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OPTIMIZATION OF THE FUNCTION INJECTION MODELS IN THE MAGNETOSPHERE

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Abstract

Six function injection models in the magnetosphere are optimized. The minimum of the functional (least squares of the difference between experimental data and models) by different initial coefficient values of the studied mathematical models are found. Some examples of the model yield one minimum with the optimal coefficients.

Introduction

The carried out research on function injection is one of the main tasks of the International Programme STEP (Solar-Terrestrial Energy Programme). The thorough study of this problem is of great importance in the present decade (1995-2005; 23rd Solar cycle). The Polish astronomer Kopetckiy has qualified this decade as a "dangerous decade" due to the fact that, during it, an extremely high geomagnetic activity is expected.

Having in mind this fact, we optimized six existing models of function injection F, described in literature, [Feldstein et all., 1990, 1989, Dremuhyna et all., 1990, Ivanova P., 1992, Murayama, 1986, Bargadze, 1986, Akasofu, 1981], using one of the numerical methods, namely the simplex method (well known in experiment planning).

Models of the function injection F have been made by a lot of authors. For example, in [Feldstein et all., 1990], linear regression equations for a are obtained, which connect the velocity of entering energy to the ring current with various combinations of geoeffective parameters of the Solar Wind (SW) and the Interplanetary Magnetic Field (IMF). The highest correlation coefficient is equal to 0.8 and it is characteristic of the correlation between the magnetic field of the ring current of the function injection in the magnetosphere F_{exp} calculated by ground observations and its model F_{mod}.

Estimation method

We have studied six models of function injection F [Feldstein et all., 1990, 1989, Murayama T., 1986, Bargadze L. F. et all., 1986, Akasofu S. I., 1981], which are shown on Table No 1, where x_1, x_2, x_3 are their coefficients. SW and IMF take part in the models. The conditional designations are V and D - the velocity and the density of the SW. B, By, Bz are the module, the azimuthal and the vertical component of the IMF, ε is the power function of Akasofu and τ is the ring current decay constant.

We have improved the models by optimizing their coefficients. For this task we used the simplex method [Nelder J.A. et all., 1964] because of the simplicity and synonymy of its mechanism:

Let's take functional (1)

(1) $U=\sum_{i=1}^{N}(DR-DRM)^2$, where DR are the experimental values of the ring current, where DRM is the mathematical expression of the model M=1, 2, 3, 4, 5 and 6 respectively. M stands for the number of the optimized model.

The essence of the method lies in the fact that we make a random simplex (a body with N+1 pecks, k=1, 2,..., N+1, N are the parameters) of the computed value of functional U.

Further it changes under the influence of three operations:

a) reflection $P^*=(1-\alpha)P-\alpha P_k$, where $\alpha\in(0,1)$, P_k are the pecks of the simplex, k=1,...,N+1 ; $U_h=\max(U_k)$ for P_h, where U_h is the maximal value of the functional in pecks P_k. P is the central point of the simplex, α is a reflection coefficient, P_l is the simplex peck with minimal value of the functional U or we have the condition $U_L=\min(U_k)$ for P_L.

b) contraction $P^{**}=\beta P^*+(1-\beta)P$, where $\beta\in(0,1)$ is the contraction coefficient.

c) extension $P^{**}=\gamma P^*+(1-\gamma)P$, where $\gamma\in(0,1)$ is the simplex extension coefficient and P is its center. The simplex goes in the global minimum of the functional U with these operations, where its pecks are in one point, which gives the optimal values of our parameters.

We consider DRM-ring current for the investigation models in every iteration by the folowing expression:

$$DRMj = (2\ FM_{j-1}+DRM_{j-1})[2-(1/\tau_{j-1})]/[2+(1/\tau_{j-1})]$$

Further the consideration procedure goes to (1). The iterational process continues to the accuracye that we are expecting e. g. $U = 0,1.10^{-2}$ in our case.

Results

The results and the optimization proceses are shown in tables No 1, 2, 3 and 4. The experimental values for DR-ring current are from SSC 27 August 1978 14 UT , 30 August 1978 2UT and 23 March 1969 14UT.

On Table No 1 the investigation models are given. Given three coefficients are x_1, x_2 and x_3 , the values of which we can see in Table No 4. The value of the funcional U is shown in the last column of Table No 4 , from which the significant improvement of the studied models is seen. In all models, the value of the functional U is equal to 10^5, but in the ones obtained using a new coefficient the value of U is 10^{-2}.

