Bulgarian Academy of Sciences. Space Research Institute.
Aerospace Research in Bulgaria. 19, 2005, Sofia

MODEL OF RISK ASSESSMENT UNDER BALLISTIC STATISTICAL TESTS

Ivan Gabrovski, Juliana Karakaneva

Defence Advanced Research Institute
"Rakovski" Defence and Staff College

Abstract

The material presents the application of a mathematical method for risk assessment under statistical determination of the ballistic limits of the protection equipment. The authors have implemented a mathematical model based on Pierson's criteria. The software accomplishment of the model allows to evaluate the V50 indicator and to assess the statistical hypothesis' reliability. The results supply the specialists with information about the interval valuations of the probability determined during the testing process.

Introduction

Creating effective personal protection tools against ballistic threats is an important task because of human life value. Work in this area is ongoing, performed by both the manufacturers of body armour and the researchers who provide scientifically-grounded solutions for individual armour quality assessment.

Accounting for the fact that there is no tool ensuring 100 % protection, conventional approaches aim to provide probabilistic estimations of life-important organs' protection, evaluating risk degree.

In the military, the *V50* ballistic test is used, which is the internationally recognised standard to assess the fragmentation resistance of personal protection, particularly hard armour, helmet, and vest. The *V50* testing identifies experimentally the velocity at which a bullet has 50 percent chance of penetrating the test object.

The *V50* ballistic limit velocity for material is defined as the velocity at which the probability of penetrating the chosen projectiles is exactly 0.5 (STANAG 2920[1]).

Using the Up and Down firing method, the first round shall be loaded with the amount of propellant calculated to give a projectile a velocity equivalent to the estimated *V50* ballistic limit for armour.

After a number of projectile have been fired, *V50* is calculated as the average of the velocities recorded for six fair impacts consisting of the three lowest velocities for complete penetration and the three highest velocities for partial penetration, provided the spread is not greater than 40 m/s.

Many body armour manufacturers use a modified form of the military *V50* testing as a design tool to develop and assess new body armour products. This test identified the velocity at which specific projectile has 50-percent chance of penetrating the armour which is being tested.

The *V50* ballistic limit testing allows producers to evaluate various designs against one another to optimize their characteristics for a specific type of body armour. A trend has emerged in which manufacturers publish test data and also put *V50* test information on the labels of some of their body armour.

The *V50* ballistic limit testing is a useful and informative statistical tool to evaluate certain armour characteristics at the armour's design phase, and to evaluate armour degradation over time.

Formulation of the research

Determining the probability characteristics of personal protection means calls for elaboration of research methodology to raise information reliability. Moreover, it is necessary to choose suitable methods for data processing to allow evaluate the risk level of the taken decisions.

Except for *V50*, the following characteristics of body armour are used in practice:

- V_{np} ballistic limit - the velocity under which a bullet definitely (100 %) doesn't penetrate the test object;
- V_p ballistic limit - the velocity over which a bullet definitely (100 %) penetrates the test object.

The above characteristics are determined by the experiment of firing according to the "Up-Down" method. This method involves a set of experiments at which the velocity is decreased upon penetrating the test object and alternatively - the velocity is increased upon non-penetrating the test object. The attempts are made within a bracket of velocity interval covering the three areas – area of non-penetrating (*np*), area of penetrating (*p*) and area of mixed results.

In the introduction we examined the model of *V50* determining, according to USA standard. In [2], the authors show a probability-statistical model based on the frequencies of penetrating and non-penetrating attempts, and their distribution depending on velocity.

The initial data are several pairs of firing velocities and obtained results for *(np)* and *(p)* values. We evaluate the cumulative frequencies:

$m_1, m_2, m_3, ..., m_n,$

relevant to the event "non-penetrating" at velocities greater than respectively:

$v_1, v_2, v_3, ..., v_n,$

where v_i , $i = 1, ...,n$ are the middle points of equal-width velocity range subintervals during the experiment. This range is determined according to the above-described approach.

