Българска академия на науките. Bulgarian Academy of Sciences
Аерокосмически изследвания в България. 14. Aerospace Research in Bulgaria
София. 1998. Sofia

Application of models for comparison in aerospace experiments data processing and interpretation

Roumen Nedkov

Space Research Institute, Bulgarian Academy of Sciences

Introduction

Practically most result mistakes in aerospace investigations are provoked by the oftenly occurring moments of indefiniteness in the real process of data registration and processing. The resons such indefiniteness origin are practically from different nature and have a complex character. Principally they are: discrepancy between the theoretical models for description of space area and the real conditions, the space experiments take place, Re-covering of the investigated classes of patterns in space of instrumental measurements; Choice of wrong identification of educating patterns connected with wrong definition of discriminate functions in the area of recognition and classification of the investigated objects; principal oblated of the scientific hardware [1, 2].

The present papers are dedicated to the application of models for comparison in pairs, which result in decrease the level of indefiniteness in the educating patterns identifying.

The main aim of application of such models is to decrease the mistakes in space experiments data processing and interpretation.

Models of experiments for comparison in pairs

In result of data processing different values of estimate of the features of the images of the k objects $A_1, A_2, ..., A_k$ of one and the same class occur.

Questions whether these differences are provoked by random factors or the objects are essentially different rises.

If $[\mathbf{X}_1], [\mathbf{X}_2], ..., [\mathbf{X}_k]$, $k \in [1, K]$ are matrixes of the measured sampling values of the features and each matrix had dimension $n \times m$, $(n \in [1, N]$, $m \in [1, M])$, i.e.

(1)
$$[\mathrm{X}_i]=\begin{bmatrix} x_{11}, x_{12}, \ldots, x_{1m} \\ x_{21}, x_{22}, \ldots, x_{2m} \\ \ldots\ldots\ldots\ldots\ldots \\ x_{r1}, x_{r2}, \ldots, x_{rm} \\ \ldots\ldots\ldots\ldots\ldots \\ x_{n1}, x_{n2}, \ldots, x_{nm} \end{bmatrix}$$

then the set of k objects of one and the same class is defined by the vectors of the measured sampling values of the features of the images.

The matrix is:

(2)
$$\begin{bmatrix} \vec{\mathrm{X}}_1^{\mathrm{T}} \\ \vec{\mathrm{X}}_2^{\mathrm{T}} \\ \vdots \\ \vec{\mathrm{X}}_r^{\mathrm{T}} \\ \vdots \\ \vec{\mathrm{X}}_k^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} x_{11}, x_{12}, \ldots, x_{1m} \\ x_{21}, x_{22}, \ldots, x_{2m} \\ \ldots\ldots\ldots\ldots\ldots \\ x_{r1}, x_{r2}, \ldots, x_{rm} \\ \ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots \\ x_{k1}, x_{k2}, \ldots, x_{km} \end{bmatrix}$$

The procedure CP is:

The measured sampling values of the feature of the images $[\mathrm{X}_1], [\mathrm{X}_2], \ldots, [\mathrm{X}_k]$ are compared. Each one is compared with the other one (the objects is compared with itself). The account of the operation CP of k objects based on the matrix $[\mathrm{X}_1], [\mathrm{X}_2], \ldots, [\mathrm{X}_k]$ is $0{,}5\, N M K (K-1)$.

Analitically the procedure is described by the inequality:

(3)
$$g_m(\vec{\mathrm{X}}_{imn}) > g_m(\vec{\mathrm{X}}_{jmn}) \Rightarrow y_{ijmn} = 1, (y_{jimn} = 0),$$

(4)
$$g_m(\vec{\mathrm{X}}_{imn}) > g_m(\vec{\mathrm{X}}_{jmn}) \Rightarrow y_{ijmn} = 0, (y_{jimn} = 1),$$

where g_m, $m \in [1, M]$, y_{ijmn} are m-th discriminate function, based on the m-educating feature and the binary random value: The binary random value is a CP procedure result. $y_{ijmn} = 1$ means that the i-th object belongs to the given class. $y_{jimn} = 0$ means that the applicance of the i-th object to the given class is rejected $i, j \in [1, k]$. In result of CP procedure ((3) or (4)) a numeric series $\{s_i\}$ is set. It's terms are the weight sums of compared objects when $m \to \infty, n \to \infty$ the terms of the numeric series $\{s_i\}$ limes to it's mean values

(5)
$$s_{im} = \frac{1}{N} \sum_{n=1}^{N} y_{imn}.$$

Quantitatively y_{jirm} the indefiniteness connected with the random value of y_{jirm} can be described by means of the probability by which $y_{ijmn} = 1$, i.e.

