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CONSEQUENCES FROM CONSERVATION OF THE TOTAL DENSITY OF THE UNIVERSE DURING THE EXPANSION

Dimitar Valev

Space Research and Technology Institute – Bulgarian Academy of Sciences
e-mail: valev@gbg.bg

Abstract

The recent Cosmic Microwave Background (CMB) experiments have shown that the average density of the universe is close to the critical one and the universe is asymptotically flat (Euclidean). Taking into account that the universe remains flat and the total density of the universe Ω_0 is conserved equal to a unit during the cosmological expansion, the Schwarzschild radius of the observable universe has been determined equal to the Hubble distance $R_s = \frac{2GM}{c^2} = R \sim cH^{-1}$, *where M is the mass of the observable universe, R is the Hubble distance and H is the Hubble constant. Besides, it has been shown that the speed of the light c appears the parabolic velocity for the observable universe* $c = \sqrt{\frac{2GM}{R}} = v_p$ *and the recessional velocity* $v_r = Hr$ *of an arbitrary galaxy at a distance r > 100 Mps from the observer, is equal to the parabolic velocity for the sphere, having radius r and a centre, coinciding with the observer. The requirement for conservation of* $\Omega_0 = 1$ *during the expansion enables to derive the Hoyle-Carvalho formula for the mass of the observable universe* $M = \frac{c^3}{2GH} \sim 10^{53}\,kg$ *by a new approach.*

1. Introduction

The problem for the average density of the universe $\overline{\rho}$ acquired significance when it was shown that the General Relativity allowes to reveal the large-scale structure and the evolution of the universe by simple cosmological models [1-3]. Crucial for the geometry of the universe appears the dimensionless total density of the universe $\Omega_0 = \frac{\bar{\rho}}{\rho_c}$, where $\bar{\rho}$ is the average density of the universe and ρ_c is the critical density of the universe. In the case of $\Omega_0 < 1$ (open universe) the global spatial curvature is negative and the geometry of the universe is hyperbolic and in the case of $\Omega_0 > 1$ (closed universe) the curvature is positive and the geometry is spherical. In the special case of $\Omega_0 = 1$ (flat universe) the curvature is zero and the geometry is Euclidean. Until recently scarce information has been available about the density and geometry of the universe. The most reliable determination of the total density Ω_0 is by measurements of the dependence of the anisotropy of the Cosmic Microwave Background (*CMB*) upon the angular scale. The recent results have shown that $\Omega_0 \approx 1 \pm \Delta\Omega_0$, where the error $\Delta\Omega_0$ decreases from 0.10 [4, 5] to 0.02 [6]. The fact that Ω_0 is so close to a unit is not accidental since only at $\Omega_0 = 1$ the geometry of the universe is flat (Euclidean) and the flat universe was predicted by the inflationary theory [7]. The total density Ω_0 includes densities of baryon matter $\Omega_b \approx 0.05$, cold dark matter $\Omega_c \approx 0.22$ [8] and dark energy $\Omega_\Lambda \approx 0.73$, producing an accelerating expansion of the universe [9, 10]. The found negligible *CMB* anisotropy $\frac{\delta T}{T} \sim 10^{-5}$ indicates that the early universe has been very homogeneous and isotropic [11]. Three-dimensional maps of the distribution of galaxies corroborate homogeneous and isotropic universe on large scales greater than 100 *Mps* [12, 13].

2. Consequences from conservation of the total density of the universe during the expansion

The flat geometry of the universe allows to solve some cosmological problems in the Euclidean space. The finite time of the cosmological expansion H^{-1} (age of the universe) and the finite speed of the light *c* set a finite particle horizon $R \sim cH^{-1}$ beyond which no material signals reach the observer. Therefore, for an observer in an arbitrary location, the universe appears a three-dimensional, homogeneous and isotropic sphere having finite "radius" (particle horizon) equal to the Hubble distance $R \sim cH^{-1}$, where $H \approx 70\ km\ s^{-1}\ Mps^{-1}$ [14] is the Hubble constant and $H^{-1} \approx 1.37 \times 10^{10}$ *years* is the Hubble time (age of the universe).

The fact that the total density of the universe Ω_0 is close to a unit is fundamental since only $\Omega_0 = \frac{\overline{\rho}}{\rho_c} = 1$ supplies flat geometry of the universe. There are no arguments to assume the recent epoch privileged in relation to the other epochs; therefore, the universe always remains flat, and *the total density of the universe* Ω_0 *is conserved equal to a unit during the cosmological expansion:*

$$(1) \qquad \Omega_0 = \frac{\overline{\rho}}{\rho_c} = 1$$

The critical density of the universe [15] is determined from equation (2):

$$(2) \qquad \rho_c = \frac{3H^2}{8\pi G} \approx 9.5 \times 10^{-27}\ kg\ m^{-3},$$

where G is the universal gravitational constant.

