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NON-DIFFUSIVE MECHANISM OF CHARGED PARTICLES ACCELERATION UNDER THE ACTION OF AN ELECTROSTATIC WAVES PACKAGE

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Abstract

A new non-diffusive mechanism of charged particles acceleration is considered. The latter is conditioned by the wave-particle interaction in the resonance of second order that corresponds to the nonlinear oscillator excitation by an external force. The calculations show that a leap of the accelerating particle can be observed in the process of the resonance interaction, from one potential well to another that moves with a greater velocity. A sequence of such leaps through out the separatrix leads to particles acceleration with multiple increasing of their kinetic energy. The mechanism of charged particles acceleration under consideration is realizing when the conditions are fulfilled as follows. For the charge that has been captured in the potential well of the wave package n-th harmonics with a frequency ω_n*, wave vector* κ_n *and amplitude* E_n*, a resonance of the second order with (n+1)-th harmonics should be fulfilled. The harmonics phase velocity* ω_n / κ_n *is to increase with the increasing of n, i.e.* $\omega_{n+1} / \kappa_{n+1} > \omega_n / \kappa_n$*. The regions of the captured particles velocities must adjoin for neighboring harmonics. The waves amplitudes are sufficiently enough for nonlinear oscillator excitation and throwing it throughout the separatrix. In this way, a new mechanism has been found of non-diffusive charged particles acceleration by a package of electrostatic waves with small but finite amplitude. A procedure of parameters selection is formed for the sequence of harmonics in the package that take part in the charged particle acceleration process. The effect under consideration is of interest, particularly, for the problem of cosmic rays generation and interpretation of origin mechanisms of accelerated particles flows (of electrons and ions) that are observed in the space plasma.*

1. Problem formulation and basic equations

Let us consider a charge interaction having a mass m with a package of electrostatic waves consisting $(n+1)$ modes with frequencies ω_n, wave vectors κ_n and amplitude E_n. Considering one-dimensional case, the equation of the charged particle motion can be presented in the form:

$$mx_{tt} = qE_o \sin(\kappa_o x - \omega_o t + \psi_o) + \sum_{n=1}^{N} E_n \sin(\kappa_n x - \omega_n t + \psi_n) \quad (1)$$

where ψ_n is the wave phase, $\omega_n = \omega(\kappa_n)$ is determined by the dispersion equation. We assume that charge velocity x_t is close to the phase velocity ω_o / κ_o in the initial moment of time. For the numerical study it is suitable to pass over a counting system that moves with velocity of ω_o / κ_o and to introduce dimensionless time

$$x = \frac{\omega_o}{k_o} t + \frac{y}{k_o}, \quad \tau = \omega_{nonlin} t, \quad A_n = E_n / E_o$$

$$\omega_{nonlin} = (qk_o E_o / m)^{1/2}, \quad q_n = k_n / k_o, \quad \omega_{nonlin} = (k_o \omega_n - k_n \omega_o) / k_o \omega_{nonlin} \quad (2)$$

where ω_{nonlin} is bounce-frequency of the oscillations that have been captured in the potential well. Varying the initial counting point of time we can make that $\psi_o \pi$. As a result, substituting (2) into Eq. (1) we obtain finally

$$\frac{d^2 y}{d\tau^2} + \sin y = \sum_{n=1}^{N} A_n \sin(q_n y - \Omega_n \tau + \Psi_n) \equiv f(y, \tau) \quad (3)$$

Equation (3) describes the nonlinear oscillator motion in the potential well $U_o(y) = (1 - \cos y)$ when interacting with the force $f(y, \tau)$. If $f = 0$, the oscillator energy preserves, $\mathcal{E} = 0.5 y_\tau^2 + U(y)$, and $0 \le \mathcal{E} < 2$, $|y_\tau| < 2$ corresponds to the particle that has been captured in the well $-\pi < y < \pi$. The spectrum $E_n(k_n)$ that is considered to be given determines the wave amplitudes.

