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SOME APPROACHES TO THE RESEARCH MODELS OF CHAOTIC PHENOMENA IN THE SOLAR SYSTEM

Kostadin Sheiretsky

Space Research Institute-Bulgarian Academy of Sciences

Abstract

Our solar system provides a plethora of examples of chaotic motion. The giant planets in our solar system are chaotic, as are the inner planets (independently). In extreme cases, chaos can disrupt some orbital configurations, resulting in the loss of a planet. The spin axes of planets may also evolve chaotically. Despite the variety and complexity of applications, we can introduce many of the concepts in solar system dynamics using the pendulum: phase space structure, periodic motion, and stability.

The physical basis of chaos in the solar system is now better understood: in all cases investigated so far, chaotic orbits result from overlapping resonances.

A series of remarkable features in the asteroid belt vividly illustrates the importance of dynamical chaos in the solar system. The distribution of semi-major axes of asteroid orbits contains a number of distinct gaps. These are called Kirkwood gaps, in honor of Daniel Kirkwood, who first identified them and noted that they occur at locations where the orbital period, T, which depends on the semi-major axis, would be of the form $(p/q)T_J$, where T_J is the orbital period of Jupiter and p and q are integers. The paper that ignited the modern era of work on the Kirkwood problem was Jack Wisdom's (1982) - first contribution to the study of the 3:1 mean-motion resonance at a = 2.50 AU. His startling results showed that an orbit at this resonance could remain quiescent, with a low eccentricity, e < 0:1, for more than 100,000 years, but also showing occasional surges lasting for about 10,000 years that would lift to a maximum value of about 0.35. Such a value is just sufficient to allow crossing of Mars' orbit, resulting in an eventual collision or a close encounter.

Further afield, about one new "short-period" comet is discovered each year. They are believed to come from the "Kuiper Belt" (at 40 AU or more) via chaotic orbits produced by mean-motion and secular resonances with Neptune. Finally, the planetary system itself is not immune from chaos. For example, Mercury, in 10^{12} years, may suffer a close encounter with Venus or plunge into the Sun. In the outer solar system, three-body resonances have been identified as a source of chaos, but on an even longer time scale of 10^9 times the age of the solar system.

The first striking example of chaotic behavior in the solar system was given by the chaotic tumbling of Hyperion, a small satellite of Saturn whose strange rotational behavior was detected during the encounter of the Voyager spacecraft with Saturn.

The equations of motion for the orientation of a satellite S orbiting around a planet P on a fixed elliptical orbit of semi-major axis a and eccentricity e (Figure 1) are given by the Hamiltonian:

$$H = \frac{y^2}{2} - \frac{3}{4}\frac{B-A}{C}\left(\frac{a}{r(t)}\right)^3 \cos 2(x - v(t))$$

(1)

where $r(t)$ is the distance from the planet to the satellite, x gives the orientation of the satellite with respect to a fixed direction, $y = \frac{d}{dt}x$ is its conjugate variable, ꞉is the true anomaly of the satellite, and A < B < C are the principal inertia moments of the satellite

When expanding the Hamiltonian with respect to eccentricity (e), which is supposed to be small, and retaining only the first order terms in eccentricity, one obtains:

$$H = \frac{y^2}{2} - \frac{\alpha}{2}\cos 2(x-t) + \frac{\alpha e}{4}\left[\cos(2x-t) - 7\cos(2x-3t)\right],$$

$$\alpha = \frac{3(B-A)}{2C}.$$

(2)

As a result of the transition between librational motion and rotational satellite motion, small chaotic zones appear. When perturbation size □e increases, resonant zones corresponding to the various possible resonant terms *cos* 2(*x-t*), *cos* (2*x-t*), *cos* (2*x*-3*t*) will overlap, giving rise to large-scale chaotic motion.

This is the case for Hyperion, where □c = 0,039. The resulting effect is that the rotational motion of Hyperion is not regular, and it becomes impossible to adjust any periodic or quasi-periodic model to its light curve.

