

## POSSIBILITIES FOR APPLYING PHASE-MANIPULATED COMPLEMENTARY SIGNALS IN SPACECRAFT-BASED RADARS

*Borislav Bedzhev*

*National Military University, Artillery Department,  
Air-Defence and Communication and Information Systems*

### **Abstract**

*In spacecraft-based radar systems, high-quality images of the planet, satellite, and comet surface are obtained using the Synthetic Aperture Radar (SAR) method. Moreover, to provide both wide performance range and great distance resolution, complex radar signals with inner pulse modulation are applied.*

*Currently, space SARs use mostly linear frequency modulation (LFM) signals. However, alongside with their positive features, they also possess some disadvantages. At the same time, in the recent dozen years or so, signals with internal pulse phase manipulation (PM) became widely applied.*

*The results of the paper may be summarized as follows:*

*1. A mathematical method for synthesizing a new class of PM signals called generalized complementary signals (GCS), whose ACF features close-to zero side lobe level has been developed.*

*2. Through computer simulation of radiolocation systems with inverse synthesized aperture, the applicability of GCS in prospective spacecraft-based radar systems has been proven.*

### **1. Introduction.**

As it is known [1], in spacecraft-based radar systems, the method of synthetic aperture is widely used in order to obtain high-quality images of planet, satellite and comet surfaces. In this process, complex radar signals with inner pulse modulation are applied because they provide both wide performance range and great distance resolution.

The most important feature of the autocorrelation function (ACF) of synthetic aperture radar (SAR) signals is the level of their side lobes, because they determine the dynamic range of the image and the possibility to identify small objects. At present, in spacecraft-based radars, signals with linear frequency modulation are widely applied. They were proposed fifty

years ago, but despite their positive qualities, they have some disadvantages as well, such as a relatively high level of ACF's side lobes and complex generating and processing hardware [2]. In this regard, it should be emphasized that, recently, signals with inner pulse phase manipulation (PM) have found wide application in wireless communications. For instance, PM signalling is the basis of the so-called "Direct Sequence Spread Spectrum" techniques [3]. As a result, the PM signal generating and processing hardware has improved drastically.

In view of the above-mentioned facts, this paper aims:

- to suggest mathematical methods for synthesis of a new class of PM signals, named generalized complementary signals, featuring ACF with close-to-zero level of the side lobes;

- to illustrate the advantages of the usage of generalized complementary signals in prospective SARs.

## 2. Method of phase manipulated complementary signals synthesis

It is known [4], that PM (or DS-SS) signals represent sequences of  $n$  equivalent elementary pulses which are described by:

$$(1) \quad v(t) = \sum_{j=1}^n U_j \cdot u_0(t - t_j) \cdot \cos[\omega_0 \cdot (t - t_j) + \theta_j],$$

where:

-  $U_j$  are the amplitudes of the elementary pulses;

-  $\omega_0 = 2\pi f_0$ ;  $f_0$  is the carrier frequency;

$$u_0(t) = \begin{cases} 1, & \text{if } 0 \leq t \leq \tau_0 \\ 0, & \text{if } t < 0, \text{ or } t > \tau_0 \end{cases}.$$

To simplify the practical accomplishment of the complex process of PM signal receiving, the following limitations in formula (1) are made [3, 4]:

-  $\tau_0 = \text{const}$ ;  $U_j = U_0 = \text{const}$ ;  $j = 1, 2, \dots, n$ ;

-  $\theta_j \in \{(2\pi l) / m$ ;  $l = 0, 1, \dots, m - 1\}$ .

In this case, the PM signal can be described as a sequence of complex amplitudes of elementary signals [4]:

$$V(t) = \sum_{j=1}^n U_0 \cdot \zeta(j) \cdot u_0(t - t_j),$$

where  $\{\zeta(j)\}_{j=0}^{n-1}$  is the set of complex amplitudes of the elementary pulses and the elements of the set are the  $m$ -th roots of the unity:

$$(2) \quad \zeta(j) \in \{\exp(2\pi i l / m); l = 0, 1, \dots, m - 1\}.$$

It is known that complementary series are a pair of two special PM signals, whose aggregated non-periodical auto-correlation function (ACF) resembles a delta pulse. The classical Golay's definition of the complementary series [5] is:

**Definition 1:** The sequences  $\{\mu(j)\}_{j=0}^{n-1}$ ,  $\{\eta(j)\}_{j=0}^{n-1}$ , consisting of  $n$  elements with values  $+1$  and  $-1$ :  $\mu(j) \in \{-1, +1\}$ ;  $\eta(j) \in \{-1, +1\}$ ;  $j = 0, 1, \dots, n-1$ , are called pair of complementary series, if:

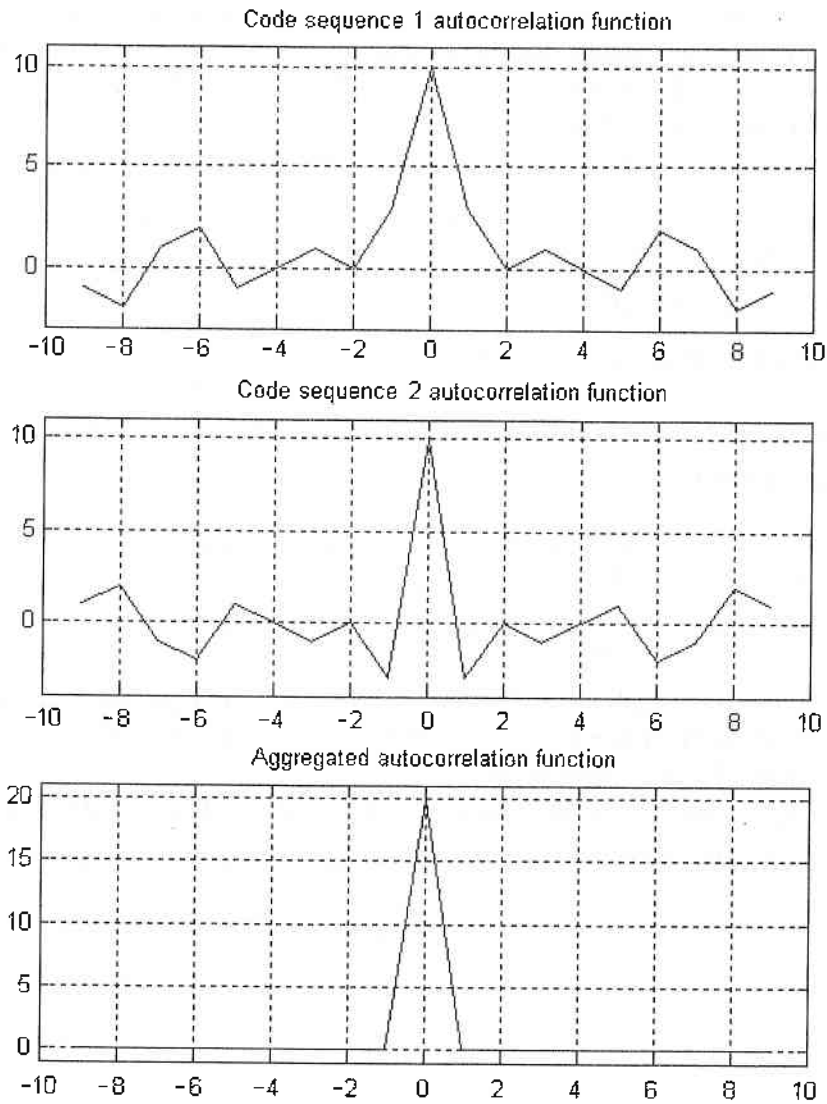
$$(3) \quad R_c(k) = R_\mu(k) + R_\eta(k) = \begin{cases} 2n; & k = 0 \\ 0; & k = \pm 1, \pm 2, \dots, \pm(n-1) \end{cases}$$

