Bulgarian Academy of Sciences. Space Research Institute
Aerospace Research in Bulgaria. 17. 2003. Sofia

SOME FEATURES OF α DISC AND ADVECTIVE-DOMINATED ACCRETION DISC. SELF-SIMILAR SOLUTIONS AND THEIR COMPARISON

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Abstract

A brief review of the features of Standard Shakura - Sunyaev Disc (SSD) and Advection - dominated Accretion Disc (ADAD) is discussed. In this paper, it is presented the physical bases, which we use to obtain the parameters, describing two models. The built theoretical systems are transformed in a suitably for operation view.

1. Introduction

The new, more functional theory about disc accretion - the advection theory [10], has appeared in the last years.

It has arisen because of that the standard theory gives common view on accretion flows, but couldn't explain any observant phenomena as: very high effective temperature (in standard theory disc is unstable - transforms to tore); non - thermal spectrum with power dependence of luminosity L from accretion rate $\dot{M}$ ($\sim M^2$ in two - temperature model); jets and s.o.

Other priority is that the advection - dominated flows may occur in both cases of optical depth - very large or very small it's value [10], which extend the volume of studying objects: active galactic nuclei, elliptical galactics, X-ray binaries and cataclysmic variables.

The conditions of transition between standard Shakura - Sunyaev disc and Advection - dominated disc are discussed by Abramowicz and Igumenchev [1]. They used a simple two - dimensional hydrodynamical model, assuming an instant destruction of SSD by some unknown physical process at radius r_{trns}. The result of their investigation shows that flux of matter from the destroyed SSD expands and forms thick disc (ADAF). The energy, which is necessary for expansion, is supplied locally by viscous heating. So expanded matter flows in all direction from source of matter and forms a geometrically thick disc.

Yamasaki [13] investigates the stability of two - dimensional ADAD against local thermal perturbations - for optically thin discs. In result he obtains that weakly unstable modes exist due to radiation effects, but the mode is stable when the thermal conduction is efficient. Because of turbulent heat diffusion, in two - temperature ADAF thermal perturbations damp.

Wu [12] proved, that in the case of very small advection, thermal instability exist when the disc is geometrically thin. If consider thermal diffusion, however it disappears. More than if the disc is advective - dominated thermal instability doesn't exist. There are enough dates that advection and thermal diffusion have significant effect on the stability of hot optically thin disc. The detail stability analysis of Wu shows that only two stable thermal equilibria of accretion disc exist. One of them is optically thin advection - dominated and the other is optically thick gas - dominated.

The family of self - similar solutions [10], where the temperature of accreting gas is almost virial and flow is quasi - spherical, define some of properties of the ADAF, as:

- the angular velocity of the flow Ω is less then Keplerian angular velocity Ω_k.
- ADAF is convective instable, because convection transfers energy from small to large radius.
- Bernoulli parameter b (scale changed) is positive in self - similar ADAF for wide range of parameters, e.g. gas may spontaneously expands to infinity.

Nakamura [9] elaborates global steady models of two - temperatures, advection - dominated accretion flows around black holes, as he pays attention to transonic region near black hole.

Chen and Abramowicz [4] present optically thin ADAD, described by full system of differential equations. They obtain global transonic solutions. As a result from this follows that far from sonic point, self - similar solutions is a good approximation to global structure of the flow. That is true if accretion rate is close to maximum value, above, which the solutions for optically thin disc don't exist. The simple self-similar solutions nowhere approach to complete solution [11].

In recent work we consider optically thick advection - dominated flows. The mainly aim of the paper is to show that the optically thin disc remains geometrically thin because of the advection [3]. It is known that when α increases the sonic point removes outward [4]. That is why such advective flow is supersonic, when viscosity parameter α is large for optically thick disc.

This letter is built as follows: It consists of two parts.

Part I: In § 2 we present the main physically characteristics of two flows. In § 3 is present the vertical structure. Comments of part I.

Part II: Paragraph 4 describes the equations of evolution of both discs. In § 5 we have obtained the self-similar solutions. Discussion of part II.