They briefly recall a mathematical model introduced in (Celletti, 1990) to describe the "spin–orbit" interaction in Celestial Mechanics. Let *S* be a tri-axial ellipsoidal satellite orbiting around a central planet *P* (Figure1). Let T_{rev} and T_{rot} be the periods of revolution of the satellite around *P* and the period of rotation about an internal spin–axis. A *p*:*q* spin–orbit resonance occurs whenever:

$T_{rev} / T_{rot} = p / q,\ p, q \in N, q \neq 0.$

The equation of motion may be derived from the standard Euler's equations for a rigid body. In normalized units (i.e. assuming that the mean motion is one, 2□ / T_{rev} = 1) we obtain:

$$\frac{d^2x}{dt^2} + \varepsilon\left(\frac{1}{r}\right)^3 \sin(2x - 2v) = 0, \varepsilon = \frac{3(B-A)}{2C}. \qquad (3)$$

We investigate numerically the stability of the periodic orbits. We denote by □*(*p*/*q*) the value of the perturbing parameter at which transition occurs.

The plot of □ (*p*/*q*) (Figure 2) shows that for low eccentricity values there is only a marked peak corresponding to the 1:1 commensurability (occurring when the periods of revolution and rotation are the same). Increasing the eccentricity, other resonances appear. Indeed, the 3:2 resonance can be observed at eccentricities larger than 0.01.

Prof. Damgov introduces a heuristic model for the discrete distribution of Solar system planets and the satellites' mean distances from the primaries.

Herewith, as a general model we take a periodical motion, that is the rotation of a charge at which an electromagnetic wave falls along the x-axis.

The charge motion equation is

$$\ddot{x} + 2\beta\dot{x} + \omega_o^2 x = eE_x \sin(\nu t - kx), \qquad \mathbf{(4)}$$

where E_x is the x-axis electrical field component and *k* is the wave modulus.

The solution of Eq. (10), describing a linear oscillator under wave action, is written in the form of a quasi-harmonic function

$$x(t) = a\sin(\omega t + \alpha),$$

where $a(t)$ and $\alpha(t)$ are the slowly changing amplitude and phase, $\omega = \nu / N$ is the charge periodic motion frequency, and N =1,2,3,... is an integer.

The examination of the stability solutions reveals that the stable oscillations amplitude a satisfies condition $J'_N(ka) = 0$.

Hence, the stationary charge oscillations can be realized for amplitudes a_i, belonging to a strictly defined set of amplitude values. The a_i values are determined by the Bessel function extremes, and may be presented as $ka_i = j_{N,i}$, where $j_{N,i}$ is the N-th order Bessel function argument value ka_i at the i-th extremum point. The Solar system planets' mean distances are presented on Table 1.

The unsolved problems in solar system chaotic dynamics are many. As yet, we have no relation for secondary resonances, where ejection time is a function of the Lyapunov time. Unlike the outer planets, the source of inner planets chaos has not been convincingly established.

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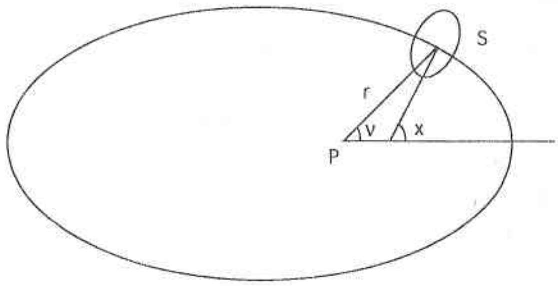

Fig. 1: The spin–orbit geometry.