By analogy, the cumulative frequencies are evaluated:

$k_1, k_2, k_3, ..., k_n,$

relevant to the event "penetrating" at velocities less than respectively:

$v_1, v_2, v_3, ..., v_n.$

For statistics: M = $\{ m_1, m_2, m_3, ..., m_n \}$ and K = $\{ k_1, k_2, k_3, ..., k_n \}$, we define the smoothed probability distributions, respectively $P_m(x)$ and $P_k(x)$, corresponding to normal distribution. The probability $P_m(x)$ is related with the "nonpenetrating" event when the velocities $\xi \geq x$, and the probability $P_k(x)$ is related with the "penetrating" event when the velocities $\xi \leq x$.

Thereby *V50* is defined as the value ***x***, for which:

$P_m(x) = P_k(x).$

Under the above-formulated conditions, the authors develop the probability model and determine the approximated probability distributions [3]. Based on data from real experiments they evaluate quintiles through linear regression equations and draw up the distribution image for $P_m(x)$ and $P_k(x)$ within the range of the velocity ***v***.

Determination of risk level under the probabilistic-statistical estimations

The previous research continues in this paper with the objective to verify the statistical hypothesis for conforming with the normal distribution law after the Pearson's criterion - χ^2 and determining the confidence intervals of the penetration probability.

A Basic Statistical Method in Hypothesis Testing

The verification of the hypothesis is made separately for the cases of non-penetrating and penetrating firing. Below is given the method which describes the first case.

We evaluate the frequencies of non-penetrating attempts:

$f_i = m_i - m_{i-1}$ за $i = 1,2,\dots,n,$ where $m_0 = 0.$

According to the probabilistic model, the next estimations are the mean $\bar{x}$ and variance s of velocity, concerning the event "non-penetrating":

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} v_i f_i, \qquad s^2 = \frac{1}{n-1}\sum_{i=1}^{n} f_i (v_i - \bar{x})^2 .$$

The theoretical probabilities $p_1, p_2, p_3, \dots, p_n$ of random variables

$$t_i = \frac{v_i - \bar{x}}{s}$$

are $\quad p_i = |F(t_i) - F(t_{i-1})|$ for $i = 1,2,3,\dots,n$ and $p_1 = F(t_1)$.

The statistics χ^2:

$$\chi^2 = \sum_{i=1}^{n} \frac{r_i^2}{r p_i} - r , \tag{1}$$

where $r_i / r = f_i$ and $r = \sum_{i=1}^{n} r_i$,

features Pearson distribution:

$$P(\chi^2 \geq \chi_0^2) = \frac{1}{2^{\frac{n-1}{2}} \Gamma(\frac{n-1}{2})} \int_{\chi_0^2}^{\infty} x^{\frac{n-1}{2}-1} e^{-\frac{x}{2}} dx , \tag{2}$$

and $\Gamma(n) = \int e^{-z} z^{n-1} dz$

is a well known *gamma* function.

This test based on statistics χ^2 is called chi-square test. The hypothesis' examination comprises the following steps:

- Choosing level of importance $q\%$, for example $q = 5$, and following (1) evaluating χ_q^2, such that $P(\chi^2 > \chi_q^2) = \frac{q}{100}$;
- Calculating the value of χ^2 according to (2);
- There are two cases for χ^2:

1) $\chi^2 > \chi_q^2$ i.e. we get into the critical region and therefore, the non-conformity between the observed results and theoretical distribution data is considerable, so, the hypothesis is rejected, or

2) $\chi^2 \le \chi_q^2$ i.e. the non-conformity is not considerable and the hypothesis is accepted.

Determination of the confidence intervals of unknown probability

The determination of the confidence interval of unknown probability concerning the event "non-penetrating", for example at the interval $(v_i - \frac{h}{2}, v_i + \frac{h}{2})$ for r_i "non-penetrating" firings and n_i common implemented shots during this interval, under confidence level of at least *1-2*α, is carried out as follows:

- Evaluating the root $\tilde{p}_{r_i\alpha}$ of equation:

$S_{n_i r_i}(\tilde{p}_{r_i\alpha}) = \alpha$, where

$$S_{n_i r_i}(p) = \sum_{j=0}^{j=r_i} C_{n_i}^j p^j (1-p)^{n_i - 1} \text{ и } C_n^j = \frac{n!}{j!(n-j)!};$$

- Evaluatinge the root $p_{r_i\alpha}$ of equation:

$S'_{n_i r_i}(p_{r_i\alpha}) = \alpha$, where

$$S'_{n_i r_i}(p) = \sum_{j=r_i}^{j=n_i} C_{n_i}^j p^j (1-p)^{n_i - 1}.$$

Then the interval $p_{r_i\alpha}$ < p < $\tilde{p}_{r_i\alpha}$ covers the unknown probability **p** of the event "non-penetrating" within the interval $(v_i - \frac{h}{2}, v_i + \frac{h}{2})$ under confidence level *1-2*α.