Below, a mechanism of charged particle interaction with waves package will be considered as follows. At the first stage, the force $f_1 \equiv A_1 \sin(q_1 y - \Omega_1 \tau + \psi_1)$ swings the oscillator in the potential well $U_0(y)$ on the basis of the resonance of the second order, when $\Omega_2 \approx 2$. The later causes a transition throughout the separatrix $\varepsilon = 2$. Further on, the charge is captured by the wave $E_1 \sin(q_1 x - \omega_1 \tau + \psi_1)$. At the second stage, the force $f_2 \equiv A_2 \sin(q_2 y - \Omega_2 \tau + \psi_2)$ accelerates the charge due to the resonance of the second order at the frequency Ω_2. The charge makes a transition throughout the separatrix into the potential well U_1, that is created by a wave moving with a phase velocity $v_\phi^{(1)} = \omega_1 / k_1$. There upon a capture by the wave 2 occurs (the wave 2 moves with phase velocity of ω_2 / k_2), and so on, up to the phase velocity of $v_\phi^{(N)} = \omega_N / k_N$. Thus, we consider a package in which the phase velocity $v_\phi^{(N)}$ increases when the number n increases while the charged particle acceleration by velocity is $\Delta v_f = (\omega_N / k_N) - (\omega_0 / k_0)$. In the case of electronic Lengmuir's waves in the plasma without magnetic field, the dispersion equation has the form: $\omega^2(k) = \omega^2{}_{pe}(1 + 3k^2 r_D^2)$, where ω_{pe} is the electron Lengmuir's frequency, $r_D = (T_e / m_e \omega^2{}_{pe})^{1/2}$ is the Debay radius. Hence, for the elementary Lengmuir waves the phase velocity increases when the vector k decreases. Taking into account this circumstance, further on we can, e.g., assume that $k_N = k_o + n\Delta k$, where $\Delta k = (k_N - k_0)/n < 0$. Then, $q_N = 1 + n\Delta q$, $\Delta q = (q_N - 1)/N$.

As we have already mentioned above, in the mechanism of charge acceleration under consideration the charge energy increasing is due: *i*) to the growing of its oscillations in the potential well U_N on the account of second order resonance with the mode (N+1), *ii*) to the transition throughout the separatrix and *iii*) to the capture into the potential well U_{N+1}. Further on the process repeats. For securing the process to be uninterrupted and cyclic it is necessary to have mating of the particles velocities intervals that are captured by neighbouring modes n and $n+1$.The necessary condition could be obtained by the way as follows. Let ε_n are some numbers from the interval (0,1), i.e. $0 < \varepsilon_n < 1$. The particle that has been captured in the potential well of the mode n, when crossing the separatrix in the well U_n will have a velocity $(\Omega_n / q_n) + 2(A_n / q_n)^{1/2}$. This velocity should correspond to the particle velocity that has been captured by the $(n+1)$ mode $(\Omega_{n+1} / q_{n+1}) - 2\varepsilon_{n+1}(A_{n+1} / q_{n+1})^{1/2}$. Hence, when choosing the mode parameters it is necessary to have in mind the condition of the mating of the captured charges velocities intervals,

$$\left(\Omega_n / q_n\right) + 2\left(A_n / q_n\right)^{1/2} > \left(\Omega_{n+1} / q_{n+1}\right) - 2\varepsilon_{n+1}\left(A_{n+1} / q_{n+1}\right)^{1/2} \tag{4}$$

Even, for exciting the particles oscillations by $(n+1)$ mode – particles that has been captured into a potential well U_n - it is necessary to ensure a second – order resonance, i.e. to satisfy the condition

$$\Omega_{n+1} - \Omega_n \frac{q_{n+1}}{q_n} \leq 2\sqrt{q_n A_n} \tag{5}$$

at the frequency of the $(n+1)$ mode in a system of coordinates that moves with phase velocity Ω_n / q_n and also at the bounce – frequency of particles oscillations captured in the potential well U_n.