Therefore, the obtained models are significantly improved and specified and they model the ring current function injection in the magnetosphere with really higher accuracy. Another contribution of the present studies is in the effective application of the optimization methods in this sphere of the space physics.

Table N^0 3 illustrates the results of the method. All examples begin from different initial values of the parameters. In the end of the optimization they yield the same value for the point in which the simplex is contracted.

This represents the solution of the task.

Conclusions

From the results we can draw the following conclusions:

1. Using the algorithm and program suggested in this paper, all numerical models of the function injection F in the magnetosphere producing the magnetic variations on the Earth's surface can be optimized.
2. The optimal models produce the best mathematical approximation of F_{exp} by F_{mod}.
3. The new models improve the coefficients of the correlation r between F_{exp} and F_{mod} (for example, $r_1 = 0,91$ $r_{1\ opt} = 0,97$ by F_1).

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Table 1
Optimization models
$F1=x1.10^{-3}B_zV + x2$; if $B_z.V<1$ mV/m and $F1==-x3.10^3(V-300)$; if $B_z.V>-1$ mV/m;
$F2=-x1.B_T.V.\sin^2(Q/2).10^{-3} - x2$ if $B_T.V.\sin^2(\theta/2).10^{-3}>0,1$ mV/m. $F2=-x3.(V-300).10^{-3}$ if $B_T.V.\sin^2(\theta/2).10^{-3}.10^{-3}<0,1$ mV/m; where $\Theta=\mathrm{arcth}\ B_y/B_z$; $B_T=(B_z^2+B_y^2)^{1/2}$
$F3=-x1.10^{-6}.F_{mur} - x2$; if $F_{mur}>10^6$ $F3=-x3(V-300).10^{-3}$; if $B_z>0$ where $F_{mur}=B_s^{1,09}.V^{2,06}.D^{0,38}$; where $B_s=B_z<0$. D is density
$F4=-x1.10^{-3}.F_{bar} - x2$; where $F_{bar}=(D.V^2)^{1/6}.V.B_T.\sin^4(0/2)$; $B_T=(B_z^2+B_y^2)^{1/2}$; $\theta=\mathrm{arctg}(B_y/B_z)$
$F5=-x1.10^{-18}\varepsilon - x2$; where $\varepsilon=2.10^{14}.B^2.V.\sin^4(\Theta/2)$;
$F6=x1.V.B_z.10^{-3}$; if $B_z<0$; and $(B_z+\sigma)<0$; $F6=x2.V.(B_z-\sigma)/2).10^{-3}$; if $B_z<0$ and $(B_z+\sigma)>0$; $F6=x2.V((B_z-\sigma)/2).10^{-3}$; if $B_z<0$ and $(B_z-\sigma)<0$; $F6=x3$; if $(B_z-\sigma)>0$; $B_z>0$; σ - dispersion of the IMF V, D, Bx, By, Bz - parameters of the SW and IMF.

Table 2
Optimal models
$F1=8,8.10^{-3}B_zV - 16$; if $B_zV<1$ mV/m and $F1=68,0.(V - 300).),002$; if $B_zV> -1$ mV;
$F2=-10,3.B_T.V.\sin^2(Q/2).10^{-3}+5,0$; if $B_TV\sin^2(\theta/2).10^{-3}>0.1$ mV/m; $F2=113.(V-300).10^{-3}$; if $B^TV\sin^2(0/2).10^{-3}<0,1$ mV/m.
$F3=-10,3.10^6.F_{mur}+5,1$; if $F_{umr}>10^6$ $F3=112.(V-300).10^{-3}$; if $B_z>0$ $F_{mur}=B_s^{1,09}.V^{2,06}.D^{0,38}$
$F4=-1,2.10^{-3}.F_{Bar}-30,4$; $F_{Bar}=(DV)VB\sin(4Q/2)$
$F5=-2,2.\varepsilon.10^{-4}+9,7$; $\varepsilon=2.10^{14}.B^2.V.\sin^4(Q/2)$
$F6=10,7.V.B_z.10^{-3}$; if $B_z<0$; and $(B_z+\sigma)<0$; $F6=9,1.V.((B_z-\sigma)/2).10^{-3}$; if $B_z<0$; and $(B_z+\sigma)>0$; $F6=9,1.V.((B_z-\sigma)/2).10^{-3}$; if $B_z<0$; and $(B_z-\sigma)<0$; $F6=0$; $(B_z-\sigma)>0$ and $B_z>0$.

