ISOPERIMETRICAL TASK FOR OPTIMIZATION OF THE CUMULATIVE CHARGE FOR PSEUDOMETEORITE PARTICLES

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A factor enhancing the effect of pseudometeorite particles when the shell of a space body is fired at in laboratory conditions is the maximal utilization of the net volume of the charge forming an elongated cumulative cloud of particles. Moreover, the cloud should also have maximal velocity and the needed length, with size and mass restrictions on the cumulative charge.

The symmetry of the cumulative charge structure with respect to the axis Ox reduces the solution of the problem for optimization with respect to the dynamic load factor to determination of the maximal area of a rotating figure $ABCD$ about the axis Ox (fig.1). The figure $ABCD$ is limited by the straight lines $x=0$, $x=H$ and the plane curves $y = \Phi(x)$ and $y = \varphi(x)$. The function $y = \Phi(x)$ is known; while rotating, it describes the internal surface of the cumulative charge shell, while $y = \varphi(x)$ describes the external surface of the cumulative charge lining. The rotation of $ABCD$ determines the volume occupied by the explosive substance.

The physics of the process of formation of a high-velocity cloud of pseudometeorite particle imposes on the function $y = \varphi(x)$ the following restrictions: it should be smooth, continuous, and it should have positive first and second derivatives; besides:

(1) $$\varphi'(0) = \alpha_{\Gamma};\ \varphi'(H) = \alpha_{XB},$$

where α_{Γ} and α_{XB} are local static apex angles satisfying the condition for a continuous, discrete, or disperse cumulative jet of cloud particles [1, 2].

The boundary conditions are:

(2) $$y(A) = \varphi_0;\ y(B) = \varphi_H.$$

The cumulative charge length of the line profile is unknown. It can be determined as a function of the cumulative cloud length [3].

The length of a plane curve is determined by expression [4]:

(3) $$L = \int_0^H \sqrt{1 + \varphi'^2}\, dx\,.$$

The following problem is formulated: among all functions $y = \varphi(x)$, satisfying conditions (1), (2) and (3), it is necessary to select the one for which the area S_D of the figure *ABCD* attains maximum value:

(4) $$S_D = \int_0^H (\Phi - \varphi) dx\,.$$

For the purpose, Euler's equation [5] must be solved for function:

$$J^* = \varphi + \lambda^* \sqrt{1 + \varphi'^2}$$

where λ^* is a Lagrangian multiplier.

Using intermediate integral [5]

$$J - J'_{\varphi'}\, \varphi' = C_1$$

we obtain [5]:

$$\varphi + \lambda\sqrt{1 + \varphi'^2} - \lambda \frac{\varphi'}{\sqrt{1 + \varphi'^2}} \varphi' = C_1\,.$$

Then, we transform [5]:

$$\varphi + \frac{\lambda}{\sqrt{1+\varphi'^2}} = C_1;$$

$$\lambda^2 = (1+\varphi'^2)(C_1 - \varphi)^2;$$

$$\varphi'^2 = \frac{\lambda^2}{(C_1 - \varphi)^2} - 1;$$

$$\frac{d\varphi}{dx} = \pm \frac{\sqrt{\lambda^2 - (\varphi - C_1)^2}}{\varphi - C_1}\,.$$

But $\varphi > 0$, then:

$$\frac{(\varphi - C_1)d\varphi}{\sqrt{\lambda^2 - (\varphi - C_1)^2}} = dx;$$

$$\sqrt{\lambda^2 - (\varphi - C_1)^2} = x + C^2;$$

$$(x + C_2)^2 + (\varphi - C_1)^2 = \lambda^2 .$$

Equation (6) is the equation of a circumference, i.e. the solution of the problem is a circumference arc. Since the arc length is not determined by the problem's condition, we take into account conditions (1). The center of the wanted circumference with radius λ is determined as the interception point O_1 of normals $\eta_Г$ and η_{XB} to the tangents of function $\varphi(x)$ at points A and B respectively where, for geometrical reasons related with the given structure, the radius λ is equal to:

(7)
$$\lambda = \frac{\sqrt{H^2 + (\varphi_H - \varphi_0)^2}}{2\sin(\alpha_{XB} - \alpha_Г)} .$$

Besides:

$$C_1 = \frac{\cos\alpha_Г}{2\sin(\alpha_{XB} - \alpha_Г)} \sqrt{H^2 + (\varphi_H - \varphi_0)^2} ;$$

$$C_2 = C_1 tg\alpha_Г .$$

As a result, an analytical expression for the cumulative charge line profile is obtained, ensuring maximal utilization of the free design space and accounting for the physics of the process and the additional integral condition (3).

The problem is a simple example of the use of variation calculus in cumulative charge optimization. It should be noted that here, the effect's optimization on the parameter of maximal utilization of the net volume is partial. To provide adequate description of the process, its kinematics and dynamics must be taken into account as well.