Considering $\overline{\rho} = \frac{3M}{4\pi R^3}$, where M and R are the mass and the Hubble distance ("radius") of the observable universe, and replacing ρ_c with expression (2) in (1) we obtain:

$$(3) \qquad \frac{2MG}{R^3 H^2} = 1$$

Replacing $H \sim cR^{-1}$ in (3) we obtain:

$$(4) \qquad R = \frac{2GM}{c^2}$$

Obviously, (4) appears the formula for the Schwarzschild radius of the mass of the observable universe M [16]. Therefore, *the Schwarzschild* *radius of the observable universe* R_s *is equal to the Hubble distance* $R_s = R \sim cH^{-1} \sim 1.37\times 10^{10}$ *light years*.

From (4) we find:

(5) $$c = \sqrt{\frac{2GM}{R}}$$

Evidently, (5) is the formula of the parabolic velocity for the Hubble sphere, i.e. the sphere having mass M and a radius, equal to the Hubble distance $R \sim cH^{-1}$. Therefore, *the speed of the light c appears the parabolic velocity* v_p *for the observable universe*.

Below, we find that the recessional velocity $v_r = Hr$ of an arbitrary galaxy at a distance $r > 100$ *Mps* from the observer is equal to the parabolic velocity of a sphere, having radius r and a centre, coinciding with the observer. As mentioned at the end of the Introduction, the universe is homogeneous and isotropic on large scales greater than 100 *Mps*. Therefore, the average density ρ_r of a sphere having radius $r > 100$ *Mps* is equal to the average density of the universe $\overline{\rho}$:

(6) $$\rho_r = \frac{3m}{4\pi r^3} = \overline{\rho} \approx \rho_c = \frac{3H^2}{8\pi G},$$

where m is the mass of the total matter in the sphere.

We find from equation (6):

(7) $$H = \sqrt{\frac{2Gm}{r^3}}$$

Replacing H in the Hubble law $v_r = Hr$ we obtain the recessional velocity of a galaxy:

(8) $$v_r = Hr = \sqrt{\frac{2Gm}{r}}$$

Equation (8) coincides with the formula for the parabolic velocity of a sphere, having radius r and a centre, coinciding with the observer.

Finally, the requirement for conservation of the total density of the universe equal to a unit during the expansion allows to estimate the total mass of the observable universe M. Actually, replacing $R \sim cH^{-1}$ in (3) we find:

$$(9) \qquad M = \frac{c^3}{2GH} \approx 8.8 \times 10^{52}\ kg$$

Obviously, this mass is close to the mass of the Hubble sphere M_H:

$$(10) \qquad M_H = \frac{4}{3}\pi R^3 \overline{\rho} \sim \frac{4\pi c^3 \rho_c}{3H^3} = \frac{c^3}{2GH}$$

Formula (9) has been derived independently by dimensional analysis without consideration of the average density of the universe in [17, 18] and practically coincides with the Hoyle-Carvalho formula for the mass of the universe [19, 20], obtained by a totally different approach.

3. Conclusions

The recent *CMB* experiments have shown that the average density of the universe is close to the critical one and the universe is asymptotically flat. The flat geometry of the universe allows to solve some cosmological problems in the Euclidean space. Taking into account that the universe remains flat and the total density of the universe Ω_0 is conserved equal to a unit during the expansion, the Schwarzschild radius of the observable universe has been determined equal to the Hubble distance $R_s = \frac{2GM}{c^2} = R \sim cH^{-1}$, and the speed of the light c appears the parabolic velocity for the observable universe $c = \sqrt{\frac{2GM}{R}} = v_p$. Besides, the recessional velocity $v_r = Hr$ of an arbitrary galaxy at a distance r > 100 *Mps* from the observer, is equal to the parabolic velocity of a sphere, having radius r and a centre, coinciding with the observer.

The requirement for conservation of $\Omega_0 = 1$ during the cosmological expansion enables to derive the Hoyle-Carvalho formula for the mass of the observable universe $M = \frac{c^3}{2GH}$ by a new approach.