2. Basic equations

Accounting for the form of accretion flows we can use cylindrical coordinate system. The acceleration created by the potential in φ has the form:

$$\frac{V_\varphi}{r} = \frac{d\Phi}{dr} \tag{2.1}$$

where V_φ is the linear velocity in φ.

$$V_\varphi = \omega r \tag{2.2}$$

We shall use the Newtonian gravitational potential for standard discs:

$$\Phi = \frac{GM}{r} \tag{2.3}$$

and pseudo- Newtonian $\Phi = \dfrac{GM}{r - r_g}$ (2.4) for advective discs,

where

$$r_g = \frac{2GM}{c^2} \tag{2.5}$$

is the gravitational radius of the black hole.

G – the gravitational constant, M – the mass of the central object, c -light velocity.

The angular velocities for both discs are:

(2.6) $$\omega_k = \sqrt{\frac{GM}{r^3}}$$

(2.7) $$\omega = \sqrt{\frac{GM}{r(r - r_g)^2}}$$

Geometrically thin discs are described by yet another parameter - surface density of the disc:

(2.8) $$\Sigma = \int_{-H}^{H} \rho dz \cong 2H\rho$$

Now we can form the basic equations of non-stationary accretion:

<u>The mass conservation law:</u>

(2.9) $$r\frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial r}(r\Sigma V_r) = 0$$

There is no difference in the equation for two discs, but V_r is much larger for the advective one.

<u>The momentum conservation law:</u>

(2.10) $$r\frac{\partial}{\partial t}(\Sigma r^2 \omega) + \frac{\partial}{\partial r}(r\Sigma V_r r^2 \omega) = \frac{1}{2\pi}\frac{\partial \theta}{\partial r}$$

θ is the momentum by viscosity forces:

(2.11) $\theta = 2\pi\, W_{r\varphi} r^2$

$W_{r\varphi}$ - vertically integrated viscosity per unit length of circumference.

(2.12) $W_{r\varphi} = \int_{-H}^{H} \omega_{r\varphi} dz = \nu \Sigma r \frac{\partial \omega}{\partial r}$

ν - kinetic viscosity;

(2.13) $\nu = \alpha V_s H$

V_s is sound velocity,

$r\frac{\partial \omega}{\partial r}$ is the displacement between two layers at differential rotation of the disc.

Thermal balance equation

The discs are optically thick. Therefore, Local Thermodynamical Equilibrium exists.

(2.14) $Q^+ \sim Q^-$

where Q^+ is the heating produced by viscosity:

(2.15) $Q^+ = \frac{1}{2} W_{r\varphi}\left(r\frac{\partial \omega}{\partial r}\right)$ and

(2.16) $Q^- = \frac{acT^4}{\tau}$ is radiated cooling

a - radiation constant.
T - effective temperature
τ - optical depth of the disc

(2.17) $d\tau = \rho \chi dz$

χ - opacity coefficient.

However, if for some reason, disc accretion rate increases and inflow time becomes shorter than photon emission time, the disc cannot reradiate generated energy. Part of radiation is caught by the flow, being thus generated, which reduces entropy gradient and the flow is directed to the center. Thereby, advection appears and the thermal balance takes the form:

$$(2.18)\quad Q_{adv} + Q^{+} = Q^{-}$$

where

$$(2.19)\quad Q_{adv} = \Sigma V_r T \frac{dS}{dr}$$

and $\frac{dS}{dr}$ radial gradient of the entropy.

Radial motion equations

$$(2.20)\quad \Sigma \frac{\partial V_r}{\partial t} + \Sigma V_r \frac{\partial V_r}{\partial r} - \Sigma \omega_k^2 r = W_{r\varphi} + G$$

$$(2.21)\quad \Sigma \frac{\partial V_r}{\partial t} + \Sigma V_r \frac{\partial V_r}{\partial r} - \Sigma\left(\omega^2 - \omega_k^2\right) r = -2\frac{\partial Hp}{\partial r} + W_{r\varphi} + G$$

$$(2.22)\quad P = \int_{-H}^{H} P dz$$

$(\omega^2 - \omega_K^2)$ – as a result of advection, disc equilibrium changes. An inertial spring is needed to keep the structure stable.