(6) $$P\{y_{ijmn}=1\}=\xi_{ijmn},\ 0\le\xi_{ijmn}\le 1,$$

(7) $$\xi_{jimn}=1-\xi_{ijmn},\ 0\le\xi_{jimn}\le 1.$$

Based on the considerations exposed above the following models of experiments for CP can be formulating [3].

MODEL A. The probability ξ_{ijmn} does not depend on the sequence of comparison, i.e. either the procedure (3) or (4) is realised, because the result is one and the same. In this case the probability ξ_{ijmn} depends only on i or only on j, i.e $\xi_{ijmn}=\xi_{imn}$. That means, that there is no effect in repeat the comparison, i.e. if one of the procedures (3) or (4) is realised it's not necessary to realise the other one.

MODEL B. The probability ξ_{imn} does not depend on the feature serial number m on which the comparison is realised, according to the procedures (3) or (4), i.e. there is no need to, repeat the comparison on definite feature. In this case the probability $\xi_{inm}=\xi_{in}$.

MODEL C. The probability ξ_{in} does not depend on the choice of the consecutive discreet value of the feature comparison n because the procedures (3) or (4) are realised for all measured discreet values of the features (proportion (5)).

In this case the probability $\xi_{in}=\xi_i$.

This work is dedicated to the application of **MODEL C.**

According to this model the definition of the models applicance to the given class or their rejection can be realised by estimation of the mean probability $\hat{\xi}_i$, or by mean probability $\bar{\xi}_i=E\{\xi_{ji}\}$ when $m\to\infty, n\to\infty$ (the script $E\{.\}$ indicates the averaging operation), where the $\hat{\xi}_i$ and $\bar{\xi}_i$ following equations:

(8) $$\hat{\xi}_i=\frac{s_{im}}{M\,(K-1)},$$

(9) $$\bar{\xi}_i=E\{\hat{\xi}_i\}.$$

Criteria for estimation the results of the statistic data processing, applying MODEL C

The criteria for estimation the result of CP procedure are:

I. Criteria for total equivalency

The zero hypotheses of these criteria are the following H_0: All objects are equal, the differences in the weight sums are provoked by random factors, i.e. $\xi_i=0{,}5$ for $\forall i$.

The alternative hypothesis is H_a: The objects are different and this is not provoked by random factors, i.e. $\bar{\xi}_i\neq 0{,}5$ for $\forall i$.

A measure of displacement $d_i=\dfrac{s_i-\bar{s}_i}{0{,}5\sqrt{mk}}$ is involved.

According to the Pirson's criteria [4,5], when $m\to\infty, k\to\infty$ the distribution of d_i of limits to normal with zero mean value and a mean quadratic displacement 1-$N(0,1)$. In this case the criteria for total equivalency analytically in described as follows:

(10) $$D_m = \sum_{i=1}^{K} d_i^2 = \frac{4}{mk} [\sum_{i=1}^{K} s_i^2 - 0{,}25\, m^2 k\, (k-1)^2].$$

In this case if H_0 is true, then the value D_m must not be great than the criterial value of the tabulated function with apriori given level of significance α and k-1 degrees of freedom, i.e.

(11) $$D_m \le D_{mc} = \chi^2_{1\text{-}\alpha}\,(k-1).$$

If the condition (11) is not true, H_0 is rejected. H_a is accepted, which means that the objects are different and the differences between them are not provoked by random factors.

The scheme for application of the criteria is the following:

1. A level of significance α is given.
2. The criterial value of the chosen level of significance is defined α,

$$D_m \le D_{mc} = \chi^2_{1\text{-}\alpha}\,(k-1).$$

3. The value of D_m according to formula (10) is calculated.
4. A check-up according to the condition (11) is done. It consist in the following:

(12) if $D_m \le D_{mc} = \chi^2_{1\text{-}\alpha}\,(k-1)$ then H_0 is true,

(13) if $D_m \ge D_{mc} = \chi^2_{1\text{-}\alpha}\,(k-1)$ then H_a is true.

II. Criteria for the special object

The zero hypotheses for this criteria is the following:

H_0: All objects are equal, the special object is as all, i.e. $\bar{\xi}_i = 0.5$ for $\forall i$.

The alternative hypothesis is H_a: The object with weight sum s_d is different and this is not provoked by random factors, i.e. $\bar{\xi}_d > 0.5\ (1 \le d \le k)$.

