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# PHENOMENOLOGICAL QUARK-LEPTON MASS RELATIONS AND NEUTRINO MASS ESTIMATIONS

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***Abstract***

*Based on experimental data and estimations of the charged leptons and quark masses, a close power law with exponent ¾ has been found, connecting charged lepton masses and up quark masses. A similar mass relation has been suggested for the masses of neutral leptons and down quarks. The latter mass relation and the results of the solar and atmospheric neutrino experiments have been used for prediction of neutrino masses. The obtained masses of $\nu_e$, $\nu_\mu$ and $\nu_\tau$ are 0.0003 eV, 0.003 eV and 0.04 eV, respectively. These values are compatible with the recent experimental data and support the normal hierarchy of neutrino masses.*

**Key words:** *mass relation; quark-lepton symmetry; neutrino mass; normal hierarchy.*

## 1. Introduction

Decades after the experimental detection of the neutrino [1], it was generally accepted that the neutrino mass $m_{o\nu}$ was rigorously zero. The crucial experiments with the 50 kton neutrino detector Super-Kamiokande found strong evidence for oscillations (and hence - mass) in atmospheric neutrinos [2]. The direct neutrino measurements allowed to find neutrino mass. The upper limit for the mass of the lightest neutrino flavor $\nu_e$ was obtained from experiments for measurement of the high-energy part of the tritium $\beta$-spectrum and recent experiments yielded ~ 2 $eV$ upper limit [3, 4]. As a result of the recent experiments, the upper mass limits of $\nu_\mu$ and $\nu_\tau$

are 170 *keV* [5] and 18.2 *MeV* [6], respectively. The Solar neutrino experiments (*SNE*) and Atmospheric neutrino experiments (*ANE*) allow to find the square mass difference $\Delta m_{12}^2 = m_2^2 - m_1^2$ and $\Delta m_{23}^2 = m_3^2 - m_2^3$, but not the absolute value of neutrino masses. The astrophysical constraint of the neutrino mass is $\Sigma m_\nu < 2\ eV$ [7]. The recent extensions of the Standard model lead to non-zero neutrino masses, which are within the wide range of $10^{-6}\ eV \div 10\ eV$.

In the classical *SU(5)* model, the mass relations between charged leptons and down quark masses are simple identities: $m_e = m_d$, $m_\mu = m_s$ and $m_\tau = m_b$. The mass relations of Georgi-Jarlskog [8] ensue from the *SO(10)* model and relate charged leptons and down quark masses: $m_e = m_d / 3$, $m_\mu = 3m_s$ and $m_\tau = m_b$. However, these mass relations deviate several times, compared to experimental data. Moreover, similar mass relations are unsuited for neutral leptons (neutrino) masses.

The seesaw mechanism naturally generates small Majorana neutrino mass $m_\nu$ from reasonable Dirac mass $m_D$ and very heavy Majorana sterile neutrino mass $M_N$, namely $m_\nu \sim \frac{m_D^2}{M_N} << m_D$. But there are many seesaw models that differ in the scale $M_N$ and Dirac mass. The Grand unified theories (*GUT*) are the main candidates for seesaw models, with $M_N$ at or a few orders of magnitude below the *GUT* scale. Successful *GUT* models should essentially generate Cabibbo-Kobayashi-Maskawa (*CKM*) quark mixing matrix [9, 10] and Maki-Nakagawa-Sakata (*MNS*) lepton mixing matrix [11] and predict results compatible with the data from *SNE* and *ANE*. Yet, it is admitted that the predictions of the quark-lepton mass spectrum are the least successful aspect of the unified gauge theory [12, 13].

The purpose of this paper is to find simple and reliable quark-lepton mass relations, based on experimental data and estimations for quark and lepton masses. The next step is to estimate neutrino masses by means of these mass relations and data from *SNE* and *ANE*.

## 2. Power law approximation for the masses of charged leptons and up quarks

According to the Standard model, the fundamental constituents of matter are 6 quarks and 6 leptons. The fundamental fermions group in three generations having similar properties and increasing masses. The three generations of the fundamental fermions and their masses are presented on Table 1. The estimations of quark masses are taken from [14] and the upper mass limits of neutrino flavors are taken from [3-6].

