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A METHOD OF PHASE-MANIPULATED COMPLEMENTARY SIGNALS APPLICATION IN SPACECRAFT-BASED RADARS

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Abstract

The radar imagery, realized by means of synthetic aperture radars (SARs) is very important in the exploring of planet, satellite and comet surfaces. The most valuable feature of the autocorrelation function (ACF) of the SAR signals is the level of their side lobes, because they determine the dynamic diapason of the image and the possibility to discover small objects. With regard to this, our paper suggests a method for applying in spacecraft-based SARs the so named generalized complementary signals, which ACF does not have any side-lobes. It uses the polarization features of the electromagnetic waves.

Key Words: *Synthetic aperture radars, ideal autocorrelation function, generalized complementary signals.*

1. Introduction

The radar imagery is very important in the exploring of planet, satellite and comet surfaces [1, 2]. It may be sketched as follows. The transmitter of the spacecraft-based radar sends electromagnetic signals. The examined objects reflect the signals, producing so named echo-signals. They are the input to the radar receiver. Mostly, in order to maximize the ratio "signal/noise", the receiver is constructed as a filter, matched to the sent signals. In this case the receiver output is the autocorrelation function *(ACF)* of the sent signals. This is clarified on Fig. 1, where a radar signal is depicted (Fig. la). The duration of the signal is Γ, but it is separated in η sub-signals (or "elementary signals") with duration r_0 (i.e. $n = T / r_0$) and different carrier frequency. This technique is named "discrete frequency shift-keying" *(DFSK)*. It allows obtaining a different echo-signal from every "reflected point" of the object. Commonly, the receiver output, produced of a single point echo, is characterized by a main peak *V* and a sequence of side-lobes with maximal amplitude *Vmax*, as shown on Fig. lb. At the end, the radar receiver output signals are sampled and processed, which lead to extracting of the object image [3].

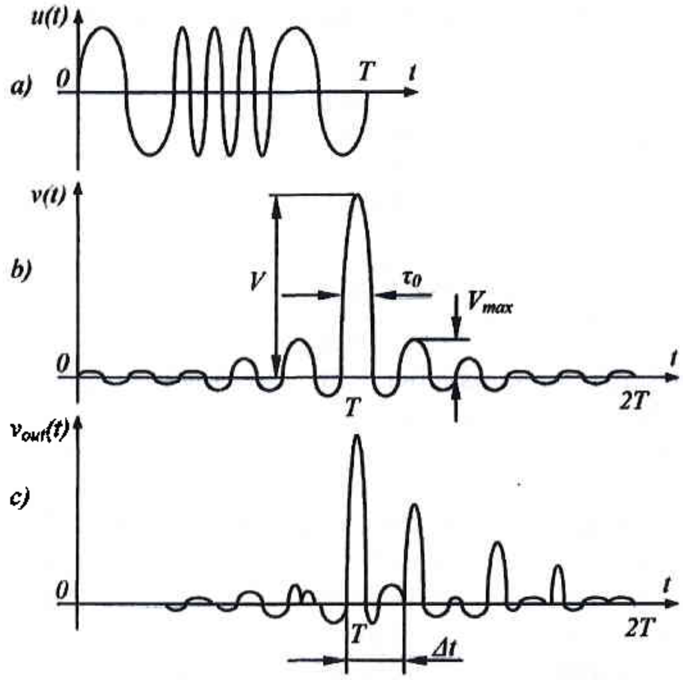

Fig. l Processing of radar signals

In general, the above described technique of complex radar signals usage guarantees simultaneously large performance range (provided by the aggregated power of elementary signals) and high distance resolution (defined by *TQ)* of the spacecraft-based synthetic aperture radars *(SARs).*

Unfortunately, the real objects comprise more than one reflected points. As a result, the echo-signals of all reflected points interfere, as depicted in Fig. lc. In this situation is hard to obtain a detailed object image, because the side-lobes of more power signals mask the main peaks of the weak signals.

With regard to this, our paper aims to suggest a method for applying of so named generalized complementary signals, which *ACF* does not have any side-lobes. It uses the polarization features of the electromagnetic waves.