In (3), the non-periodical ACFs  $R_\mu(k)$  и  $R_\eta(k)$  are defined by the well known formula [3, 4]:

$$(4) \quad R_\zeta(k) = \begin{cases} \sum_{j=0}^{n-1-k} \zeta(j) \zeta^*(j+|k|), & -(n-1) \leq k \leq 0 \\ \sum_{j=0}^{n-1-k} \zeta^*(j) \zeta(j+k), & 0 \leq k \leq n-1 \end{cases}$$

Here, the symbol "\*" means a complex conjugation. The above definition of complementary series will be clarified in Fig. 1 by an example of sequences of code-length  $n=10$ :

$\mu(0)=-1, \mu(1)=-1, \mu(2)=-1, \mu(3)=-1, \mu(4)=-1, \mu(5)=1, \mu(6)=-1, \mu(7)=1, \mu(8)=1, \mu(9)=-1$ ;  
 $\eta(0)=1, \eta(1)=-1, \eta(2)=1, \eta(3)=-1, \eta(4)=-1, \eta(5)=-1, \eta(6)=1, \eta(7)=1, \eta(8)=-1, \eta(9)=-1$ .



**Fig.1.** Autocorrelation functions of complementary sequences with  $n = 10$  elements.

The complementary series are unique among all PM signals for the following features:

- their aggregated ACF has an ideal shape resembling a delta pulse;
- if a pair of complementary series, consisting of  $n$  elements, is known, then it is easy to create an infinite set of pairs with unlimited code-length.

With respect to the second feature, it is necessary to emphasize that most PM type signals with close to ideal ACF have limited code-length. For instance, Barker codes exist only for  $n \leq 13$ , if  $n$  is an odd integer.

In the original Golay's paper, two theorems are proved [5], which show the way we can obtain derivative complementary series with code-length  $2n.r$  if two pairs of code-length  $n$  and  $r$  are known. The theorems are similar and in order to simplify the explanation, they will be combined as follows.

Theorem 1 (Golay's Theorem): If the two pairs of complementary series  $A = \{\mu(j)\}_{j=0}^{n-1}$ ,  $B = \{\eta(j)\}_{j=0}^{n-1}$ ; and  $C = \{\xi(j)\}_{j=0}^{r-1}$ ,  $D = \{\zeta(j)\}_{j=0}^{r-1}$  of code-length  $n$  and  $r$  respectively, are known, then the derivative pairs of sequences:

(5a)

$$K = \{\xi(0).A, \xi(1).A, \dots, \xi(r-1).A, \zeta(0).B, \zeta(1).B, \dots, \zeta(r-1).B\};$$

$$L = \{\zeta(r-1).A, \zeta(r-2).A, \dots, \zeta(0).A, -\xi(r-1).B, -\xi(r-2).B, \dots, -\xi(0).B\};$$

(5b)

$$M = \{\xi(0).A, \zeta(0).B, \xi(1).A, \zeta(1).B, \dots, \xi(r-1).A, \zeta(r-1).B\};$$

$$N = \{\zeta(r-1).A, -\xi(r-1).B, \zeta(r-2).A, -\xi(r-2).B, \dots, \zeta(0).A, -\xi(0).B\};$$

are complementary sequences of code-length  $2nr$ .

Complementary series of code-length  $n = 2, 10, 26$  are known at present. Using them and Golay's theorem makes it possible to create infinite number of complementary series of code-lengths:

$$(6) \quad n = 2^a \cdot 10^{b_2} \cdot 26^{b_3}.$$

It is necessary to emphasize that Golay's definition of complementary codes is not useful in some important cases. This situation has motivated some theoreticians like Tseng, Liu, Suehiro, Ignatov [ 6, 7, 8], to extend the classical definition. Namely, they have proposed the so-called "generalized complementary codes" which constitute a set of  $p$  elements (PM signals), the aggregate ACF of all sequences having an ideal shape resembling a delta pulse. Consequently, Golay's codes are a particular case of Tseng-Liu codes, where  $p=2$ . Now it ought to be seen, that:

- the PM generating hardware is drastically enhanced;
- the binary phase modulation isn't appropriate quite often, because it restricts the rate of information translation and/or the possible variants in the process of developing communication devices.

With regard to this, in the next part of the paper, we shall "correct" Definition 1 as follows.