*Table 1. Three generations of fundamental fermions and their masses (MeV)*

| Fermions | First generation | | Second generation | | Third generation | |
|---|---|---|---|---|---|---|
| Up quarks | $u$ | 3 | $c$ | $1.25\times10^{3}$ | $t$ | $1.74\times10^{5}$ |
| Down quarks | $d$ | 6 | $s$ | 122 | $b$ | $4.2\times10^{3}$ |
| Charged leptons | $e$ | 0.511 | $\mu$ | 106 | $\tau$ | $1.78\times10^{3}$ |
| Neutral leptons | $\nu_e$ | $<2\times10^{-6}$ | $\nu_\mu$ | $< 0.17$ | $\nu_\tau$ | $< 18.2$ |

A clear feature of the quark and charged lepton mass spectrum is the hierarchy of masses belonging to different generations:

$$(1) \qquad m_u << m_c << m_t\,,\; m_d << m_s << m_b\,,\; m_e << m_\mu << m_\tau$$

Most likely, a similar hierarchy of the masses of neutral leptons (neutrinos) could be anticipated $m_{\nu e} << m_{\nu\mu} << m_{\nu\tau}$. Based on experimental data, we search for a simple relation between the masses of charged leptons ($m_{cl}$) and the respective up quarks ($m_{uq}$) by the least squares. Although the linear regression $m_{cl} \approx 0.0102\, m_{uq}\;\; eV$ shows close correlation, it yields electron mass many times lower than the experimental value. After examination of other simple approximations (logarithmic, exponential and power law), we found out that the power law fits best experimental data:

$$(2) \qquad m_{cl} = k_0 m_{uq}^{\alpha} \quad eV$$

where $k_0$ = 9.33 and $\alpha$ = 0.749 ≈ ¾.

Despite the great uncertainty of u-quark mass (from 1.5 *MeV* to 5 *MeV*) and d-quark mass (from 3 *MeV* to 9 *MeV*), the slope remains within the narrow interval from 0.683 to 0.782. Although only three points make the approximation, the correlation coefficient is very high ($r$ = 0.993) and the maximal ratio of the predicted mass in relation to the respective experimental value $\Delta = m_{pr} / m_{ex}$ is 1.74 (for muon). The predicted masses of the electron and tau lepton differ from the respective experimental values by less than 40%. Therefore, mass relation (2) could be accepted as satisfactory.

### 3. Mass relation for neutral leptons and down quarks and estimations of neutrino masses

We suggest that a mass relation similar to (2) connects the masses of neutral leptons ($m_{nl}$) and the respective down quarks ($m_{dq}$):

(3) $$m_{nl} = k m_{dq}^{\alpha} \quad eV$$

where $\alpha = 0.749 \approx$ ¾ and $k$ is an unknown constant.

For $k = k_0 = 9.33$, formula (3) yields $m_{\nu e} \approx$ 1.13 *MeV*, $m_{\nu\mu} \approx$ 10.84 *MeV* and $m_{\nu\tau} \approx$ 153.66 *MeV*. These values are several orders of magnitude bigger than the experimental upper limits of neutrino masses (See Table 1), therefore $k << k_0$. Astrophysical constraints allow to limit more closely the value of $k$, since they give $m_{\nu} < \Sigma m_{\nu} <$ 2 *eV*. Thus, from equation (3) we obtain:

(4) $$k = \frac{m_{\nu\tau}}{m_b^{3/4}} < \frac{\Sigma m_{\nu}}{m_b^{3/4}} \sim 1.21 \times 10^{-7}$$

*ANE* [15] determines the squared mass difference:

(5) $$m_{\nu\tau}^2 - m_{\nu\mu}^2 \approx 2.2 \times 10^{-3} eV^2$$

Relation (3) yields:

(6) $$\frac{m_{\nu\mu}}{m_{\nu e}} \sim \left(\frac{m_s}{m_d}\right)^{3/4} \approx 9.60$$

(7) $$\frac{m_{\nu\tau}}{m_{\nu\mu}} \sim \left(\frac{m_b}{m_s}\right)^{3/4} \approx 14.17$$

Solving system (5)–(7), we obtain $m_{\nu e} \approx 3.4 \times 10^{-4}$ *eV*, $m_{\nu\mu} \approx 3.3 \times 10^{-3}$ *eV* and $m_{\nu\tau} \approx 4.7 \times 10^{-2}$ *eV*. These results support the normal hierarchy of neutrino masses.

