Българска академия на науките. Bulgarian Academy of Sciences
Аерокосмически изследвания в България. 14. Aerospace Research in Bulgaria
София. 1998. Sofia

Qualitative analysis of the free processes in a generalized linear oscillating circuit with periodic parameters
Part 2. Stability of the cannonical systems and a generalized linear resonance circuit[1]

Nikolai D. Birjuk, Vladimir N. Damgov***

* *Department of Physics, Voronej State University, 394693 Voronej, Russia*

** *Space Research Institute, Bulgarian Academy of Sciences, 6 Moscowska Str., 1000 Sofia, Bulgaria*

Introduction

Qualitative analysis assumes considerable importance in the investigation of complex oscillating systems, since it allows of identifying the most general features of the system behaviour. The paper reveals a general method for analyzing linear systems with periodic and almost periodic parameters.

The Part 1 of the present paper [1] has quoted a basic system of two linear differential equations of the generalized parametric oscillating circuit. The reasonable areas of applying different form of equations have been discussed. A three-dimensional cylindrical space has been put in correspondence to the set of equations describing every possible oscillating systems with periodic parameters. Such an approach has allowed to make a methodologically consistent classification of the oscillating circuits with periodical parameters in accordance with the most important indication, namely the stability and unstability according to Lyapunov's propoundings.

In the present Part 2 of the paper, the attention is mostly focused on the problem of stability of the cannonical systems in a general form. Criteria for the stability or instability of a general linear resonance circuit are formed.

[1] An investigation supported by the "Scientific Research" Bulgarian National Fund under Contract No TH–549/95.

Stability of the cannonical systems

Let us assume that we have the following cannonical system:

(1) $$\frac{d}{dt}x = JH(t)x,$$

where $H(t)=\begin{pmatrix}\alpha & \beta\\ \beta & \gamma\end{pmatrix}$

An important role in the stability investigation is played by the rotation angle of vector $x(t)=X(t)\,C$, C is a constant vector.

We shall denote the rotation angle of a given vector $z(t)=\begin{pmatrix}p\\ q\end{pmatrix}$ as $\varphi_z(t)$. Obviously,

(2) $$\frac{d\varphi_z(t)}{dt}=\frac{d}{dt}\operatorname{arctg}\frac{q}{p}=\frac{\dot{q}p-\dot{p}q}{p^2+q^2}=\frac{1}{p^2+q^2}\operatorname{Det}\begin{pmatrix}p & \dot{p}\\ q & \dot{q}\end{pmatrix}$$

i.e. $\varphi_z(t)-\varphi_z(0)=\int\limits_0^t \frac{\operatorname{Det}\ z/\dot{z}}{(z,z)}$.

By using this formula, as well as the relation $\operatorname{Det} a|J b=(a,b)$, we obtain the following equation for the rotation angle of the cannonical system (1)

$$\varphi_x=\int\limits_0^T\frac{(Hx,x)}{(x,x)}\,dt.$$

We designate the characteristic number $h_{max}(t)$ and $h_{min}(t)$ of matrix $H(t)$ as follows:

$$h_{\substack{\max\\ \min}}(t)=\frac{\alpha+\gamma}{2}\pm\sqrt{\frac{(\alpha-\gamma)^2}{4}+\beta^2}.$$

Since

$$h_{\min}(t)\le\frac{(Hx,x)}{(x,x)}\le h_{\max}(t)$$

it follows that

$$\int\limits_0^T h_{\min}(t)\,dt\le\varphi_x\le\int\limits_0^T h_{\max}(t)\,dt.$$

Let us assume that $H=\begin{pmatrix}\alpha_0\beta_0\\ \beta_0\gamma_0\end{pmatrix}$ and α_0, β_0, γ_0, are arbitrary numbers, satisfying the conditions that $\alpha_0>0$, $\gamma_0>0$, $\alpha_0\gamma_0-\beta_0^2=1$ while h_{min}, h_{max} are roots of the equation:

$$\operatorname{Det}\left[H(t)-hH_0\right]=0.$$

If inequalities

(3) $$n\pi<\int\limits_0^T h_{\min}(t)\,dt\le\int\limits_0^T h_{\max}(t)dt<(n+1)\pi$$

are satisfied for a definite $n=0, \pm1, \pm2, \pm\ldots$, equation (1) will be stable and $H(t)\in O_n$ (see [1]).

