an adequate condition required for the subsequent birth of a solar system – this is an interaction between two similar hierarchical levels in the fractal structure of the *Milky Way*.

3.4.4. The Universe as a self-reproducing fractal – the theory of Linde, Penrose and Gurzadian

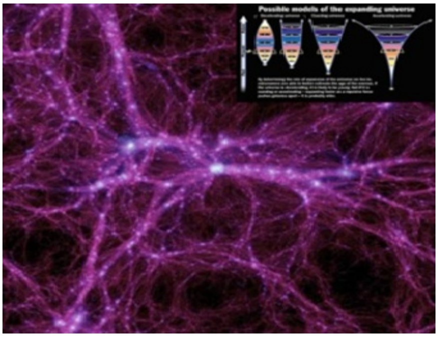

Fig. 12. Fractal pattern of the Universe – cosmic net[26,27]

It was mentioned in the foregoing section that the Universe represents a macrocosmic fractal structure. This is due to the fact that the fractal forms have an exceptionally wide distribution in the Universe. This can be noticed not only in the observed levels close to us, but also in its grand structure appearing at the level of super-galactic clusters[25](ref. *Fig.12*).

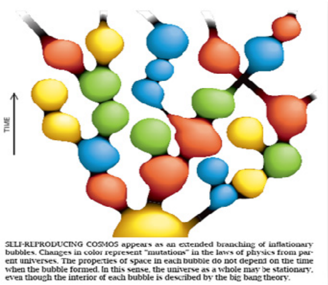

Fig. 13. Visualized fractal structure of the Multiverse: published in Scientific American - "The Self-Reproducing Inflationary Universe", A.Linde [29]

Some basic questions can be asked: *why is there such a densely saturated dissemination of fractal forms in the Universe and why they emerge in the first place*? This inevitably leads contemporary cosmologists to fractal scenarios of the global evolution and organization of the Universe. The simplified idea of the so called *Big Bang* has been put to intensive criticism because it cannot explain a number of phenomena observed in the astrophysics and physics of elementary particles, including the fact that seen on a grand scale the visible space exhibits clearly discernible fluctuations in the distribution of matter, easily definable by fractal geometry. Therefore Andrej Linde, professor in Physics at Stanford University, proposed to examine the Universe as a constantly evolving self-reproducing fractal[28]. This concept developed on the basis of the so called *inflationary universe scenario*, directly relevant to its accelerated inflation, assumes that the Universe consists of multiple big explosions each representing a different sub-universe with specific laws of physics. According to Professor Linde, however, the entire structure is united in a self-reproducing tree-like fractal (ref. *Fig.13*). This was substantiated by Penrose and Gurzadian in 2010 when they announced that structures were discovered in the cosmic microwave background which are most probably related to events preceding the Big Bang.[30]

4. Conclusion

This paper demonstrated the mathematical tools of the fractal theory. Thanks to the introduced notation of the fractal processes and the mathematical concept of the *degree of self-similarity* the complex dynamic processes in the Universe, as well as their trajectory of appearance, can be analysed structurally. This is to pinpoint the significance of the fractal theory in the field of astrophysics and in the investigation of the nonlinear properties and manifestations of some key objects and processes in space.

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ФРАКТАЛЕН ПОДХОД КЪМ КОСМИЧЕСКИ СТРУКТУРИ, ОБЕКТИ И ДИНАМИЧНИ ЯВЛЕНИЯ

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Резюме

Представя се фракталната теория и определението за това що е фрактал. Показани са основните свойства на абстрактните и естествени фрактали, както и математическия инструментариум, използван за описание на тези сложни, комплексни и нелинейни форми и явления във Вселената. Демонстрират се някои от първите опити за приложение на фракталната теория в областта на астрофизиката и по-специално процесите, свързани с гравитационните сингулярности, формата на спиралните галактики и динамичната организация и структура в Космоса.