Besides the conditions (4) and (5) let determine the amplitude A_n choice. First of all we notify that usually the amplitudes $E(k)$ change considerably at $\Delta k \sim k$ for an sufficiently wide wave spectrum. In the case under consideration $\Delta n \sim 1$ is connected with a lower change of the wave vector $\Delta k << k$. That is why A_n could be regarded to be a slowly changing functions of the mode number *n*, i.e. $|A_{n+1} - A_n| << |A_n|$. Depending on the conditions of wave package generation, A_n could decrease as well as increase when increasing *n*. Here we will study the case of A_n decreasing when increasing *n*. The rate of A_n decreasing is determined by the number of modes that take part in the charged particles acceleration as the minimum A_n will not ensure yet enough exciting of the nonlinear oscillator in the condition of the second order resonance.

The last question left is the choice of the mode phases Ψ_n. Below, they have been chosen by an experimental way from the interval $(-\pi, \pi)$. But we should notice that, accordingly to the calculations made for each mode *n*, there exist several subintervals of favorable phase Ψ_n that further charges acceleration.

2. Numerical study of charge acceleration by wave packages

In the course of numerical study of Eq.(3), the starting point was the consideration of the influence on the oscillator $y(\tau)$ with two modes $n = 1$ and $n = 2$ having parameters as follows:

$$\begin{aligned} &A_1 = 0.8, \quad q_1 = 0.85, \quad \Omega_1 = 1.87, \quad \Psi_1 = \pi/4, \quad \Omega_1/q_1 \approx 2.2 \\ &A_2 = 0.6, \quad q_2 = 0.65, \quad \Omega_2 = 3.08, \quad \Psi_2 = \pi/2, \quad \Omega_2/q_2 \approx 4.738 \end{aligned} \tag{6}$$

The initial conditions for the oscillator *y* have been taken to be

$y(0)=\dot{y}(0)=0$. The calculations have showed that the mode 1, exciting the oscillator due to the second order resonance, throws it away from the potential well U_o. We have an analytical approximation for the average position $\langle y(\tau)\rangle$ in the time period $\tau \le 380$ presented in the form: $\langle y(\tau)\rangle \cong 722+2.2(\tau-342)$. This corresponds to the capture of the particle by the mode 1, i.e. into the potential well $U_1(y,\tau)$. Further on, the particle is captured by the mode 2 and the averaged by the fast oscillations charge position is described by an approximation as follows: $\langle y(\tau)\rangle \cong 703.25+4.7384(\tau-356)$, where $\tau \in (380,1700)$.

Taking into account the dispersion, in the onset of the mentioned interval by τ it was $|(y(\tau)-<y(\tau)>| \le 0.4$ while by the end $\tau \sim 1700$ the oscillation amplitude in the potential well U_2 was decreased down to the level $|(y-<y>| \le 0.26$.

Afterwards, taking into account the mode 3, the Eq.(3) became to be characterized with parameters

$A_3=0.5,\quad q_3=0.55,\quad \Omega_3=3.855,\quad \Psi_3=23\pi/80,\quad \Omega_3/q_3 \approx 7.009$.

Parameters (6) have been used for the modes 1 and 2. The calculations have confirmed the acceleration scenario presented above: gaining energy in the potential wells U_o,U_1,U_2, thereafter charge is captured by the mode 3. For $\tau>936$ its average position can be determined by an approximation as follows: $\langle y(\tau)\rangle \cong 2026.23+7.009(\tau-947)$. At $\tau>1274$ the amplitude of frequency deviation in the potential well, created by the mode 3, is not large: $|(y(\tau)-<y(\tau)>| \le 0.5$. The diagram presented in Fig. 1 illustrates the charge position $y(\tau)$ when acting with three modes having parameters indicated above and at the interval $\tau<2600$. The character of charge oscillations after its caption by the mode 3 is shown in Fig 2. The phase plane (y,p), where $p=\dfrac{dy}{d\tau}$, is presented in Fig. 3 for the interval $\tau<200$ and in the Fig .4 – for the end of the calculations interval $\tau \in (1700,2000)$.