Table 3

Examples, illustrating the optimization process:
initial and optimal values of the model coefficients

	x1	x2	x3	U
Initial values example 1 for M1	10,9	7,9	14,5	$0,27.10^4$
Initial values example 2 for M1	10,0	0,8	13,5	$0,5.10^4$
Initial values example 3 for M1				
Optimal values for all three examples	8,8	-16,0	-68,0	$0,3.10^{-2}$
Initial values example 1 for M2	19,4	0,4	14,4	$0,46.10^5$
Initial values example 2 for M2	15,0	0,8	13,0	$0,5.10^5$
Initial values example 3 for M2	19,0	1,0	15,0	$0,15.10^6$
Optimal values for all three examples	10,3	-5,0	-113,0	$0,1.10^{-1}$
Initial values example 1 for M3	5,8	0,5	14,5	$0,1.10^5$
Initial values example 2 for M3	5,0	0,8	13,0	$0,3.10^5$
Initial values example 3 for M3	4,0	1,0	14,0	$0,5.10^5$
Initial values example 4 for M3	4,4	1,5	15,0	$0,6.10^5$
Optimal values for all three examples	10,3	-5,1	-112,0	$0,3.10^{-2}$
Initial values example 1 for M4	6,4	7,4		$0,9.10^5$
Initial values example 2 for M4	5,0	4,8		$0,7.10^5$
Initial values example 3 for M4	4,0	5,0		$0,2.10^5$
Initial values example 4 for M4	8,5	4,0		$0,4.10^6$
Optimal values for all three examples	1,2	30,0		$0,4.10^{-1}$
Initial values example 1 for M5	6,4	7,4		$0,8.10^6$

Initial values example 2 for M5	5,0	4,8		$0,1.10^{7}$
Initial values example 3 for M5	4,0	5,0		$0,7.10^{6}$
Initial values example 4 for M5	8,5	4,0		$0,4.10^{7}$
Optimal values for all three examples	2,2	-2,7		$0,2.10^{-1}$
Initial values example 1 for M6	6,8	7,5		$7,5.10^{4}$
Initial values example 2 for M6	5,9	5,0		$0,2.10^{5}$
Initial values example 3 for M6	5,4	5,3		$0,3.10^{5}$
Initial values example 4 for M6	10,4	4,5		$0,1.10^{5}$
Optimal values for all three examples	10,7	9,1		$0,3.10^{-1}$

Table 4

Coefficients of the old and the new (optimal) models

Old coeff.	x1	x2	x3	U
M1	8,9	7,0	14,1	$0,3.10^{4}$
M2	19,8	0,6	14,1	$0,5.10^{5}$
M3	3,7	0,4	14,1	$0,3.10^{5}$
M4	3,8	2,8	-	$0,6.10^{5}$
M5	7,2	3,1	-	$0,4.10^{5}$
M6	5,4	5,4	-	$0,2.10^{5}$
New (optimal) coefficient				
M1	8,8	-16,0	-68,0	$0,3.10^{-2}$
M2	10,3	-5,1	112,0	$0,1.10^{-1}$
M3	10,3	-5,1	112,0	$0,7.10^{-2}$
M4	1,2	30,5	-	$0,5.10^{-1}$
M5	2,2	9,7	-	$0,2.10^{-1}$
M6	10,7	9,1	-	$0,3.10^{-1}$

R e f e r e n c e s

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ОПТИМИЗАЦИЯ НА ФУНКЦИОНАЛНИ ИНЖЕКЦИОННИ МОДЕЛИ В МАГНИТОСФЕРАТА С ПОМОЩТА НА СИМПЛЕКС МЕТОДА

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Резюме

Оптимизирани са шест инжекционни модела в магнитосферата. Използвайки различни стойности на началните коефициенти на изследваните математически модели е намерен минимума на функционала (метод на най-малките квадрати на разликата между експерименталните и моделните данни). Някои примери на модела дават един минимум с оптимални коефициенти.