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The isoperimetrical task for optimization of the cumulative charge of pseudometeorite particles on the parameter of maximal utilization of net volume is formulated and an analytical solution in the form of an equation of a circumference is obtained for the cumulative charge lining profile.

ИЗОПЕРИМЕТРИЧНА ЗАДАЧА ЗА ОПТИМИЗАЦИЯ НА КУМУЛАТИВЕН ЗАРЯД ЗА ПСЕВДОМЕТЕОРИТНИ ЧАСТИЦИ

Христо Христов, Виктор Баранов

Формулирана е изопериметрична задача за оптимизация на кумулативен заряд за псевдометеоритни частици по параметър - максимално използуване на полезния обем и е получено аналитично решение във вид на уравнение на окръжност за образуващата на профила на кумулативната облицовка.

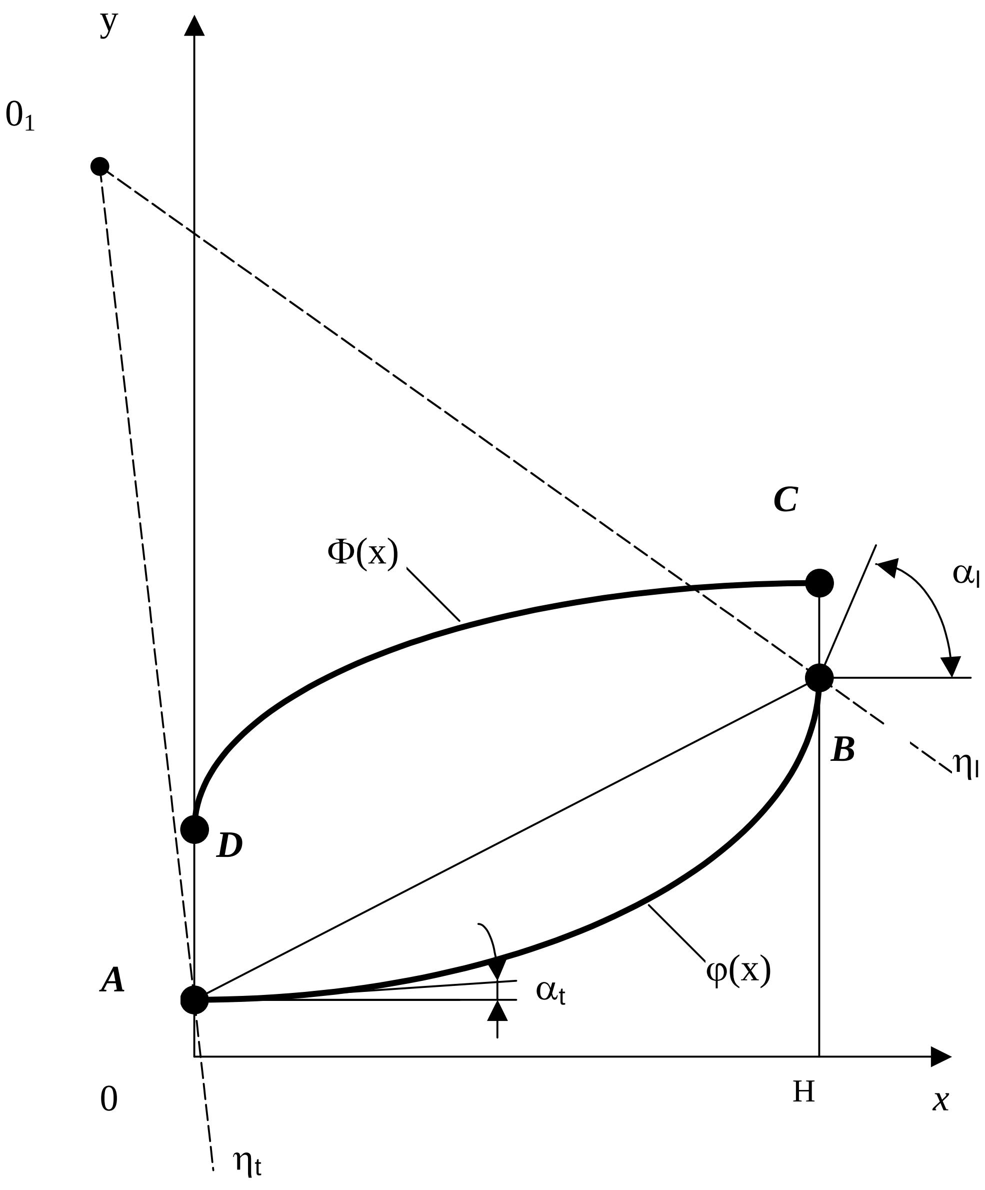

Fig.1. The diagram of cumulative line profile definition $y = \varphi(x)$.