The method of wave parameters selection that has been described above convincingly demonstrates the possibility of increasing of the mode number that take part in the process of charged particles acceleration and furthers increasing of accelerated charges energy. In the following paper we intend to analyze a non-diffusive acceleration by a package of ten modes.

3. Conclusion

The results of the analysis presented can be summarized as follows. A mechanism has been proposed of non-diffusive charged particles acceleration with a package of electrostatic waves. The mechanism is caused by a process of growing oscillations of the particle that has been captured into the potential well. The latter is due to the second order resonance at the neighbor mode that has a larger phase velocity with a transition throughout the separatrix, and particle capturing by another wave. The process described repeats cyclicly so as the consequence of resonance interactions to correspond to the ever increasing waves phase velocities. A method is formulated of selecting parameters of the wave sequence that realizes the acceleration mechanism described above. Supporting numerical calculations have been made. The developed method for analysis allows adding by a simple enough way new modes to the rest and in this way – increasing the charged particles energy. For the mechanism under consideration the nonlinearity of dispersion equation $\omega(k)$ is of a principle importance.

$$\frac{d^2}{dt^2}y(t) + \sin(y(t)) + b\cdot\frac{d}{dt}y(t) = \left(0.1 + \alpha\cdot\frac{t}{\beta + t}\right)\cdot\sin\left[k\cdot y(t) - \left(\Omega - 0.07\cdot\frac{t}{100 + t}\right)\cdot t + \phi\right]$$

$\phi := \frac{\pi}{1n}$ $\quad A := 0.1$ $\quad k := 0.6$ $\quad \Omega := 1.98$ $\quad b := 0.005$ $\quad \alpha := 0.5$ $\quad \beta := 100$ $\quad y(0) = 0$ $\quad y'(0) = 0$