Definition 2: *The set of  $p$  sequences (PM signals), whose elements are complex numbers, belonging to the multiplicative group of the  $m$ -th ( $m > 2$ ) roots of unity:*

$$(7) \quad \{A_1 = \{\xi_1(j)\}_{j=0}^{n_1-1}; A_2 = \{\xi_2(j)\}_{j=0}^{n_2-1}; \dots; A_p = \{\xi_p(j)\}_{j=0}^{n_p-1}\};$$

$$\xi_k(j) \in \{\exp(2\pi i l / m_k); l = 0, 1, \dots, m_k - 1\}; k = 1, 2, \dots, p.$$

*are a set of generalized complementary codes (sequences) if and only if their aggregate ACF has the ideal shape, resembling a delta pulse:*

$$(8) \quad R_c(r) = \sum_{k=1}^p R_{A_k}(r) = \begin{cases} n = n_1 + n_2 + \dots + n_p; & \text{if } r = 0; \\ 0; & \text{if } r = 1, 2, \dots, \max\{n_k\}. \end{cases}$$

This definition will be illustrated by the following so-far-unknown complementary codes ( $m = 4$  in (2),  $p = 2$ ):

$$(9) \quad \{\mu(j)\}_{j=0}^2 = \{1, i, 1\}; \quad \{\eta(j)\}_{j=0}^2 = \{1, 1, -1\};$$

$$\{\mu(j)\}_{j=0}^4 = \{-i, i, 1, 1, 1\}; \quad \{\eta(j)\}_{j=0}^4 = \{1, i, -1, 1 - i\}.$$

Now we shall see whether it is possible to create sets of complementary sequences of unlimited code-length, using some initial sets of complementary sequences of short code-length. In order to reach this goal, we shall prove a theorem, whose particular case is Golay's theorem.

It should be emphasized that a common theorem couldn't be developed based on Theorem 1, because the method of Golay, Tseng and Liu is not applicable for the situation of generalized complementary codes, according to Definition 2. Therefore, we shall use a new algebraic method [9]. At the beginning, we shall introduce some terms, used in the common theorem.

Definition 3: The matrix  $H_{q,p} = \{h_{k,l}(x)\}; k = 1, 2, \dots, q; l = 1, 2, \dots, p$  will be called *generalized column orthogonal matrix*, if:

$$(10) \quad \sum_{k=1}^q h_{k,j}(x) h_{k,s}^*(x^{-1}) = \begin{cases} c_j; & c_j = \text{const}, \quad \text{if } j = s; \\ 0, & \text{if } j \neq s; j = 1, 2, \dots, p; s = 1, 2, \dots, p. \end{cases}$$

*The entries of the matrix  $H_{q,p}$  in (10) are polynomials with equivalent maximal power of  $x$  in each column:  $\deg h_{k,j}(x) = r_j - 1$ .*

It is easy to see that the column orthogonal matrices satisfy the equation:

$$(H^*)^T \cdot H = \begin{bmatrix} c_1 & 0 & \dots & 0 \\ 0 & c_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & c_p \end{bmatrix}.$$

Here:

- the superscript "T" means "changing the places of the rows and columns";

- the subscripts are omitted in order to simplify the expression.

Examples of the matrix  $H$  when  $\deg h_{j,k}(x) = 0$  are:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}, \begin{bmatrix} \omega & 1 & 1 \\ 1 & \omega & 1 \\ 1 & 1 & \omega \end{bmatrix}, \text{ i.e., where: } \omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2};$$

$$\omega^2 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}, 1 + \omega + \omega^2 = 0.$$

Now we are able to prove the following common theorem.

Theorem 2: Let  $H_{p,q}$  be generalized orthogonal column matrix, and let  $\{A_k = \{\xi_k(j)\}_{j=n_1+\dots+n_{k-1}}^{n_k-1}\}_{k=1}^p$  be a set of generalized complementary sequences. Then the set:

(11)

$$\{h_{11} \cdot A_1, h_{12} \cdot A_2, \dots, h_{1p} \cdot A_p\}; \{h_{21} \cdot A_1, h_{22} \cdot A_2, \dots, h_{2p} \cdot A_p\}; \dots; \{h_{q1} \cdot A_1, h_{q2} \cdot A_2, \dots, h_{qp} \cdot A_p\};$$

is a set of generalized complementary codes, too. Here, multiplications in (11) mean:

$$(12) \quad h_{ij} \cdot A_k = \zeta_{ij}(0) \cdot A_k, \zeta_{ij}(1) \cdot A_k, \dots, \zeta_{ij}(r_{ij}-1) \cdot A_k,$$

if:

$$(13) \quad h_{ij}(x) = \zeta_{ij}(r_{ij}-1) \cdot x^{r_{ij}-1} + \zeta_{ij}(r_{ij}-2) \cdot x^{r_{ij}-2} + \dots + \zeta_{ij}(1) \cdot x + \zeta_{ij}(0).$$

*Proof:* The proof will be made using the so-called creative functions method [10]. According to this method, the ACF of the PM signal will be presented by the following polynomial:

(14)

$$P(x) = F(x).F^*(x^{-1}) = R_{\zeta\zeta}^{*}(-(n-1)).x^{-(n-1)} + R_{\zeta\zeta}^{*}(-(n-2)).x^{-(n-2)} + \dots + R_{\zeta\zeta}^{*}(0) + \dots + R_{\zeta\zeta}^{*}(n-2).x^{n-2} + R_{\zeta\zeta}^{*}(n-1).x^{n-1} = \sum_{k=-(n-1)}^{n-1} R_{\zeta\zeta}^{*}(k).x^k$$

Here:

$$(15) \quad F(x) = \zeta(n-1).x^{n-1} + \zeta(n-2).x^{n-2} + \dots + \zeta(1).x + \zeta(0),$$

is the polynomial, corresponding to the sequence  $\{\zeta(j)\}_{j=0}^{n-1}$  of complex amplitudes of the elementary pulses of the PM signal,  $R_{\zeta\zeta}^{*}(k)$  are the ACF values and  $F^*(x^{-1})$  is the polynomial:

(16)

$$F^*(x^{-1}) = \zeta^*(n-1).x^{-(n-1)} + \zeta^*(n-2).x^{-(n-2)} + \dots + \zeta^*(1).x^{-1} + \zeta(0).$$

Applying (14)-(16) in (11) yields:

(17)

$$R_c(x) = \sum_{i=1}^q \left[ \sum_{j=1}^p h_{ij}(x^{n_j}) . x^{r_{i1}n_1 + r_{i2}n_2 + \dots + r_{ij}n_j} . A_j(x) \right] \left[ \sum_{j=1}^p h_{ij}^*(x^{-n_j}) . x^{-(r_{i1}n_1 + r_{i2}n_2 + \dots + r_{ij}n_j)} . A_j^*(x^{-1}) \right] = \\ = \sum_{i=1}^q \sum_{j=1}^p h_{ij}(x^{n_j}) h_{ij}^*(x^{-n_j}) A_j(x) . A_j^*(x^{-1}) + \sum_{i=1}^q \sum_{j=2}^p \sum_{k=1}^{j-1} \left[ h_{ij} h_{ij-k}^* . x^{r_{j-k}n_{j-k} + \dots + r_{ij}n_j} A_j(x) A_{j-k}^*(x^{-1}) + \right. \\ \left. + h_{ij-k} h_{ij}^* . x^{-(r_{j-k}n_{j-k} + \dots + r_{ij}n_j)} A_{j-k}(x) A_j^*(x^{-1}) \right].$$

It is easy to see, that in (17):

$$\sum_{i=1}^q \sum_{j=1}^p h_{ij}(x^{n_j}) h_{ij}^*(x^{-n_j}) A_j(x) . A_j^*(x^{-1}) = \sum_{j=1}^p A_j(x) . A_j^*(x^{-1}) \sum_{i=1}^q h_{ij}(x^{n_j}) h_{ij}^*(x^{-n_j}) = \\ = \sum_{j=1}^p c_j . A_j(x) . A_j^*(x^{-1}) = (c_1 n_1 + c_2 n_2 + \dots + c_p n_p),$$

$$\sum_{i=1}^q \sum_{j=2}^p \sum_{k=1}^{j-1} \left[ h_{ij}(x^{n_j}) h_{ij-k}^*(x^{n_{j-k}}) . x^{r_{j-k}n_{j-k} + \dots + r_{ij}n_j} A_j(x) A_{j-k}^*(x^{-1}) + \right. \\ \left. + h_{ij-k}(x^{n_{j-k}}) h_{ij}^*(x^{n_j}) . x^{-(r_{j-k}n_{j-k} + \dots + r_{ij}n_j)} A_{j-k}(x) A_j^*(x^{-1}) \right] = 0,$$



because:  $\sum_{i=1}^q h_{ij}(x^{n_i}) \cdot h_{ij-k}^*(x^{-n_{j-k}}) = 0$ ,  $\sum_{i=1}^q h_{ij-k}(x^{n_{j-k}}) \cdot h_{ij}^*(x^{-n_i}) = 0$ , according to (10).

The last equations show that in (11), condition (8) is satisfied. Hence, set (11) is a set of generalized complementary sequences.

Theorem 2 reduces the problem for synthesis of complementary sequences to two steps: first, finding an arbitrary "initial" complementary set  $\{A_k\}_{k=1}^p$ , and, second, constructing an appropriate "creative" matrix  $H_{p,q}$ . As the above-shown sets (9) are appropriate initial sets, it is enough to concentrate our efforts on solving the second problem. With regard to this, it is easy to verify that the following matrices  $H(x)$  satisfy condition (10) [9]:

$$(18) \quad H(x) = \begin{bmatrix} B_1(x^n); B_2(x^n) \\ \tilde{B}_2^*(x^n); -\tilde{B}_1^*(x^n) \end{bmatrix}, \quad H(x) = \begin{bmatrix} B_1(x^{2n}); & B_2(x^{2n}) \\ x^n \cdot \tilde{B}_2^*(x^{2n}); & -x^n \cdot \tilde{B}_1^*(x^{2n}) \end{bmatrix},$$

where:

$$(19) \quad B_k(x) = \zeta_k(r-1) \cdot x^{n(r-1)} + \zeta_k(r-2) \cdot x^{n(r-2)} + \dots + \zeta_k(1) \cdot x^n + \zeta_k(0),$$

$$(20) \quad \tilde{B}_k^*(x^{-n}) = \zeta_k^*(r-1) \cdot x^{-n(r-1)} + \zeta_k^*(r-2) \cdot x^{-n(r-2)} + \dots + \zeta_k^*(1) \cdot x^{-n} + \zeta_k^*(0),$$

and  $\{\zeta_k(j)\}_{j=0}^{r-1}; k=1,2$  is an arbitrary set of generalized complementary sequences of length  $r$  and  $p=2$ .

Construction (11), proven in the paper, will be illustrated by two examples, where sets (9) of generalized complementary series, will be used. Let the first one be as follows:

$$A_1 = B_1 = \{\mu(j)\}_{j=0}^2 = \{1, i, 1\}; \quad A_2 = B_2 = \{\eta(j)\}_{j=0}^2 = \{1, 1, -1\}.$$

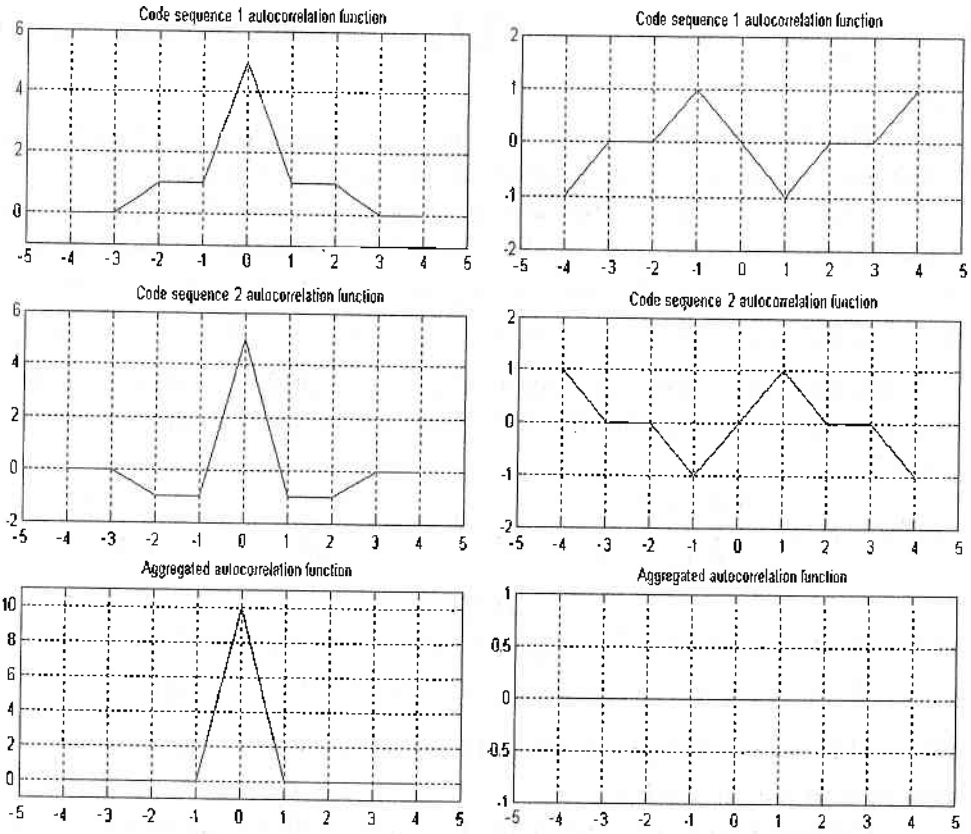
Applying the construction of Theorem 2, the derivative set of generalized complementary series of length  $2 \cdot n \cdot r = 2 \cdot 3 \cdot 3 = 18$  is obtained:

$$K = \{1, i, 1, i, -1, i, 1, i, 1, 1, 1, -1, 1, 1, -1, -1, -1, 1\}; \quad L = \{-1, -i-1, 1, i, 1, 1, i, 1, -1, -1, 1, i, i, -i, -1,$$

In the second example, the sets of generalized complementary series:

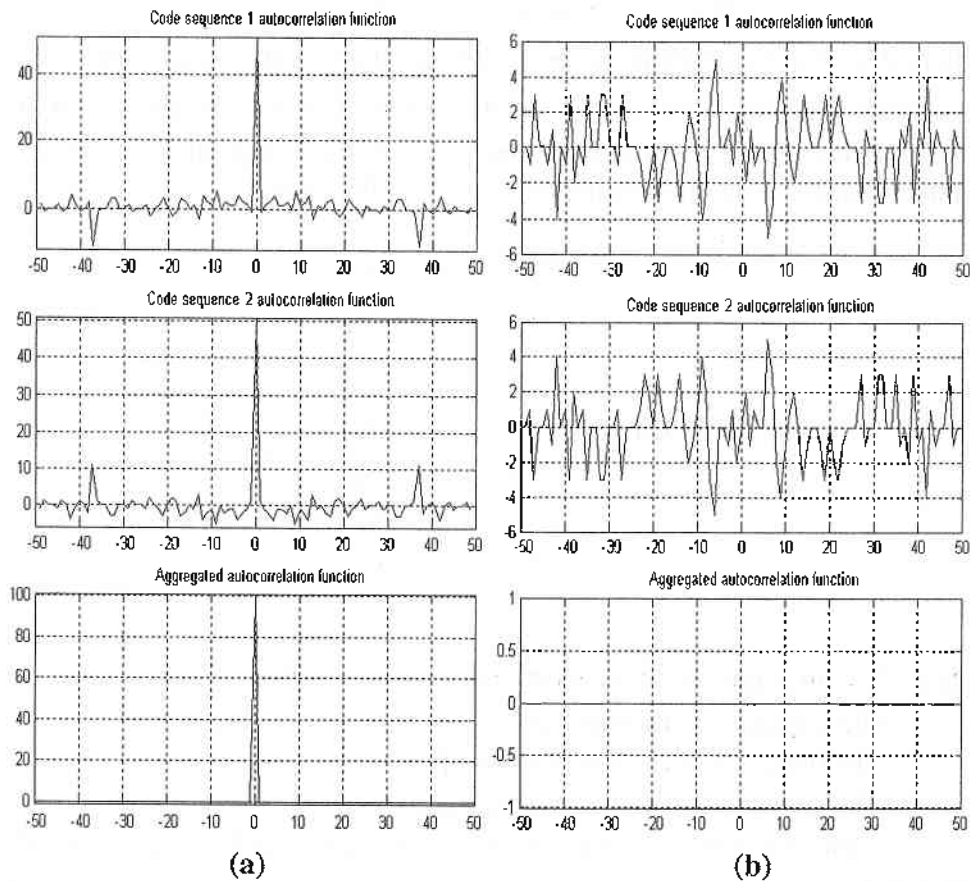
$$A_1 = B_1 = \{\mu(j)\}_{j=0}^4 = \{-i, i, 1, 1, 1\}; \quad A_2 = B_2 = \{\eta(j)\}_{j=0}^4 = \{1, i, -1, 1, -i\},$$

are used to develop a set of generalized complementary series of length  $2 \cdot n \cdot r = 2 \cdot 5 \cdot 5 = 50$ . The complex parts of the initial and derivative sequences are shown in Fig.2 and Fig. 3, respectively.



(a)

(b)  
**Fig.2.** Real (a) and imaginary (b) components of the autocorrelation functions of complementary sequences with  $n = 5$  elements and  $m = 4$ .



**Fig.3.** Real (a) and imaginary (b) components of the autocorrelation functions of complementary sequences with  $n = 50$  elements and  $m = 4$ .

### 3. Possibilities for applying phase-manipulated complementary signals in spacecraft-based radars

The possible advantages of applying sets of generalized complementary sequences are verified by computer experiments. The operation of a radar using the method of inverse synthetic aperture in the process of target identification is simulated.

The experiments are based on the geometrical model, proven in [11]. It is assumed that the object is radiated by a pulse sequence of pulse duration  $T$ , repeat period  $T_p$  and carrier frequency  $f_0$ . During the experiments, the impact of the type of the inner pulse phase modulation on the quality of the object image is examined. Namely, the pulses of the radar transmitter are described by the following formula:

$$(21) \quad u(t) = U_0 \exp\{i[\omega t + b\pi + \varphi_0]\},$$

where  $U_0$  is the amplitude of the carrier frequency,  $\omega = 2\pi f_0$  - the angle frequency;  $\varphi_0$  - the initial phase, and the parameter  $b$  determines the type of phase modulation. For instance, in the case of phase modulation with 13-element Barker code, the parameter  $b$  assumes the values:

$$(22) \quad b = \begin{cases} 0 & , \quad t = \overline{1,5} \tau_0 ; \\ 1 & , \quad t = \overline{6,7} \tau_0 ; \\ 0 & , \quad t = \overline{8,9} \tau_0 ; \\ 1 & , \quad t = 10 \tau_0 ; \\ 0 & , \quad t = 11 \tau_0 ; \\ 1 & , \quad t = 12 \tau_0 ; \\ 0 & , \quad t = 13 \tau_0 . \end{cases}$$

where  $\tau_0$  is the duration of the elementary pulses (chips).