Software instrument

Characteristic of the application software

Based on the probabilistic statistical method, the authors coded a software application. It is a *Windows Forms* style application and works under *.NET Framework* (Fig. 1). The software environment is *MS Visual Studio .NET* and the language in which it was coded is *C#*. The application's implementation under *.NET* has several advantages compared to the old *Windows* style (COM) applications.

First, the whole information about the application and the components used by it is saved in configuration files. These files use *XML* syntax to save hierarchical data which makes them more flexible than the old *.ini* files.

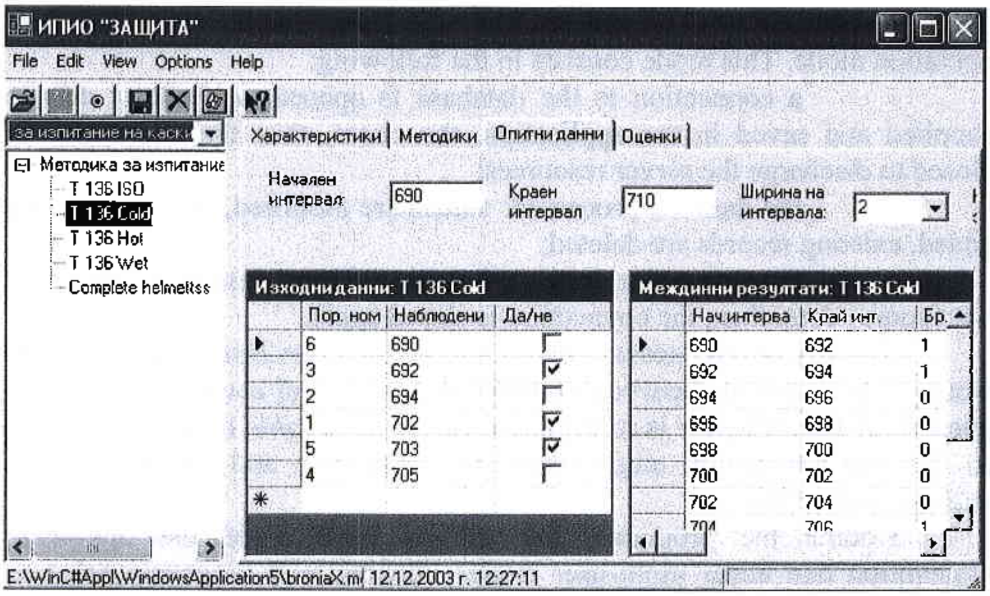

Fig. 1

The other advantage is the object-oriented approach. In contrast to *COM*, *.NET Framework* is designed under the inheritance concept. All objects of the *.NET Framework* form a single-root hierarchy, the class *System.Object* to which all other classes are successors. These classes ensure functionality in all possible areas, including user interface, data access, security, *Internet* programming, communications between devices, etc.

And third, *.NET* provides a solution for the version problem. The particular components of the application are saved in an application directory, or in its sub-directory. Different applications can use different versions of the components because they are saved in different directories. Each component of the global cash is saved in a separate subdirectory and different versions of the same component can exist simultaneously on the same computer device. Each application coded under a given component version continues working correctly even when the user installs a new (or older) version of this component.

Working with database

All data of the estimation model are saved into a database, the *Microsoft Access.*

The *System.Data* classes build the part of the *.NET Framework* known as *ADO.NET* for working with the database (DB). *ADO.NET* use *.NET* controlled data providers. The application uses *read-only* data operation mode. This mode consists in the following:

- a connection to the database is opened, a block of data is supplied and saved in the application, the client, then the connection is closed to discharge the server resources;
- the data are processed: values are modified, new records are added, existing records are deleted;
- the connection is opened again and local data are harmonized with source data, then, the connection is closed again.

The key of data access is an object of the class *DataAdapter*, which works as a connector between *DataSet* and the actual source of data. The purpose of *DataAdapter* is to fill one or more *DataTable* objects with data, so that the application might close the connection and operate thereon without connection.