Using a similar system (2.9, 2.10, 2.18, 2.21), Narayan and Yi [10] have obtained that in advective discs:

$$(2.23)\quad \begin{aligned} V_r &= -c_1^{'} \omega_k r \\ \omega &= \omega_k c_2 \\ V_s &= c_3 \omega_k^2 r^2 \end{aligned}$$

where $c_1^{'}$ c_2 c_3 are dimensionless constants.

3. Vertical structure [8].

Equations of hydrostatical equilibrium:

$$(3.1)\quad \frac{1}{\rho}\frac{\partial P}{\partial z} = -\omega_k^2 z$$

$$(3.2)\quad \frac{1}{\rho}\frac{\partial P}{\partial z} = -\omega^2 z$$

Equation of continuity:

$$(3.3)\quad \frac{\partial \Sigma}{\partial z} = \rho$$

$$\Sigma = 2H\rho$$

Equation of radiation transfer:

$$(3.4)\quad \frac{c}{3\chi\rho}\frac{\partial\left(aT^4\right)}{\partial z} = -Q^+$$

$$(3.5)\quad -\Sigma V_r T\frac{\partial S}{\partial z} + \frac{c}{3\chi\rho}\frac{\partial\left(aT^4\right)}{\partial z} = -Q^+$$

and we take into account (2.23):

$$(3.6)\quad \frac{c_1^{\cdot}}{\sqrt{c_3}} V_s \Sigma T\frac{dS}{dz} + \frac{c}{3\chi\rho}\frac{\partial\left(aT^4\right)}{\partial z} = -Q^+$$

where

$$(3.7)\quad S = c_p \ln T - R\ln P$$

is the entropy of ideal gas, R is the gas constant.

The vertical gradient of radiating fluctuation is equal to energy released in
disc:

$$(3.8)\quad \frac{\partial Q}{\partial z} = \varepsilon$$

Equation of ideal gas:

$$(3.9)\quad P = \rho \frac{RT}{\mu}$$

$$\frac{P}{\rho} = V_s^2$$

χ is opacity coefficient:

$$(3.10)\ \chi = \frac{\chi_0 \rho^a}{T^b}$$

χ_0 Thompson's opacity coefficient:
a, b are constants.

The obtained differential system will be transformed by an appropriate group, corresponding to the approximation for a slim disc:

(3.11) ΔP ~ -P; ΔQ ~ Q; ΔT ~ T; ΔZ ~ H

This allows us to receive the solution in power dependences of independent variables or their dimensionless combinations – that is the self-similar solution [2].

To obtain a complete algebraic system we must also include the specific moments in the discs:

$$(3.12)\quad h_* = \omega r^2$$

as well as the average momentums of viscosity power between the disc's adjacent payers.

The following system of equations is obtained for both discs:

Standard disc	Equations holding for both discs	Advection-dominated disc
$h = \sqrt{GMr}$ (3.13)		$h_* = \frac{\sqrt{GMr^3}}{r - r_g}$ (3.14)
$\omega_k = \sqrt{\frac{GM}{r^3}}$ (3.15)		$\omega = \sqrt{\frac{GM}{r\left(r - r_g\right)}}$ (3.16)
$V_s = \omega_k H$ (3.17)	$P = \frac{\Sigma}{2H} V_s^2$ (3.18)	$V_s = \omega H$ (3.19)
$W_{r\varphi} = k \Sigma T$ (3.20) $k = -\frac{3}{2} \alpha \frac{R}{\mu}$	$F = W_{r\varphi} r^2$ (3.21)	$W_{r\varphi} = k' \Sigma T$ (3.22) $k' = -\left(\frac{1}{2} + c_2\right) \alpha \frac{R}{\mu}$
$Q^+ = -\frac{3}{4} W_{r\varphi} \omega_k$ (3.23)		$Q^+ = -\frac{1}{2}\left(\frac{1}{2} + c_2\right) W_{r\varphi} \omega$ (3.24)
$\varepsilon = aT^4 = -\frac{3\chi\Sigma}{2c} Q^+$ (3.25)		$\frac{c_1'}{\sqrt{c_3}} \frac{V_s \Sigma T}{H}\left(c_p - R\right) +$ $+ \frac{2acT^4}{3\chi\Sigma} = -Q^+$ (3.26)
$\chi = k_1 \Sigma^{a_1} T^{b_1} H^{c_1}$ (3.27) $k_1 = \frac{\chi_0}{2^{a_1}}$		$\chi = k_{1a} \Sigma^{a_{10}} T^{b_{10}} H^{c_{10}}$ (3.28) $k_{1a} = \frac{\chi_0}{2^{a_{10}}}$