On the other hand, the Large mixing angle (*LMA*) of Mikheyev-Smirnov-Wolfenstein (*MSW*) solution for *SNE* yields [16]:

$$(8) \qquad m_{\nu\mu}^2 - m_{\nu e}^2 \approx 7.9\times10^{-5} eV^2$$

This equation, together (6) and (7), yield $m_{\nu e} \approx 9.3\times10^{-4}$ *eV*, $m_{\nu\mu} \approx 8.9\times10^{-3}$ *eV* and $m_{\nu\tau} \approx 0.13$ *eV*. These values are almost three times bigger than the values obtained by Super Kamiokande data, therefore, they do not fit well with the latter. However, the Small mixing angle (*SMA*) *MSW* solution for *SNE* [17] yields:

$$(9) \qquad m_{\nu\mu}^2 - m_{\nu e}^2 \approx 6\times10^{-6} eV^2$$

This equation, together (6) and (7), yield $m_{\nu e} \approx 2.6\times10^{-4}$ *eV*, $m_{\nu\mu} \approx 2.5\times10^{-3}$ *eV* and $m_{\nu\tau} \approx 3.4\times10^{-2}$ *eV*. These values differ by less than 25% from the results obtained by Super Kamiokande data, which show that according to the suggested approach, *SMA MSW* solution fits better with the *ANE* than *LMA MSW*.

Thus, the obtained quark-lepton mass relations and the results of the solar and atmospheric neutrino experiments provide to estimate the masses of $\nu_e$, $\nu_\mu$ and $\nu_\tau$ of $(2.6\div3.4)\times10^{-4}$ *eV* $(2.5\div3.3)\times10^{-3}$ *eV* and $(3.4\div4.7)\times10^{-2}$ *eV*, respectively. These values are close to the neutrino masses ($2.1\times10^{-4}$ *eV*, $2.5\times10^{-3}$ *eV* and $5.0\times10^{-2}$ *eV*) found in [18] by the mass relation connecting the masses of four stable particles and the coupling constants of the fundamental interactions.

We could calculate constant *k* using the most trustworthy data for neutrinos and down quark masses, namely $\nu_\tau$ and b-quark masses:

$$(10) \qquad k = \frac{m_{\nu\tau}}{m_b^{3/4}} \sim 2.42\times10^{-9}$$

The obtained mass relations (2) and (3) are shown in Fig. 1. It shows that the neutrino masses estimated by *SMA MSW* are close to the neutrino masses estimated by *ANE*, i.e. two sets of estimations are compatible.