Coding the programs, the authors have made the optimistic assumption that under multi-user mode collisions will be occasional, i.e., most likely the application will work on the client workstation with client database. For the cases when the database is on the server and many users work with, ways to prevent collisions have been provided.

The program provides user interface to introduce data and functions, to process the results from experiments, and to present output results in the form of histograms and documents.

Besides the previous research of the authors, namely determining *V50* ballistic limits of body armour (Fig. 2), the software instrument presents specified-above characteristics. The results are evaluated based on the given algorithm and relevant program module. According to the above-procedure, the calculations yield the values of χ^2 - crucial, χ^2 of “non-penetrating” and χ^2 of “penetrating’ shots (Fig. 2). The output report shows whether the hypothesis is accepted or rejected (Fig. 3).

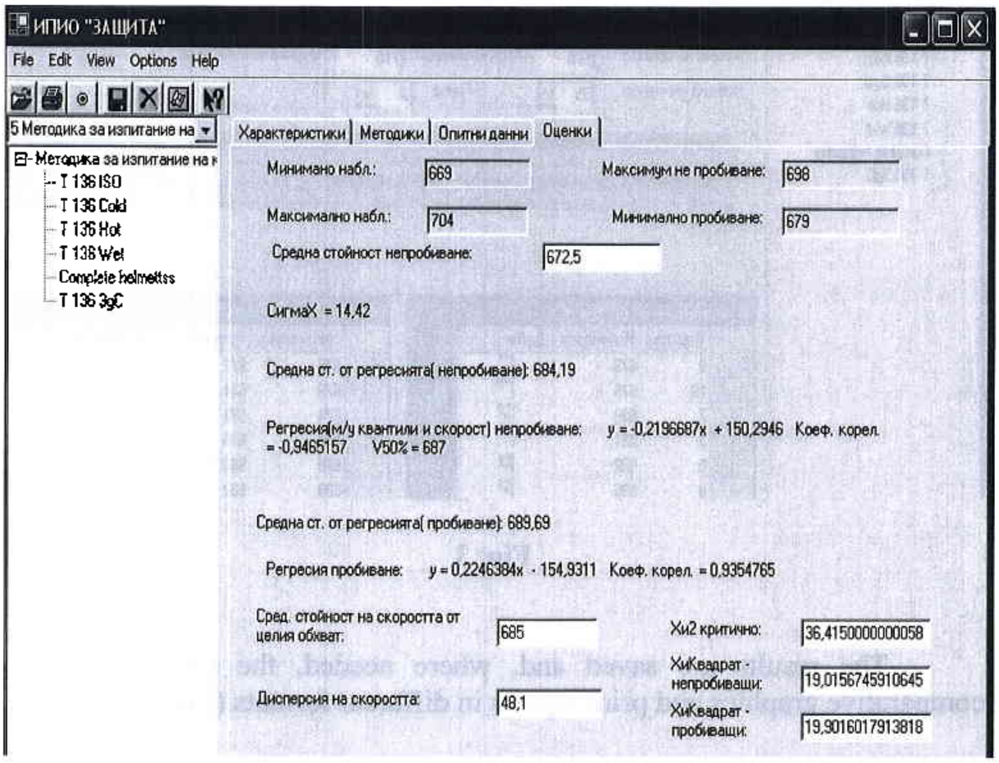

Fig. 2

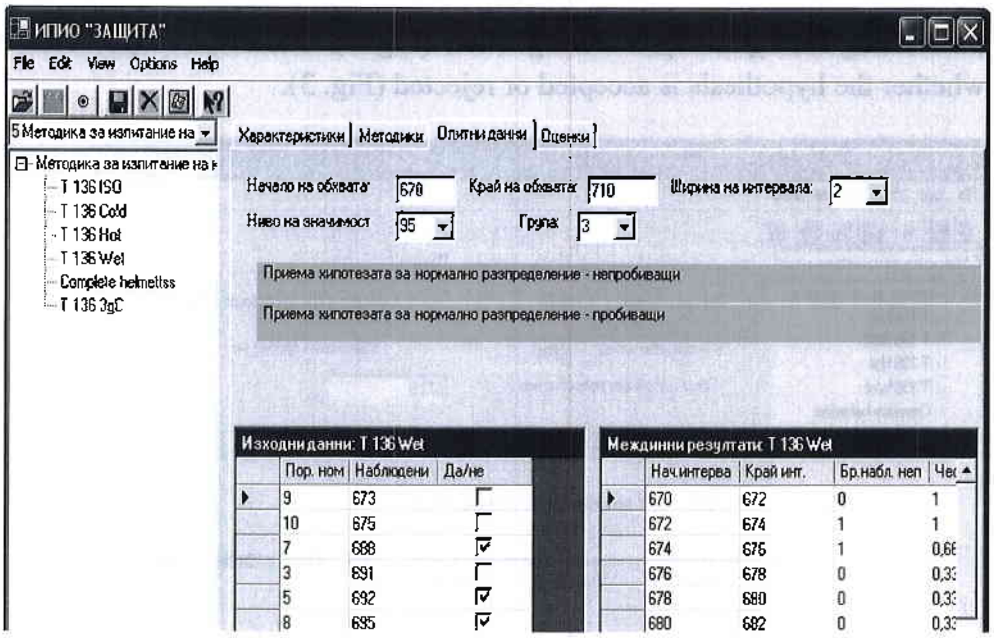

Fig. 3

The results are saved and, where needed, the user can obtain comparative graphics and print reports in different formats (Fig. 4).

Conclusions

The authors' efforts are focused on the creation of mathematical model and procedure to estimate the basic characteristics of body protection armours. The application of probabilistic-statistical approach provides to make much estimation based on real experiments with helmets and armoured vests. The developed software application is a very useful tool in this process. The obtained results conform to the theoretical hypothesis. This assumed approbation manner confirms the approach's correctness and validates the chosen method.