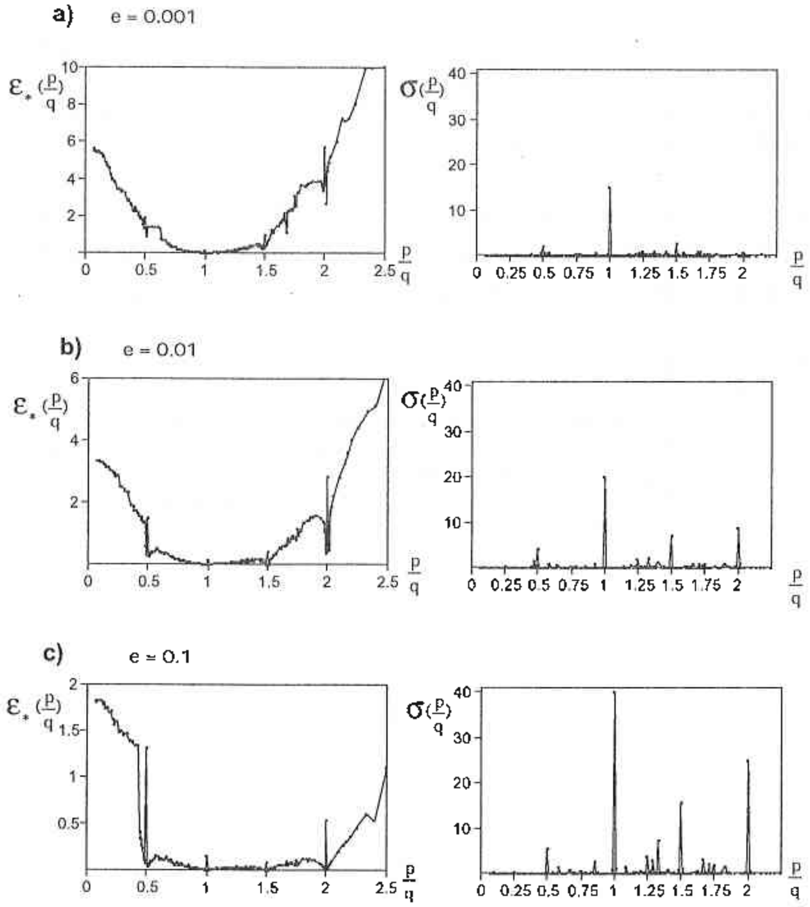

Fig. 5: Left panel: plot of □*(p/q) vs. p/q for the frequencies listed in the text; right panel: plot of the CSI □(p/q) vs. p/q for the same frequencies as in the left panel. a) c = 0.001, b) e = 0.01, c) e = 0.1.

Planets in the Solar system	Data from direct astronomical measurements of planet distances from the Sun (Allen, 1973) A.U.	Computed planet distances using Equation (65) of the "Oscillator-wave" model	
		i	a/j
Mercury	0.39	1	0.392
Venus	0.72	3	0.723
Earth	1.00	5	1.000
Mars	1.52	9	1.530
Asteroids	2.78	19	2.824
Jupiter	5.2	37	5.132
Saturn	9.55	71	9.474
Chiron (Collewll's objects)	13.71	104	13.689
Uranus	19.18	147	19.180
Neptune	30.03	232	30.035
Pluto	39.67	307	39.598

Tab. 1: Mean planet distances in the Solar system

НЯКОИ ПОДХОДИ ПРИ МОДЕЛНОТО ИЗСЛЕДВАНЕ НА ХАОТИЧНИТЕ ЯВЛЕНИЯ В СЛЪНЧЕВАТА СИСТЕМА

Костадин Шейретски
Институт за Космически изследвания – БАН

Резюме

Нашата слънчева система представя множество от примери за хаотични движения. Гигантските планети, както и вътрешните планети са подвластни на хаоса. В някои случаи хаосът може да разруши някой орбитални конфигурации, водейки до загуба на планета. Оста на въртене на планетите може също да еволюира хаотично. Независимо от разнообразието и сложността на хаотичните явления, ние можем да представим много от концепциите на динамиката на слънчевата система използвайки махало: Структурата на фазовото пространство, периодично движение и стабилност.