2. A Method of Phase Manipulated Complementary Signals Applying in Spacecraft-based Radars

It is known [4] that discrete phase and frequency modulated signals may be presented as the real part of the complex-valued function:

$$(1)\qquad V(t)=\sum_{j=1}^{n}\{U_j.\exp(i\theta_j).\exp[2\pi i(f_0+f_j)t]\}u_0(t-j\tau_0),$$

where / = v-T; *Uj* is the amplitude of the j* elementary impulse y=1,2,...,«; /$_0$ is the carrier frequency; {/ь/2,—,/«} are real time functions, which expresses the frequency modulation; *{Θ j*; 0 < *θ j* < *2π; j* = 1,2, ...*n]* is the set of numbers, describing the phase modulation and:

$$(2)\qquad u_0(t)=\begin{cases}1, \text{ if } 0\le t\le\tau_0\\ 0, \text{ if } t<0, \text{ or } t>\tau_0\end{cases}.$$

To maximize the transmitter efficiency and to simplify the practical realization of the process of signal receiving, the so-named uniform signals with r$_0$ = *const; Uj* = *const; j* = 1,2,..., *n;; 0j* e {(2л/) *Im; 1* = 0,1,...,*m* - 1}are most widely applied.

In this case and if only discrete phase shift keying *(DPSK)* is applied, the signal is named "discrete phase manipulated *(PM)* signal". It can be comprehensively described by the sequence *{ζθ')}"*=\ *of* normalized complex amplitudes of elementary signals [41:

$$(3)\qquad \zeta(j)=\exp(i.\theta_j);\quad \zeta(j)\in\{\exp(2\pi il/m); l=0,1,...,m-1\}.$$

As above mentioned, the signals, which *ACF* has close-to-zero level of the side-lobes, are the most attractive for implementation in spacecraft based *SARs.* With regard, in the rest part of the paper our attention shall be focused on the so-named generalized complementary signals, which *ACF* is free of any side-lobes. It is known that a single radar signal does not have non-periodical *ACF* with zero level of the side-lobes. Moreover, the classes of single uniform discrete radar signals with small level of their side-lobes seem to be very rare. Due to this reason, Golay introduced [5] the so-named complementary series (or signals (CSs)). They are a pair of two uniform binary phase manipulated signals, which aggregated non-periodical *ACF* is similar to a delta pulse.

It is necessary to emphasize, that Golay's definition of *CSs* is not useful in some important cases. This situation has motivated some theoreticians to extend the classical definition as follows [6, 7,8].

Definition 1: *The set of p sequences (PM signals), which elements are complex numbers, belonging to the multiplicative group of the m-th (m>2) roots of unity:*

$$(4)\quad \{A_1 = \{\xi_1(j)\}_{j=1}^{n_1}; A_2 = \{\xi_2(j)\}_{j=1}^{n_2}; ...; A_p = \{\xi_p(j)\}_{j=1}^{n_p}\};$$
$$\xi_k(j) \in \{\exp(2\pi i l / m_k); l = 0,1,...,m_k - 1\}; k = 1,2,...,p.$$

*are **a set of generalized complementary signals** (GCCs) if and only if their aggregated ACF has an ideal shape, similar to delta impulse:*

$$(5)\quad R_c(r) = \sum_{k=1}^{p} R_{A_k}(r) = \begin{cases} n = n_1 + n_2 + ... + n_p; & if \ r = 0; \\ 0; \ if & r = 1,2,...,\max\{n_k\}. \end{cases}$$

In (5) the non-periodical *ACF R^,* (r) are defined with the well known formula [4]:

$$(6)\quad R_\zeta(r) = \begin{cases} \sum_{j=1}^{n-|r|} \zeta(j)\zeta^*(j+|r|), & -(n-1) \le r \le 0 \\ \sum_{j=1}^{n-r} \zeta^*(j)\zeta(j+k), & 0 \le r \le n-1. \end{cases}$$

Consequently, Golay's codes are a particular case of the GCCs, when *ρ = 2, m = 2.* The CCs and GCCs are unique among all *PM* signals with their following features:

- their aggregated *ACF* has an ideal shape, similar to a delta pulse;
- if a pair of GCCs, consisting *η* elements, is known, then it is easy to create an infinite set of pairs with unlimited code-length.

It ought to emphasize that the most type uniform *PM* signals with close to ideal *ACF* have limited code-length. For instance, Barker codes exist only for *η* < 13, if η is an odd integer.

With regard to the GCCs positive features, they are studied very intensively and a quick reference showed more than 200 conference reports and magazine articles, related to this theme, during the past ten years.

