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THE INFLUENCE OF SOME SPECIFIC TYPES OF INSTABILITY ON STRUCTURE FORMATIONS IN ACCRETION DISCS

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Abstract

When we investigate the different structures in accretion flows, we reveal the relation between the generation of a certain type of instability and the arising of a structure formation mechanism, when all required conditions are available.Here, we shall consider the effect of some instabilities and the formations generated by them.

Magnetohydrodynamical instabilities

a) Balbus-Hawley instability

There are places in hydrodynamical flows, where the velocity field changes abruptly (the shock fronts). In these places, as a result of the differential rotation of the parts of the flow with great differences in density and velocity, conditions are generated for magnetic shear instability, which is known as Balbus-Hawley instability. In the presence of a magnetic field, destabilization effect of the differentially rotating flows is available, and this instability is the mechanism generating flow turbulence [7].

b) Kelvin - Helmholtz instability

This instability acts on the boundary of two fluid flows, which in our case could be the two parts of an accretion flow. If the boundary is weakly perturbed, velocities increase and at different densities, the following instability condition is derived:

$$(1) \qquad \rho_1\rho_2\left((v_1 - v_2)k\right)^2 \leq (\rho_1 + \rho_2)(\rho_1 - \rho_2)k_\lambda g$$

where v_1 and v_2 are the velocities of two flows

and ρ_1 , ρ_2 are their densities.

The relation between frequency ω and wave vector k_λ is given by the dispersion relation [4]:

$$(2) \quad \frac{\omega}{k} = \frac{\rho_1 v_1 + \rho_2 v_2}{\rho_1 + \rho_2} \pm \left[\frac{g}{k}\left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right) - \frac{\rho_1 \rho_2 (v_1 - v_2)^2}{(v_1 + v_2)^2} \right]^{1/2}$$

The Kelvin - Helmholtz instability is generated when the expression in the square brackets of the above relation is negative and there is a difference in the two flows' velocities. In the presence of this instability, undulations are formed at the boundary; as a result of this, with further progressing of this instability, vortices are formed.

Other instabilities

The generation of structures in accretion discs is caused not only by magnetohydrodynamical instability. In the fluid media of accretion discs, conditions for other types of instabilities are observed.

a) Turing instability

In some systems, the coupling between two transport processes generates instability mechanism. Then the development of this instability is determined by the difference of the diffusion coefficients along the different directions of the transport acting there [1].

The diffusion coefficient participates in the reaction-diffusion equation, which has the following standard form [2]:

$$(2) \quad \frac{\partial C}{\partial t} = F(C) + D\nabla^2 C$$

where the first term in the right-hand side is reaction and the second is diffusion. D is the diffusion coefficient (or matrix of the transport coefficient), C - a concentration of matter.

The reaction-diffusion systems are a manifestation of spatial or temporal patterns if they are far from thermo-dynamical equilibrium [2], which is an important condition for dissipative structures' formation [8]. An key aspect of all application of the reaction-diffusion equation, such as partial differential equation, is this simple combination of reaction and diffusion in the right-hand side of the equation.

Then, taking into account the condition for Turing instability, the reaction-diffusion equation takes the form:

$$(3)\qquad \frac{\partial C}{\partial t} = F(C) + D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2}$$

This difference between the diffusion coefficients of the two components is the necessary restriction for generation of Turing instability [2].

This assertion can be derived by the following representations and transformations.

The accretion disc is considered as a hydro-dynamical system and is described by hydro-dynamical equations [3].

First, this is the continuity equation, which expresses mass conservation:

$$(4)\qquad \frac{\partial \rho}{\partial t} + \nabla.(\rho v) = 0$$

The conservation of momentum for each gas element is represented by Euler equation:

$$(5)\qquad \rho \frac{\partial v}{\partial t} + \rho v.\nabla v = -\nabla P + F$$

where for both equations, the quantities ρ, v, P, F are respectively: density, velocity, pressure and a certain force.