The trajectory parameters are:

- the object velocity  $V = 400 [m/s]$ ;

- the course parameter  $\alpha = \pi [rad]$ ;

- the measure of the object space grid cells  $\Delta X = 0.6 [m]$ ;

$\Delta Y = 0.6 [m]$ ;

- the initial coordinates of the object  $x_0 = 0 [m]$ ;  $y_0 = 5 \cdot 10^4 [m]$ .

The parameters of the radiated pulse sequence are:

- the duration of an PM radiated pulse (in the case of Barker code, phase modulation)  $T = 5 \cdot 2 \cdot 10^{-8} [s]$ ;

- the duration of an elementary pulse (chip)  $\tau_0 = 4 \cdot 10^{-9} [s]$ ;

- the carrier frequency  $f = 10^{10} [Hz]$ ;

- the number of pulses reflected from the object  $N_p = 500$ .

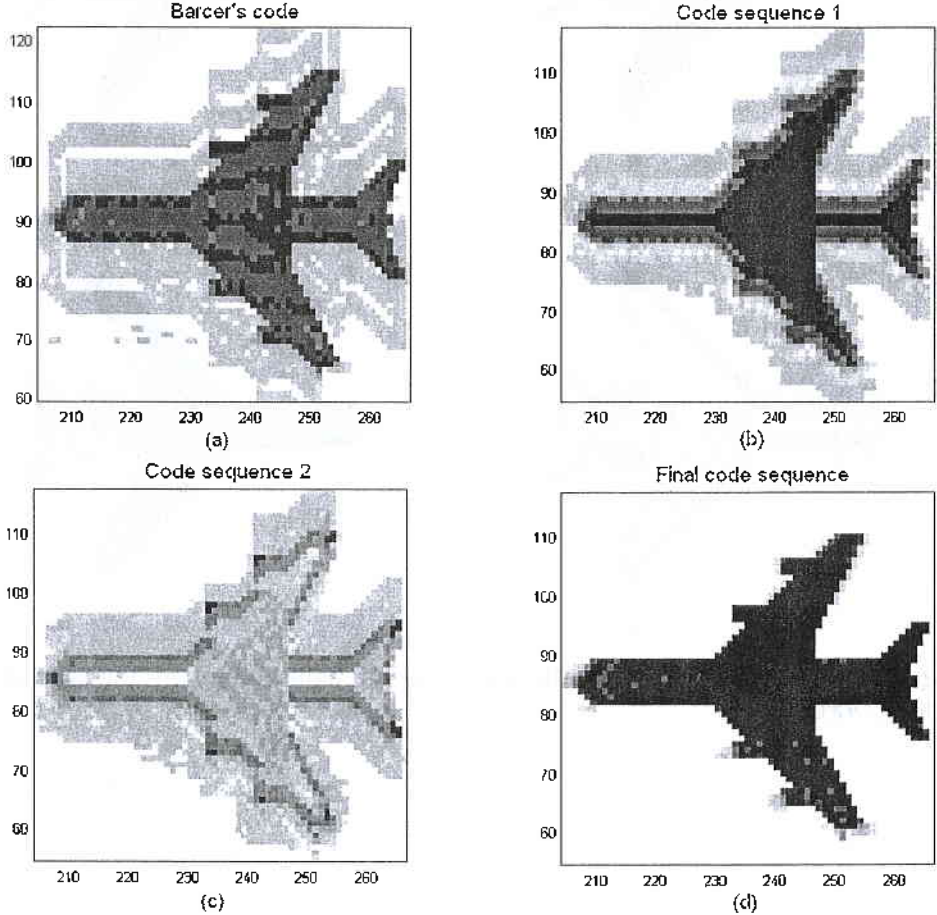
In the case of modulation with complementary codes, the following sequences are applied:

$$(23) \quad \{\mu(j)\}_{j=0}^7 = \{+1, +1, +1, -1, +1, +1, -1, +1\};$$

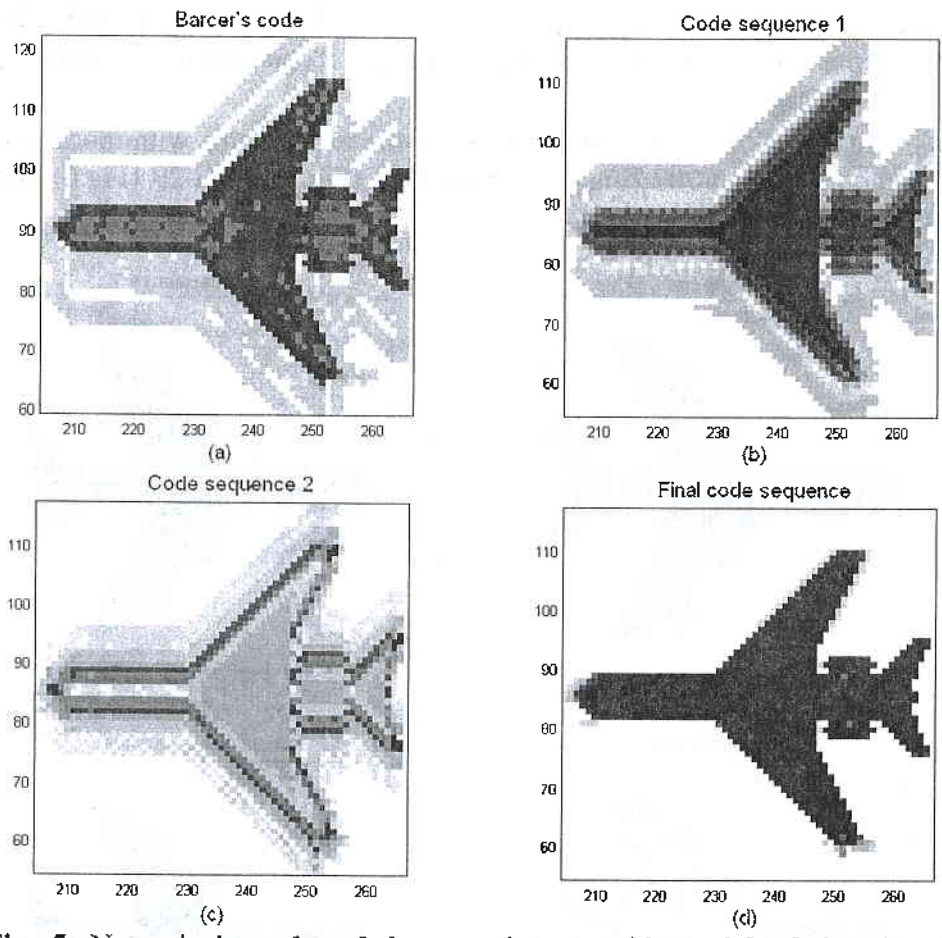
$$\{\eta(j)\}_{j=0}^7 = \{+1, +1, +1, -1, -1, -1, +1, -1\}.$$