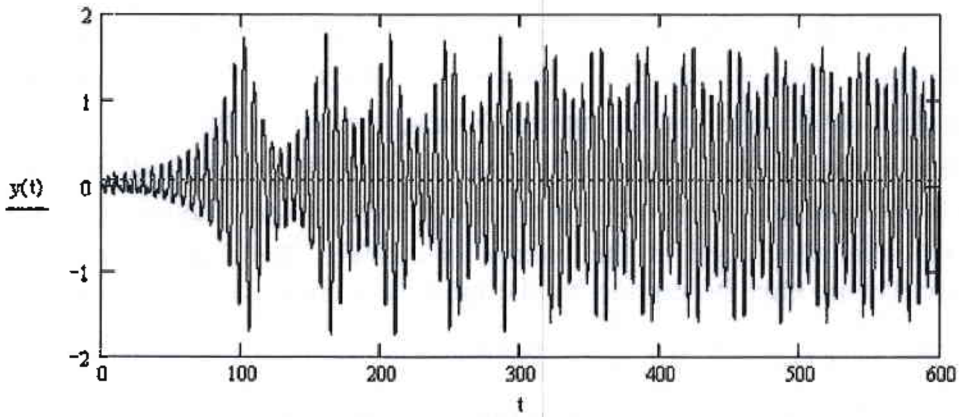


Fig. 1

$\alpha := 0.7$

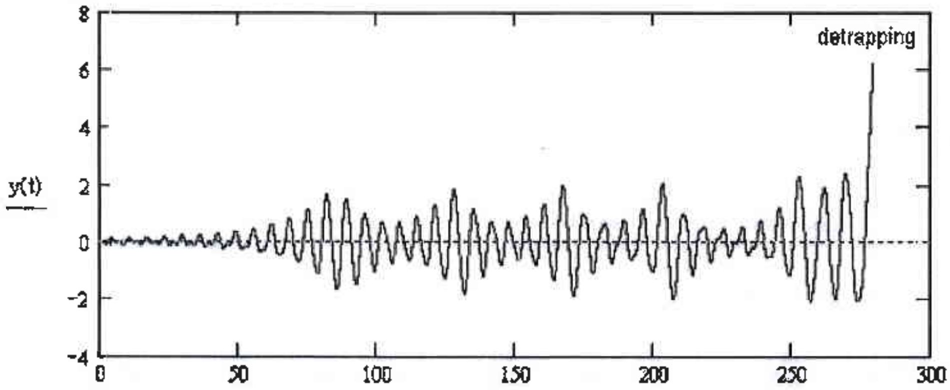
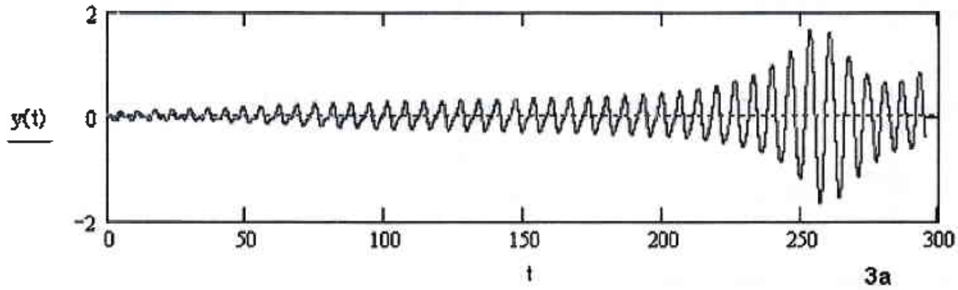


Fig. 2

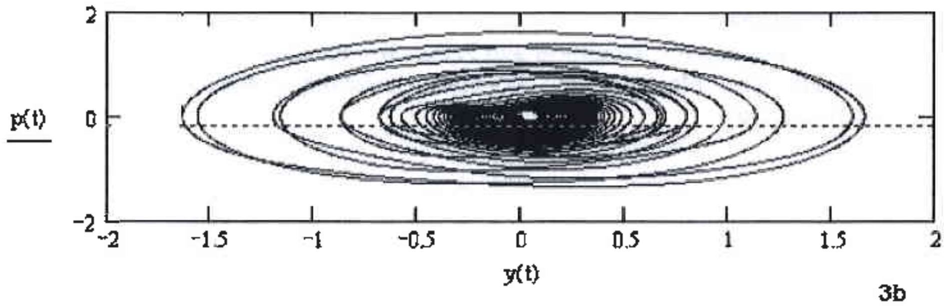
$\phi := \frac{\pi}{10}$ $\quad A := 0.1$ $\quad k := 0.6$ $\quad \Omega := 2$ $\quad b := 0.01$ $\quad \alpha := 0.3$ $\quad \beta := 130$ $\qquad y(0) = 0$ $\quad y'(0) = 0$

$$\frac{d^2}{dt^2}y(t) + \sin(y(t)) + b\cdot\frac{d}{dt}y(t) = \left(0.1 + \alpha\cdot\frac{t}{\beta + t}\right)\cdot\sin\left[k\cdot y(t) - \left(\Omega - 0.11\cdot\frac{t}{100 + t}\right)\cdot t + \phi\right]$$