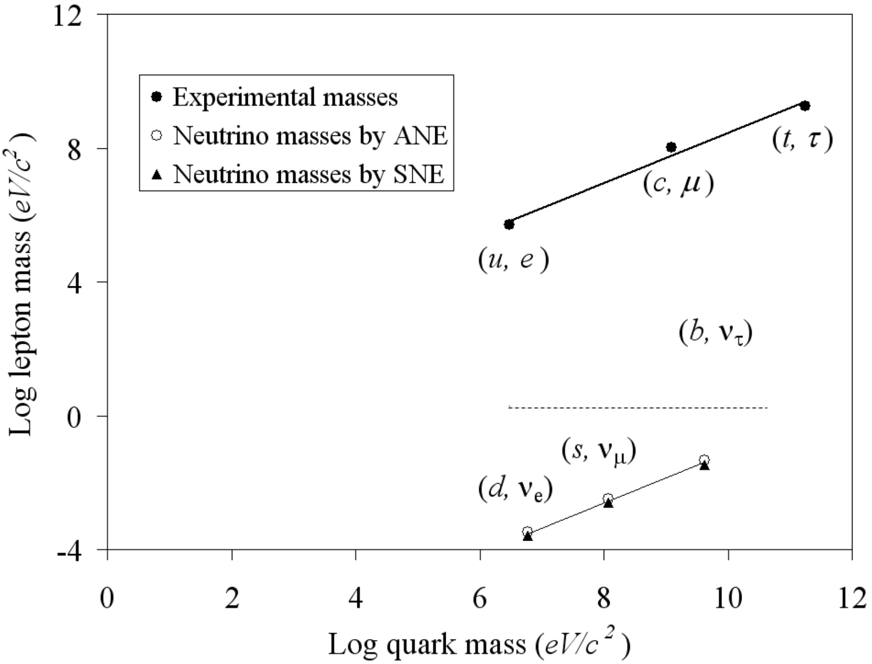


*Fig. 1. Mass relations for charged leptons and up quark masses (thick solid line) and for neutral leptons and down quark masses (thin solid line). The dashed line shows the upper limit of the neutrino masses obtained by astrophysical constraints*

The attempt to relate the masses of charged leptons with the masses of down quarks and the masses of neutral leptons with the masses of up quarks did not yield satisfactory results, since the data from *SNE* and *ANE* did not fit within the framework of the suggested approach. Besides, the respective mass relations predict a muon mass which is nearly three times less than the experimental value (see Table 2) and an electron neutrino mass which is less than $10^{-7}$ *eV*.

Table 2 shows the masses of charged leptons calculated by different approaches and experimental values. The last row of the table shows the maximal ratio (deviation) of the masses predicted by the respective approach in relation to the experimental values $\Delta = m_{pr} / m_{ex}$. Clearly, the power law relating to the masses of charged leptons and up quarks fits best experimental data.

*Table 2. Masses of charged leptons calculated by various approaches and experimental values (MeV)*

| Model | *SU(5)* | *SO(10)* | Linear (Down quarks) | Power law (Down quarks) | Linear (Up quarks) | Power law (Up quarks) | Exp. data |
|---|---|---|---|---|---|---|---|
| Electron | 6 | 2 | 2.545 | 0.905 | 0.031 | **0.663** | **0.511** |
| Muon | 122 | 366 | 51.8 | 36.9 | 12.8 | **60.7** | **105.7** |
| Tau | 4200 | 4200 | 1782 | 2877 | 1775 | **2449** | **1777** |
| Maximal deviation | 11.74 | 3.91 | 4.98 | 2.86 | 16.48 | **1.74** | **1** |

## 4. Conclusions

Based on experimental data and estimations of charged leptons and quark masses, a power law with exponent ¾ has been found, connecting charged lepton masses and up quark masses. It has been shown that this approximation is considerably better than any known approach. A similar mass relation has been suggested for neutral leptons and down quarks. The latter mass relation and the results of *ANE* and *SNE* have been used for estimations of neutrino masses. The values of neutrino masses obtained by *ANE* are close to the ones obtained by the *SMA MSW* solution. The masses of $\nu_e$, $\nu_\mu$ and $\nu_\tau$ are estimated to $(2.6 \div 3.4)\times 10^{-4}$ *eV*, $(2.5 \div 3.3)\times 10^{-3}$ *eV* and $(3.4 \div 4.7)\times 10^{-2}$ *eV*, respectively, and they support the normal hierarchy of neutrino masses.

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# ФЕНОМЕНОЛОГИЧНИ КВАРК-ЛЕПТОННИ МАСОВИ ВРЪЗКИ И ОЦЕНКИ НА МАСИТЕ НА НЕУТРИНОТО

***Д. Вълев***

## Резюме

На базата на експерименталните данни и оценки за масите на заредените лептони и кварките бе намерен степенен закон с показател ¾, свързващ масите на заредените лептони и up-кварките. Подобна масова връзка бе предложена за масите на неутралните лептони и down-кварките. Последната масова връзка и резултатите от слънчевите и атмосферните неутринни експрименти бяха използвани за предсказване на масите на неутриното. Получените маси на $\nu_e$, $\nu_\mu$ и $\nu_\tau$ са 0.0003 *eV*, 0.003 *eV* и 0.04 *eV*, съответно. Тези стойности са съвместими със съвременните експериментални данни и подкрепят нормалната йерархия на масите на неутриното.