In the odd periods, the first sequence is used, and in the even periods, the second one. The duration of each sequence (“concatenated” pulse) is  $T = 3,2 \cdot 10^{-7}$  [s].

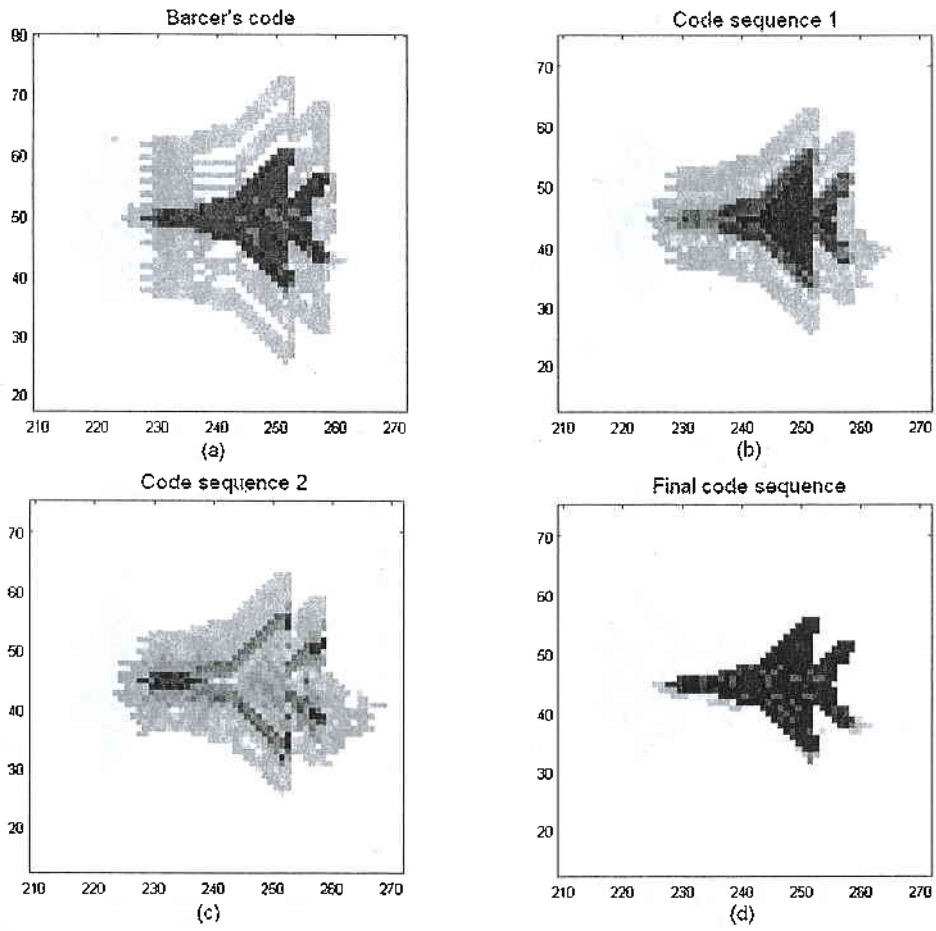
In Fig. 4-7, the numerical results of the experiments with models of the aircrafts Boeing-707 (Fig.4), Falcon-2000 (Fig.5), MiG-29 (Fig.6) and MiG-35 (Fig.7) are shown.



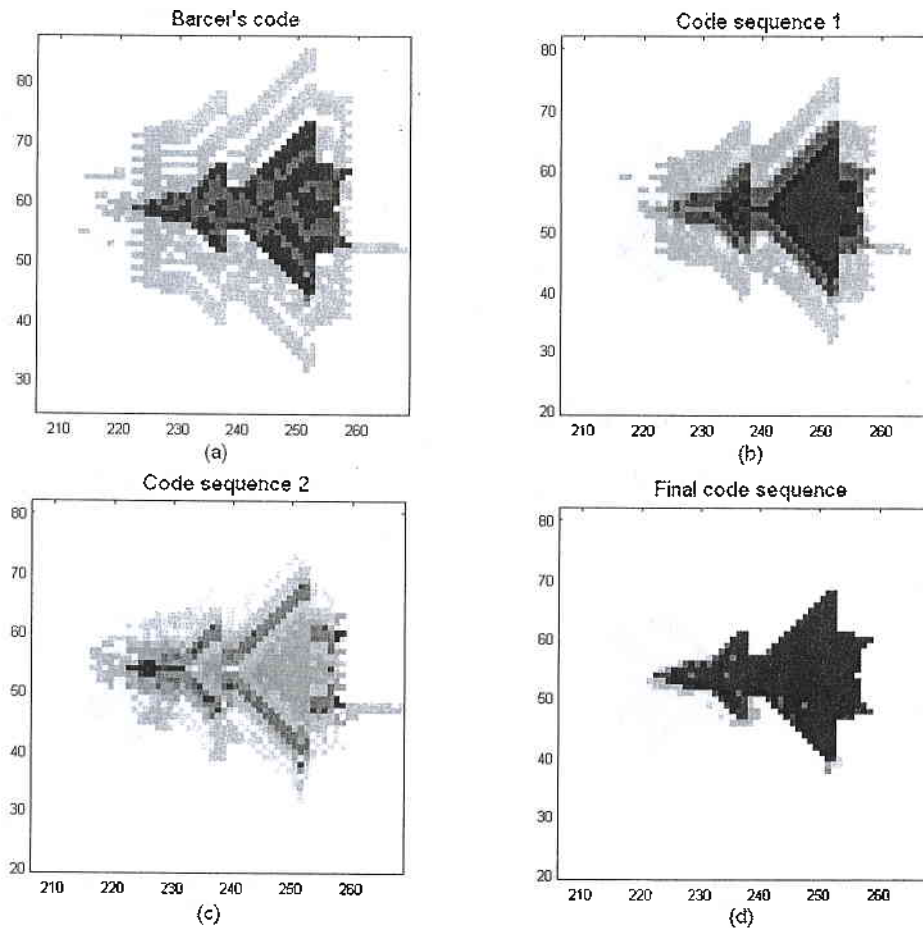
**Fig. 4.** Numerical results of the experiments with model of the aircraft Boeing-707.