$$p(t) := \frac{d}{dt}y(t)$$



3a



3b

Fig. 3

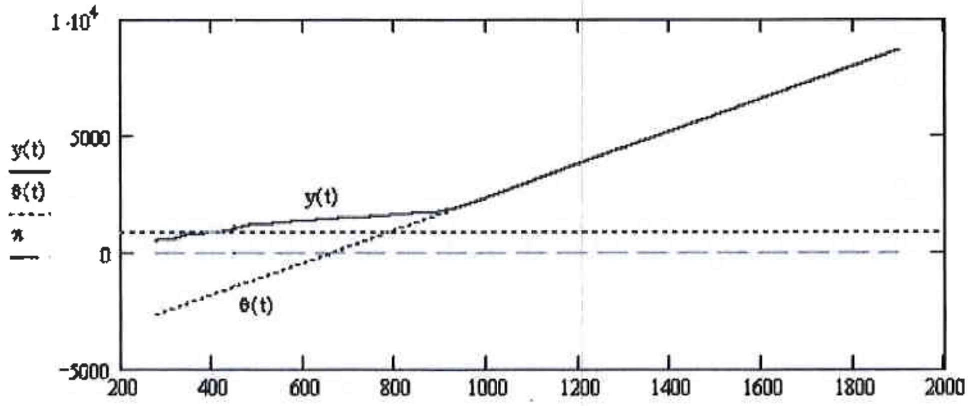


Fig. 4

Particle acceleration by waves

$$\frac{d^2}{dt^2}y(t) + \sin(y(t)) + b\cdot\frac{d}{dt}y(t) = A2\cdot\sin(k2\cdot y(t) - \Omega2\cdot t + \phi2) + A3\cdot\sin(k3\cdot y(t) - \Omega3\cdot t + \phi3) + A4\cdot\sin(k4\cdot y(t) - \Omega4\cdot t + \phi4)$$

$\phi2 := \frac{5\pi}{20}$ $A2 := 0.8$ $k2 := 0.85$ $\Omega2 := 1.87$ $\frac{\Omega2}{k2} = 2.2$ $b := 0.01$

$\phi3 := \frac{5\pi}{10}$ $A3 := 0.6$ $k3 := 0.65$ $\Omega3 := 3.08$ $\frac{\Omega3}{k3} = 4.738$

$\phi4 := \frac{23\cdot\pi}{80}$ $A4 := 0.5$ $k4 := 0.55$ $\Omega4 := 3.855$ $\frac{\Omega4}{k4} = 7.009$

$\theta(t) := 2026.23 + 7.009\cdot(t - 947)$

References

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НЕДИФУЗИОНЕН МЕХАНИЗЪМ НА УСКОРЯВАНЕ НА ЗАРЕДЕНИ ЧАСТИЦИ ПОД ДЕЙСТВИЕ НА ЕЛЕКТРОСТАТИЧЕН ВЪЛНОВ ПАКЕТ

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Резюме

Разгледан е нов механизъм за недифузионно ускоряване на заредени частици от пакет електростатични вълни, обусловен от взаимодействието вълна-частица при резонанс от втори ред, съответстващ на възбуждането на нелинейния осцилатор от външна сила. С числени пресмятания е показано, че в процеса на резонансното взаимодействие се получава прехвърляне на ускоряваната частица от една потенциална яма в друга, движеща се с по-голяма скорост. Поредицата от такива прехвърляния през сепаратрисата допринася за ускоряването на зарядите с многократно увеличение на тяхната кинетична енергия. Този механизъм за ускоряване на заредените частици се реализира при изпълнение на следните условия. Ако зарядът е прихванат в потенциалната яма на n - ната хармонична от вълновия пакет с честота ω_n, вълнов вектор κ_n и амплитуда E_n, следва да се реализира резонанс от втори ред с *(n+1) -ва* хармонична. Фазовата скорост на хармоничната ω_n/k_n следва да нараства с увеличението на и, т.е. $\omega_{n+1}/\kappa_{n+1} > \omega_n/k_n$. За съседните хармонични обхватите на скоростите на прихванатите частици трябва частично да се препокриват. Амплитудите на вълните следва да са достатъчни за възбуждане на осцилатора и прехвърляне на заредената частица през сепаратрисата. Подробно е описан този нов механизъм на недифузионно ускоряване на заряди от пакети електростатични вълни с малка, но крайна амплитуда. Формулирана е процедура за отбор на параметрите за редицата от хармонични в пакета, участващи при ускоряването на заредените частици. Даденият ефект представлява интерес, в частност, при решаване на проблема за генерацията на космически лъчи и при интерпретацията на механизмите на произход на потоци ускорени частици (електрони и йони), наблюдавани в космическата плазма.