Let us present the motion equation for viscous fluid (Navier-Stokes eq.) in cylindrical coordinates. Because averaging takes place along the z-direction, we shall express all derivatives in the terms of the coordinates (r, φ):

$$(6)\qquad \begin{aligned} &\frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + \frac{V_\varphi}{r}\frac{\partial V_r}{\partial \varphi} - \frac{V_\varphi{}^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{1}{\rho} F_r + \\ &\nu\left(\frac{\partial^2 V_r}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \varphi^2} + \frac{1}{r}\frac{\partial V_r}{\partial r} - \frac{2}{r^2}\frac{\partial V_\varphi}{\partial \varphi} - \frac{V_r}{r^2}\right) \end{aligned}$$

$$(7)\quad \frac{\partial V_\varphi}{\partial t}+V_r\frac{\partial V_\varphi}{\partial r}+\frac{V_\varphi}{r}\frac{\partial V_\varphi}{\partial \varphi}-\frac{V_r V_\varphi}{r}=-\frac{1}{\rho r}\frac{\partial P}{\partial \varphi}+\frac{1}{\rho}F_\varphi+ \nu\left(\frac{\partial^2 V_\varphi}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 V_\varphi}{\partial \varphi^2}+\frac{1}{r}\frac{\partial V_\varphi}{\partial r}-\frac{2}{r^2}\frac{\partial V_r}{\partial \varphi}-\frac{V_\varphi}{r^2}\right)$$

Here ν is kinematic viscosity, V_r and V_φ are the two velocity components.

The energy transfer equation can be represented as follows:

$$(9)\quad \frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2+\rho\varepsilon\right)+\left[\left(\frac{1}{2}\rho v^2+\rho\varepsilon+P\right)v\right]=f.v-\nabla.F_{rad}$$

where $\frac{1}{2}\rho v^2$ - the kinetic energy per unit volume,

$\rho\varepsilon$ - internal or thermal energy per unit volume.

The last term in the square brackets represents the so-called pressure work.

On the right-hand side:

F_{rad} - the radiative flux vector;

$-\nabla.F_{rad}$ - expresses the rate at which radiant energy is being lost by emission, or increased by absorption.

In an accretion disc we consider the transport of "vortical" function or vorticity, which may be denoted by Ψ. This term is provided by vortical equation [5]:

$$(10)\quad \left(\frac{\partial}{\partial t}+\vec{v}.\nabla\right)\frac{\nabla\times\vec{v}}{\rho}=0$$

which is obtained combining the rotation of the momentum equation and the continuity equation. Thus, $\Psi=\nabla\times\vec{v}$ and eq. (10) yields:

$$(11)\quad \left[\frac{\partial\Psi}{\partial t}+\nabla.(\Psi\vec{v})\right]\frac{1}{\rho}=f$$

We express this equation in cylindrical coordinates in terms of the r,φ again:

$$(12)\quad [\ \frac{\partial \Psi_r}{\partial t} + V_r \frac{\partial \Psi_r}{\partial r} + \frac{\Psi_\varphi}{r}\frac{\partial V_r}{\partial \varphi} - \frac{\Psi_\varphi^{\ 2}}{r}\ ]\frac{1}{\rho} = f$$

$$(13)\quad [\ \frac{\partial \Psi_\varphi}{\partial t} + V_r \frac{\partial \Psi_\varphi}{\partial r} + \frac{\Psi_\varphi}{r}\frac{\partial V_\varphi}{\partial \varphi} - \frac{\Psi_r \Psi_\varphi}{r}\ ]\frac{1}{\rho} = f$$

Here, f expresses the transport mechanism of the vortex or this is the diffusion from eq. (3) and in our consideration, f has the form: $D\nabla^2\Psi$.

Taking into account vortical equation (11) and expressions (12) and (13), the reaction-diffusion equation (3) takes the form [8]:

$$(14)\quad \frac{\partial \Psi_r}{\partial t} = h(r,\varphi) + D_r \nabla^2 \Psi_r$$

$$(15)\quad \frac{\partial \Psi_\varphi}{\partial t} = g(r,\varphi) + D_\varphi \nabla^2 \Psi_\varphi$$

where h and g are the source functions, having the form: $(\Psi.\nabla)v$

Thus we obtain two equations with different diffusion coefficients, expressed for both components.