In this case according to the Pirson's criteria the distribution of the weight sums limits to the normal with mean value $E\{s_i\}$ = 0,5 m (k-1) and a mean quadratic displacement $\sigma_s = 0{,}5\sqrt{m(k-1)}$.

The criterial value of the weight sum s_c is defined by means of the equations

(14) $$s_c = [0{,}5m\,(k-1) + \chi^2_{1\text{-}\alpha}(k-1)\,\sigma_s]$$

or

(15) $$s_c = [0{,}5m\,(k-1) + \chi^2_{1\text{-}\alpha}(k-1)\,\sigma_s + 0{,}5]$$

if the data capacity is not enough.

The symbol [.] indicated the least integer, which does not exceed the numerical value of the expression in the square brackets.

The scheme for application of the criteria is the following:

1. The level of significance α is given.
2. The criterial value of the weight sum s_c for the chosen level of significance α is calculated according to the formulas (14) and (15).
3. The values of the critical weight sum s_c and the weight sum of the special object are compared. This procedure has the following view:

(16) if $s_d < s_c$ H_0 is accepted,

(17) if $s_d > s_c$ H_a is accepted.

III. Criteria for equivalence of two special objects

The task solved by these criteria is as follows:

Does the difference in the weight sums of two special objects means that the objects are from different classes.

The zero hypotheses is H_0: The difference in the weight sums of the two special objects is provoked by random factors.

The alternative hypothesis is H_a: The difference in the weight sums of the two special objects is not provoked by random factors, i.e. the objects are from different classes.

According to the Pirson's criteria the distribution of the difference of the weight sums $\Delta s_{lf} < |s_l - s_f|, (l, f \in [1, k])$ limits to normal with zero mean value and a mean quadratic displacement $\sigma_{\Delta s_{lf}} = 0{,}5\sqrt{mk}, m \in N(0, \sigma_{\Delta s_{lf}})$.

The critical value Δs_c of the difference is defined by the equations

(18) $$\Delta s_c = [\chi^2_{1-\alpha}(k-1)\sigma_{\Delta s_{lf}}]$$

or

(19) $$\Delta s_c = [\chi^2_{1-\alpha}(k-1)\sigma_{\Delta s_{lf}} + 0{,}5]$$

if the data capacity is not enough. The symbol [.] indicates the least integer, which does not exceed the numerical value of the expression in the square brackets.

The scheme for application of the criteria is the following:

1. The level of significance α is given.
2. The critical value of the difference Δs_c for the chosen level of significance α is calculated according to (18) or (19).
3. The values of $\Delta s_{lf} < |s_l - s_f|, (l, f \in [1, k])$ and Δs_c are compared. This procedure has the following view:

(20) if $\Delta s_{lf} < \Delta s_c$ H_0 is accepted,

(21) if $\Delta s_{lf} \geq \Delta s_c$ H_a is accepted.

IV. Criteria for equivalence of two objects (criteria for the least significant sum)

The task solved by this criteria is the definition of such difference in the weight sums of the two objects s_i and s_j, $\Delta s_{ij} = |s_i - s_j|$, which is statistically significant.

The zero hypotheses of these criteria are H_0: All objects are equal, the differences in the weight sums are provoked by random factors $\bar{\xi}_i = \bar{\xi}_j$ for $\forall i, j$.

The alternative hypothesis is H_a: The objects are different and this is not provoked by random factors, i.e. $\bar{\xi}_i \neq \bar{\xi}_j$ for $\forall i, j$.

The scheme for application of the criteria is the following:

1. The level of significance α is given.
2. A check-up according to the scheme of the criteria for common equivalence is done. If H_0 is accepted the procedure is cut off. If H_a is accepted the procedure continues.
3. The critical value of the diference Δs_c, according to (18) or (19) for the chosen level of significance α, is calculated.
4. A check-up according to item 3 of the scheme of criteria III is done. If H_0 is accepted, it is considered that the objects are from one and the same class. If H_a is accepted, it is considered that the objects belong to different classes, which means that their difference is statistically sighnificant.

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Received 16.I.1997

Приложение на модели за сравнение по двойки при обработка и интерпретация на данни, получени от аерокосмически експерименти

Румен Недков

(Р е з ю м е)

В работата са предложени три модела за използване на метода за сравнение по двойки при обработка на данни от аерокосмически изследвания. Целта на прилагане на конкретни вероятностни модели при обработката е намаляване на грешката при определяне на обучаващите признаци при класификация. Изложеното в статията дава основание за прилагане на конкретни статистически модели при класификация на получените аерокосмически данни още в процеса на първичната обработка.