The natural question, which arises from Definition 1, is "How can be implemented the GCCs in a real communication system?". The most obvious answer is the usage of *ρ* different frequency carriers *ffr,k = l,2,...,p,* phase manipulated according to the sequences A_k*,k = l,2,...,p.* Unfortunately, this is not the best approach, when the communication system is a spacecraft-based *SAR.* This conclusion will be clarified with following example. Suppose that spacecraft-based *SAR* exploits GCCs with *ρ* = 2 and the transmitter radiates simultaneously two uniform *PM* signals with carriers /j, /2, manipulated according to the sequences *A*, *Aj* . As a result of so-named Doppler effect, the carriers f_e^

of echo-signals will be:

$$(7) \qquad f_{ek} = f_k \frac{1-V_R/c}{1+V_R/c} \approx f_k(1-2V_R/c), \; k=1,2,$$

where *V%* is the radial velocity of the spacecraft relatively to the object and *c* is thc velocity of the electromagnetic waves propagation. If the explored object is on the earth surface, then V_R must be at least 27 360 km/h. Then the difference Δ/ = \\f_e\\ - /$_{e2}$1 will be too significant and it may lead to irreparable phase distortions between the components of the *GCCs.*

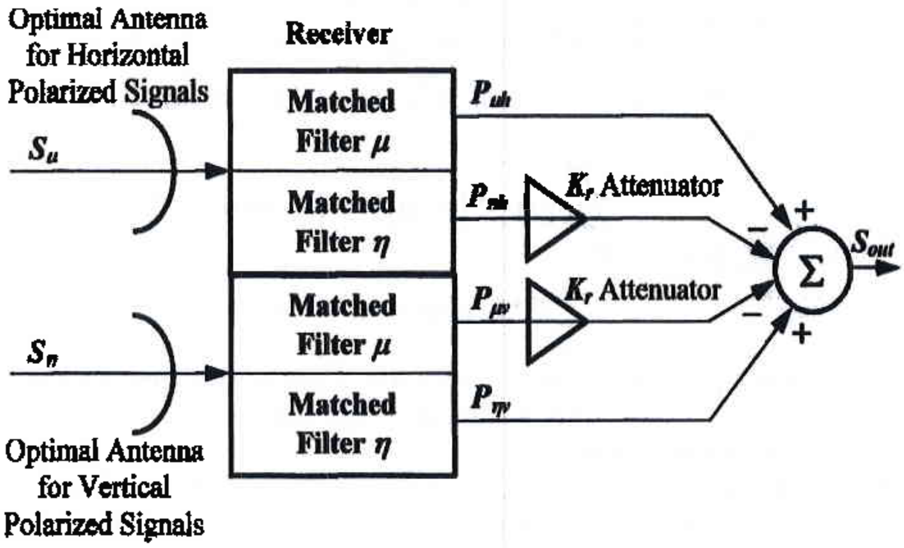

Fig. 2 Method of GCCs applying in spacecraft-based *SARs*

Due to above reason in the rest part of our report we shall prove a more appropriate approach to GCCs implementation in spacecraft-based *SARs*. Namely, we propose the two uniform *PM* signals, composing a pair of *GCCs,* to be transmitted on one frequency carrier /$_0$ simultaneously but by means of different types of polarization. Let the horizontal and the vertical polarized *PM* signals be described with the sequences *A* - *tPUyfji-i* anc* ^2 = *{n(j))* /$_=$i respectively. Then the signals, reflected by an object point, will be *S^*, S_n respectively. The reflected signals are connected with transmitted signals by following matrix equation:

$$(8) \qquad \begin{Vmatrix} S_\mu \\ S_\eta \end{Vmatrix} = \begin{Vmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{Vmatrix} \begin{Vmatrix} A_1 \\ A_2 \end{Vmatrix}; \; \|D\| = \begin{Vmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{Vmatrix},$$

where the complex valued matrix |D| is the so-named "polarization matrix of the target scattering". Its entries depend on physical features of the object, its orientation and position relatively to the radar and carrier frequency of transmitted signals. When these parameters are constant, then the entries of the matrix *WOW* are constants also and more over D17 = Z)?i. Accounting that the size of a single reflected point is small, it may be concluded that Dj j *m* D_{22} · Consequently, the matrix jJDjj can be presented in the form:

$$(9) \qquad \|D\| = \begin{Vmatrix} k_1 & k_1.k_2 \\ k_1.k_2 & k_1 \end{Vmatrix},$$

where D_u = D_{22} =*ťi *Φη* ^{lk}\) = (#21 ^{fk}\) = ^{k}2-

As above stated, the main obstacle for GCCs practical implementation by means of polarized electromagnetic waves is the fact that every echo-wave comprises the both horizontal and vertical polarized components. Due to this reason, we propose the method of signal processing, shown on Fig. 2. We shall explain it using the following notation. Let *{ζθ')}$^{\eta}$-]* be the sequence of normalized complex amplitudes of elementary signals, composing an arbitrary complex signal. As mentioned, the result of processing of this signal by its matched filter will be the *ACF* of the signal. It may be presented with the following polynomial:

$$(10) \qquad P(x) = F(x).F^*(x^{-1}).$$

Here:

$$(11) \qquad F_\zeta(x) = \zeta(n).x^{n-1} + \zeta(n-1).x^{n-2} + ... + \zeta(2).x + \zeta(1),$$