The evidence that, in accretion discs, the necessary restriction for the ratio between D_r and D_φ to be not equal to unit confirms the possibility for generation of Turing instability in this reaction-diffusion system.

Since these instabilities are the expression of a spatial pattern for the bifurcation area, as a result of them, some structures may be formed in the disc, namely: vortical structures and the so-called Rossby solitons.

The theory of the Turing structures is just one of a variety of such mechanisms of pattern structures formation; here, we showed that this mechanism holds for the hydro-dynamical system of the accretion disk.

But what do actually the Rossby vortices represent and which other instabilities give rise to them.

b) Rossby instability

In studying a non-magnetized Keplerian accretion dişc, as a result of non-axisymmetric perturbations, instability arises which generates Rossby vortices in non-linear limit. The presence of such vortices might be crucial for the hydro-dynamical transport of angular momentum in accretion discs [6]. A wave of non-linear Rossby vortices carries the mass and entropy maximum inward, exciting further vortices, which transport the angular momentum outward.

Here, we shall use again the cylindrical system of coordinate to express the basic equations of a non-barotropic disc. We shall consider surface density $\Sigma(r) = \int_{-h}^{h} dz\rho(r,z)$ and vertically integrated pressure $P(r) = \int_{-h}^{h} dzp(r,z)$. The perturbed quantities of surface density, pressure and velocity are expressed as follows:

$$
\begin{aligned}
\tilde{\Sigma} &= \Sigma + \delta\Sigma(r,\phi,t) \\
\tilde{P} &= P + \delta P(r,\phi.t) \\
\tilde{v} &= v + \delta v(r,\phi,t)
\end{aligned}
\qquad (16)
$$

Thus, we obtain the equations for the perturbed disc:

$$
\begin{aligned}
&\frac{D\tilde{\Sigma}}{Dt} + \tilde{\Sigma}\nabla.\tilde{v} = 0 \\
&\frac{D\tilde{v}}{Dt} = -\frac{1}{\tilde{\Sigma}}\nabla\tilde{P} - \nabla\Phi \\
&\frac{D}{Dt}\left(\frac{\tilde{P}}{\tilde{\Sigma}^{\Gamma}}\right) = 0
\end{aligned}
\qquad (17)
$$

where $D/Dt = \partial/\partial t + v.\nabla$ and $S = P/\Sigma^{\Gamma}$ is the entropy of the disc matter. Here, the last eq. ( ) shows the isentropic behavior of the disc matter.

Since this instability is related to the entropy behaviour, ultimately, the so-called key function $\Re(r) = \Lambda(r)S^{2/\Gamma}(r)$ is derived, which has a maximum or minimum. Then, instability is possible only provided $\ln\left(\Lambda S^{2/\Gamma}\right)$ disappears at some r.

Here, in the way described above, we obtain again the vortical equation in the form:

$$\frac{D}{Dt}\left(\frac{\Psi_z}{\Sigma}\right)=\frac{\nabla\Sigma\times\nabla P}{\Sigma^3}, \text{ where } \Psi_z=\hat{z}.\nabla\times v \text{ is the vorticity.}$$

For a barotropic flow the right-hand side of the equation is zero and each fluid element conserves its specific vorticity.

In the opposite case, the term $\nabla\Sigma\times\nabla P\propto\nabla T\times\nabla S$ destroys this conservation, providing the pressure force to generate vortices in the flow.

Comments

Here we have not mentioned all types of instabilities, which act in accretion flow in general. Our aim was to show their reference to structures formation in accretion discs. This was proven by analytical computation, based on the major accretion disc equations and the relevant instability expressions.

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ВЛИЯНИЕТО НА ОПРЕДЕЛЕН ВИД НЕУСТОЙЧИВОСТИ ПРИ СТРУКТУРООБРАЗУВАНЕТО В АКРЕЦИОНЕН ДИСК

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Резюме

От важно значение при изследването на акреционните течения е откриването на връзката между появата на вид неустойчивост и възникването на механизъм за структурообразуване, като са налице всички изисквани условия за този процес. Тук ще разгледаме проявата само на някои видове неустойчивости и зараждащите се вследствие на тях формирования.