**Fig. 5.** Numerical results of the experiments with model of the aircraft Falcon-2000 .



**Fig.6.** Numerical results of the experiments with model of the aircraft MiG-29.



**Fig.7.** Numerical results of the experiments with model of the aircraft MiG-35.

On the basis of the experimental results, it is easy to see that:

1) If sequences (23) are applied separately, the structural noise is bigger than the noise in the case of modulation with Barker's codes (Fig. 4a,b,c – Fig. 7a,b,c).

2) If sequences (23) are applied together (in “aggregate”), the structural noise decreases practically to zero, which results in ideal shape of the ACF (Fig. 4d – Fig. 7d).

#### 4. Conclusions

From everything stated above, it can be easily seen that Theorem 2 generalizes Golay's [5], Liu's, Tseng's, Suehiro's and Ignatov's constructions [6, 7, 8]. This was revealed upon investigating the common case where the phase of the PM signals can take  $m \geq 2$  values. As a result,



Theorem 2 construction accelerates the process of synthesis of complementary series with unlimited code-length  $2^s \cdot n^u \cdot r^v$  using complementary series with short lengths  $n$  and  $r$ . The advantages in the design of modern communication devices are also increased because systems with PM signals with modulated phase according to a set of generalized complementary series make effective use of the electromagnetic spectrum.

Having in mind the above-mentioned positive features, the construction for synthesis of complementary series proven in the paper could be used successfully in prospective spacecraft-based radar systems, where PM signals allow enhancing of the ranging distance without losing measurement accuracy and target resolution.

### References

1. Мардиросян Г., Аерокосмически методи и средства в екологията и изучаването на околната среда – С.: Академично издателство “Проф. М. Дринов”, 2003г., 208 с.
2. Шияновский Э. Н., Применение Д-кодов для фазовой манипуляции сигналов РСА, Радиотехника, 1998, № 4, с. 79 – 86
3. Dinan E. P., Jabbari B., Spreading codes for direct sequence CDMA and wideband CDMA cellular networks, IEEE Communications Magazine, 1998, vol. 36, № 9, pp. 48 – 54
4. Варакин Л. Е., Системы связи с шумоподобными сигналами - М.: Радио и связь, 1985. – 384 с.
5. Golay M. Y. E., Complementary series, IRE Trans. on Information Theory, 1961, vol. IT – 7, №2, pp. 82 – 87
6. Tseng C. C., Liu S. L., Complementary sets of sequences, IEEE Trans. on Information Theory, 1972, vol. IT- 18, №5, p. 644 – 652
7. Игнатов В. В., Добровольский С. А., Гужва А. Ю., Матричные системы сигналов для использования в системах CDMA, Электросвязь, 2003, № 9, с. 41-42
8. Chen H.-H., Yeh J.-F., Suchigo N., A multi-carrier CDMA architecture based on orthogonal complementary codes for new generations wideband wireless communications, IEEE Communication Magazine, 2001, vol. 39, № 10, pp. 126 – 135
9. Bedzhev B. Y., Tashva Zh. N., Mutkov V. A., CDMA codes for the next generation mobile communication systems, XII International symposium of theoretical electrical engineering ISTET 03, Warsaw, Poland, 6-9 July, 2003, Conference Proceedings, vol.I, pp. 78-82.

10. Берлскамп Э., Алгебраическая теория кодирования, Пер. с англ. - М.: Мир, 1971. – 477 с.
11. Lazarov A. D., Minchev S h. N., ISAR image reconstruction and autofocusing procedure over phase modulated signals, IEE Conference RADAR 2002, Edinburg, UK, 15-17 October, 2002
12. Гецов П., Теория на управлението полета на летателни апарати – ВТС, 215 стр., 1983.
13. Гецов П., Изследване на някои възможности за стабилизация на безпилотен самолет при полет по зададена линия на пътя, Пловдив, 1990.
14. Гецов П., Полунатурно конструиране на системи за управление полета на безпилотни летателни апарати, София, 2002.

## **ВЪЗМОЖНОСТИ ЗА ИЗПОЛЗВАНЕ НА ФАЗОВО МАНИПУЛИРАНИ КОМПЛЕМЕНТАРНИ СИГНАЛИ В КОСМИЧЕСКИТЕ РАДИОЛОКАЦИОННИ СИСТЕМИ**

*Борислав Йорданов Беджев*

В радиолокационните системи (РЛС), базирани на космически апарати, за получаване на висококачествени изображения на повърхността на планетите, спътниците и кометите, широко се прилага методът на изкуствената синтезирана апертура (Synthetic Aperture Radar - SAR). При това за едновременното реализиране на голям радиус на действие и на висока разделителна способност по разстояние се използват сложни радиолокационни сигнали с вътрешно импулсна модулация.

На настоящия етап в космическите SAR основно приложение намират сигналите с линейна честотна модулация (ЛЧМ). Въпреки положителните им свойства, за тях е характерни и някои недостатъци. В същото време сигналите с вътрешно импулсна фазова манипулация (ФМ) намериха през последните десет години изключително широко разпространение.

Предвид на изложените факти, в настоящата статия са получени следните резултати.

1. Разработен е математически метод за синтез на нов клас ФМ сигнали, наречени обобщени комплементарни сигнали (ОКС), чиято АКФ има странични лоби с практически нулево ниво.

2. Чрез компютърни симулации на работата на РЛС с инверсна синтезирана апертура е обоснована приложимостта на ОКС в перспективните РЛС с космическо базиране.