$\{\zeta(j)\}_{j=1}^{n}$, $F_\zeta^*(x^{-1})$ is the so- named "Hall polynomial", corresponding to the sequence is the polynomial:

$$(12) \qquad F_\zeta^*(x^{-1}) = \zeta^*(n).x^{-(n-1)} + \zeta^*(n-1).x^{-(n-2)} + ... + \zeta^*(1),$$

the coefficients of the polynomial *P(x)* are:

p_k=R_c(k); κ = -(n-\)-(n-2),...-\,ЪX:.,n-2,n-\,

and the values *Rr(k)* of the *ACF* are computed according to (6).

Accounting the above notation, (8) and (9), the outputs of matched filters, shown on Fig. 2, may be expressed by following polynomials:

$$
(13)\quad
\begin{aligned}
P_{\mu h}(x) &= k_1[F_\mu(x) + k_2.F_\eta(x)].F^*_\mu(x^{-1});\\
P_{\eta h}(x) &= k_1[F_\mu(x) + k_2.F_\eta(x)].[k_r F^*_\eta(x^{-1})];\\
P_{\eta v}(x) &= k_1[k_2.F_\mu(x) + F_\eta(x)].F^*_\eta(x^{-1});\\
P_{\mu v}(x) &= k_1[k_2.F_\mu(x) + F_\eta(x)].[k_r F^*_\mu(x^{-1})].
\end{aligned}
$$

In (13) k_r is a special coefficient, brought into the scheme by means of two directed attenuators. Now it is easy to see, that if attenuators on Fig.2 are regulated to obtain $k_r = k_2$, then:

$$
(14)\quad
\begin{aligned}
S_{out}(x) &= P_{\mu h}(x) - P_{\eta h}(x) + P_{\eta v}(x) - P_{\mu v}(x) =\\
&= k_1\{[F_\mu(x)F^*_\mu(x^{-1}) + F_\eta(x)F^*_\mu(x^{-1})] -\\
&- k_2^2.[F_\mu(x)F^*_\mu(x^{-1}) + F_\eta(x)F^*_\mu(x^{-1})]\} = k_1.2n.(1 - k_2^2).
\end{aligned}
$$

Formula (14) shows that the method of *GCCs* usage, depicted on Fig. 2, preserves the cancellation of the *ACF* side-lobes, despite of the harmful presence of cross-reflected signals. The signal power losses depend on the relative coefficient of the cross-polarized reflection k_2.

3. Conclusions

The method of GCCs applying in spacecraft-based radars, presented above, preserves the positive features of the *GCCs,* especially the cancellation of the *ACF* side-lobes, despite of the harmful presence of cross-reflected signals. This result is reached by small losses of signal power, because mostly $k_2 < 20\%$ and hence (1 - *Ц*) > 96%.

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МЕТОД ЗА ПРИЛАГАНЕ НА ФАЗОВО МАНИПУЛИРАНИ КОМПЛЕМЕНТАРНИ СИГНАЛИ В КОСМИЧЕСКИТЕ РАДИОЛОКАЦИОННИ СИСТЕМИ

Борислав Беджев, Жанета Ташева, Росен Богданов

Резюме

Получаването на радарни изображения на повърхността на планетите, спътниците и кометите е важен момент при тяхното изучаване. В този процес най-важното свойство на автокорелационната функция (АКФ) на радиолокационните сигнали е нивото на страничните листи на АКФ, защото то определя динамичния диапазон на изображението и възможността за откриване на малоразмерни обекти. По тази причина в статията се обосновава метод за използване на така наречените обобщени комплементарни сигнали (ОКС), чиято сумарна АКФ няма странични листи. Методът се характеризира с това, че запазва ценните свойства на ОКС въпреки ефекта на кръстосано поляризационно отражение на радарните сигнали. Този положителен резултат се постига технически просто и с минимални загуби на енергия на ехо-сигналите.