This algebraic system can be solved if in (2.12) we take $\bar{\nu}$ in the advective case and use (2.7) и (3.12). We will obtain the explicit dependence of the parameters of both discs on their Σ and ω_k as well as the dependence of the average viscosity moments on Σ and h [6].

A solution system for both discs is obtained, different for both discs:

Standard α disc	Advection-dominated disc
$T = T_0 \Sigma^{2N_1} \omega_k^{2N_2}$ (3.29) $T_0 = \left\{ \frac{-27\alpha\chi_0}{2^{a_1+4} ac} \left(\frac{R}{\mu}\right)^{\frac{c_1+2}{2}} \right\}^{\frac{2}{6-2b_1-c_1}}$ $N_1 = \frac{a_1+2}{6-2b_1-c_1}$ $N_2 = \frac{1-c_1}{6-2b_1-c_1}$	$T = T_0^a \omega_k^2 r^2$ (3.34) $T_0^a = \frac{W_{r\varphi 0}^a}{k}$
$V_s = V_{s0} \Sigma^{N_1} \omega_k^{N_2}$ (3.30) $V_{s0} = \left(\frac{RT_0}{\mu}\right)^{\frac{1}{2}}$	$V_s = V_{s0}^a \omega_k r$ (3.35) $V_{s0}^a = \left(\frac{RT_0^a}{\mu}\right)^{\frac{1}{2}}$
$W_{r\varphi} = W_{r\varphi 0} \Sigma^{2N_1+1} \omega_k^{2N_2}$ (3.31) $W_{r\varphi 0} = \frac{-3\alpha RT_0}{2\mu}$	$W_{r\varphi} = W_{r\varphi 0}^a \Sigma \omega_k^2 r^2$ (3.36) $W_{r\varphi 0}^a = \alpha c_3$
$P = P_0 \Sigma^{N_1+1} \omega_k^{N_2+1}$ (3.32) $P_0 = \left(\frac{RT_0}{4\mu}\right)^{\frac{1}{2}}$	$P = P_0^a \Sigma \omega_k^2 r$ (3.37) $P_0^a = \left(\frac{V_{s0}^a c_2}{2}\right)$
$F = F_0 \Sigma^A h^B$ (3.33) $F_0 = W_{r\varphi 0} (GM)^{-\frac{-8+4b_1-2c_1}{6-2b_1-c_1}}$ $A = \frac{10+2a_1-2b_1-c_1}{6-2b_1-c_1}$ $B = \frac{18-8b_1+2c_1}{6-2b_1-c_1}$	$F = \left(2\frac{h_*}{h} - \frac{1}{2}\frac{\partial h_*}{\partial h}\right) h \Sigma \bar{\nu}$ (3.38)

4. Discussion

In the paper are shown the main theoretical principles when there is a development of the accretion in a standart and advection regimes. It's formed the horizontal and vertical structures of the accretion discs in two regimes, when the geometrically thin disc approximation is conserved.

We have emphasized on the processes, which determine the behaviour of the disc plasma in two considered cases.

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НЯКОИ ОСОБЕНОСТИ НА α ДИСК И АДВЕКТИВНО-ДОМИНИРАН АКРЕЦИОНЕН ДИСК. АВТОМОДЕБНИ РЕШЕНИЯ И ТЯХНОТО СРАВНЕНИЕ

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Резюме

Направен е кратък обзор на особеностите на Стандартния диск на Шакура-Сюняев и Адвективно-доминиращия акреционен диск. Представена е физичната основа, която ние използваме за да получим параметрите, описващи двата модела. Построените теоретични системи са трансформирани в подходящ за изследване вид.