The presented product furnishes the responsible agency with an effective tool to estimate body armours characteristics during ballistic tests. Moreover, the software instrument, which is based on modern computer technology, can be used in the research and manufacture of new protection equipment.

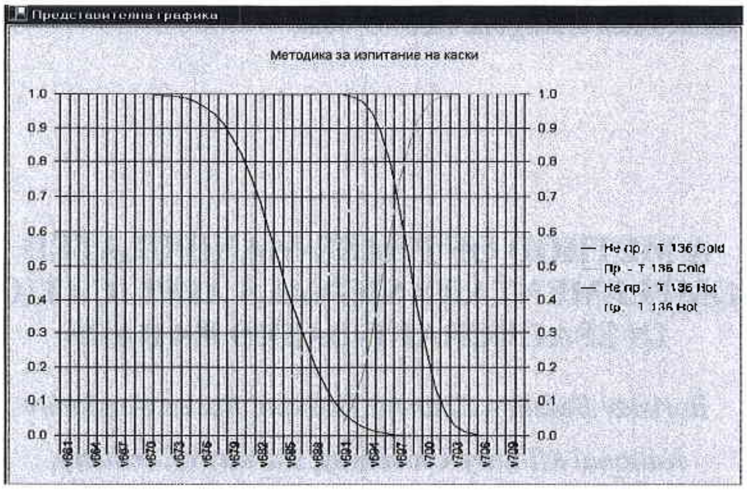

Fig.4

References

1. MILITARY STANDARD V50 BALISTIC TEST FOR ARMOR – STANAG 2920.
2. Габровски, И., Ю. Каракънева, Хр. Христов. Математически модел за определяне показателите на средствата за индивидуална балистична защита. Сборник доклади от Научната сесия на Шуменския университет, 2003. (in Bulgarian)
3. Крамер Г. Математические методы статистики. М., "МИР", 1975, (2-е изд). (in Russian)

МОДЕЛИРАНЕ НА ОЦЕНКАТА НА РИСКА ПРИ ОПРЕДЕЛЯНЕ НА БАЛИСТИЧНИТЕ ПОКАЗАТЕЛИ НА СРЕДСТВАТА ЗА ИНДИВИДУАЛНА ЗАЩИТА

Иван Габровски, Юлияна Каракънева

Резюме

В материала е представено приложението на математически модел за оценка на риска при статистическо определяне на балистичните граници на защитното оборудване. Авторите са създали математически модел на базата на критериите на Пиърсън. Софтуерната реализация на модела дава възможност да се изчисли показателят V50 и да се оцени надеждността на статистическата хипотеза. Резултатите дават на специалистите информация за оценката на интервалите на вероятността, определени в процеса на тестването.