References

1. F r i e d m a n, A., Uber die Krummung des Raumes, Z. Physik, 10, 1922, 377-386.
2. L e m a i t r e, G., Un univers Homogene de Masse Constante et de Rayon Croissant, Ann. Soc. Sci. Brux., 47A, 1927, 49-59.
3. E i n s t e i n, A., W. de S i t t e r, On the Relation between the Expansion and the Mean Density of the Universe, Proc. Nat. Acad. Sci. USA, 18, 1932, 213-214.
4. de B e r n a r d i s, P. et al., A flat Universe from High-Resolution Maps of the Cosmic Microwave Background Radiation, Nature, 404, 2000, 955-959.
5. B a l b i, A. et al., Constraints on Cosmological Parameters from MAXIMA-1, Astrophys. J., 545, 2000, L1-L4.
6. S p e r g e l, D. N. et al., First-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters, Astrophys. J. Suppl. Series, 148, 2003, 175-194.
7. G u t h, A. H., The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems, Phys. Rev. D, 23, 1981, 347-356.
8. P e a c o c k, J. A. et al., A Measurement of the Cosmological Mass Density from Clustering in the 2dF Galaxy Redshift Survey, Nature, 410, 2001, 169-173.
9. R i e s s, A. G. et al., Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant, Astron. J., 116, 1998, 1009-1038.
10. P e r l m u t t e r, S. et al., Measurements of Omega and Lambda from 42 High-Redshift Supernovae, Astrophys. J., 517, 1999, 565-586.
11. B e n n e t t, C. L. et al., Four-Year COBE DMR Cosmic Microwave Background Observations: Maps and Basic Results, Astrophys. J., 464, 1996, L1-L4.
12. S h e c t m a n, S. A. et al., The Las Campanas Redshift Survey, Astrophys. J., 470, 1996, 172-188.
13. S t o u g h t o n, C. et al., Sloan Digital Sky Survey: Early Data Release, Astron. J., 123, 2002, 485-548.
14. M o u l d, J. R. et al., The Hubble Space Telescope Key Project on the Extragalactic Distance Scale. XXVIII. Combining the Constraints on the Hubble Constant, Astrophys. J., 529, 2000, 786-794.
15. P e e b l e s, P. J. E., Physical Cosmology, Princeton University Press, Princeton, New-Jersey, 1971.
16. S c h w a r z s c h i l d, K., On the Gravitational Field of a Mass Point According to Einstein's Theory, Sitzungsberichte der Koniglich Preussischen Akademie der Wissenschaften zu Berlin, 1916, 189-196.

17. V a l e v, D., Determination of total mechanical energy of the universe within the framework of Newtonian mechanics, Suppl. Compt. rend. Acad. bulg. Sci.: 'Fundamental Space Research', 2010, 241-243; http://arxiv.org/abs/0909.2726
18. V a l e v, D., Estimations of total mass and energy of the universe, 2010, http://arxiv.org/abs/1004.1035
19. H o y l e, F., The Structure and Evolution of the Universe, Proceedings of 11th Solvay Conference in Physics, Edited by R. Stoops, 1958, Brussels.
20. C a r v a l h o, J. C., Derivation of the Mass of the Observable Universe, Int. J. Theor. Phys., 34, 1995, 2507-2509.

ПОСЛЕДИЦИ ОТ ЗАПАЗВАНЕТО НА ТОТАЛНАТА ПЛЪТНОСТ НА ВСЕЛЕНАТА ПРИ РАЗШИРЯВАНЕТО

Д. Вълев

Резюме

Съвременните изследвания на космическия микровълнов фон показват, че средната плътност на Вселената е близка до критичната, а Вселената е асимптотически плоска (Евклидова). Вземайки под внимание това, че Вселената остава плоска, а тоталната плътност на Вселената Ω_0 се запазва равна на единица при космологичното разширяване, е установено че Шварцшилдовият радиус на наблюдаемата Вселената е равен на разстоянието на Хъбъл $R_s = \frac{2GM}{c^2} = R \sim cH^{-1}$, където M е масата на наблюдаемата Вселена, R е разстоянието на Хъбъл, а H е константата на Хъбъл.

Освен това е показано, че скоростта на светлината c се явява параболична скорост за наблюдаемата Вселена $c = \sqrt{\frac{2GM}{R}} = v_p$, а скоростта на отдалечаване $v_r = Hr$ на произволна галактика на разстояние r > 100 *Мпс* от наблюдателя, е равна на параболичната скорост за сферата, имаща радиус r и център съвпадащ с наблюдателя. Изискването за запазване на $\Omega_0 = 1$ при космологичното разширяване дава възможност да се изведе формулата на Хойл-Карвальо за масата на наблюдаемата Вселена $M = \frac{c^3}{2GH} \sim 10^{53} kg$ по нов начин.