As an illustration of this stability criterion we shall consider equation $\frac{d^2y}{dt^2}+p(t)\gamma=0$, which is a particular case of the cannonical system (1). Let us set $C>0$ as a constant, for which:

$$\frac{n\pi}{T} < C < \frac{(n+1)\pi}{T}, \quad n \text{ is an integer.}$$

Provided that inequalities

$$n\pi < \frac{1}{C}\int_0^T p_C^-(t)\,dt \le \frac{1}{C}\int_0^T p_C^+(t)\,dt < (n+1)\pi, \tag{4}$$

where
$$p_C^+(t) = \begin{cases} p(t), \text{ for } p(t) > C^2 \\ C^2, \text{ for } p(t) \le C^2 \end{cases}$$

$$p_C^-(t) = \begin{cases} p(t), \text{ for } p(t) \le C^2 \\ C^2, \text{ for } p(t) > C^2 \end{cases}$$

are satisfied, the equation under consideration is stable and $p(t) \in O_n$ (see [1]).

Let us formulate a second stability criterion. Let inequality $[\boldsymbol{H}(t)\ \boldsymbol{C}, \boldsymbol{C}] \ge 0$ be valid for any t and $\boldsymbol{C}$. If inequalities $k\pi < m_- \le M_+ < (k+1)\pi$ are satisfied, equation (1) belongs to the k–th stability area (see [1]).

And if inequalities $M_- > k\pi$, $m_+ < k\pi$ are satisfied, equation (1) belongs to the k–th instability area.

If inequality $(\boldsymbol{H}(t)\ \boldsymbol{C}, \boldsymbol{C}) \le 0$ is valid for any t and $\boldsymbol{C}$, the following substitutions should be carried out in the previous inequalities: $M_\pm$ should be replaced by $M_\mp$, and $m_\pm$ by $m_\mp$.

Here:

$$M_\pm = \frac{A_\pm + C_\pm}{2} + \sqrt{\frac{(A_\pm - C_\pm)^2}{4} + B_\pm^2},$$

$$m_\pm = \frac{A_\pm + C_\pm}{2} - \sqrt{\frac{(A_\pm - C_\pm)^2}{4} + B_\pm^2},$$

$$A_\pm = \int_0^T \exp\left(\pm\int_0^t g\,dt\right)\alpha\,dt, \quad B_\pm = \frac{1}{2}\int_0^T \exp\left(\pm\int_0^t g\,dt\right)\beta\,dt,$$

$$C_\pm = \int_0^T \exp\left(\pm\int_0^t g\,dt\right)\gamma\,dt, \quad g(t) = 2\sqrt{\frac{(\alpha-\gamma)^2}{4} + \beta^2}.$$

Sometimes, owing to various reasons, equation (1) proves to be inconvenient for analyzing its stability. In such cases it is desirable to transform (1) into another equation located in the same area of stability or instability. Let us illustrate this option.

The following denotations are introduced as a supplement to (1):

$$\beta_0 = \frac{1}{T}\int_0^T \beta(t)\,dt, \quad \delta(t) = \int_0^t [\beta(t) - \beta_0]\,dt. \tag{5}$$

The variables are substituted as follows:

$$x_1 = e^{-\delta(t)} z_1, \quad x_2 = e^{-\delta(t)} z_2.$$

The result is a system of equations:

$$(6) \quad \left| \begin{aligned} \frac{dz_1}{dt} &= -\beta_0 z_1 - \gamma_1(t) z_2, \\ \frac{dz_2}{dt} &= \alpha_1(t) z_1 + \beta_0 z_2. \end{aligned} \right.$$

This is a system of a cannonical type with a matrix

$$H_1(t) = \begin{pmatrix} \alpha_1(t) & \beta_0 \\ \beta_0 & \gamma_1(t) \end{pmatrix}, \quad \alpha_1(t) = \alpha(t) e^{-2\delta(t)}, \quad \gamma_1(t) = \gamma(t) e^{2\delta(t)}.$$

Systems (1) and (6) are situated in the same areas of stability or instability, for it can be shown that the rotation angles of arbitrary vector solutions $x(t)$ and $z(t)$ for these two systems of equations are identical. System (6) however is frequently more convenient for investigation, since two elements in matrix $H_1(t)$ have proved to be equal and unchanging (constant) in time.

It is always possible to identify two constant matrices $C^{\pm}$ so as to satisfy the inequalities: $C^{-} \le H_1(t) \le C^{+}$.

These two matrices are determined in a sufficiently simple way as

$$(7) \quad C^{\pm} = \begin{pmatrix} \alpha_0^{\pm} & \beta_0 \\ \beta_0 & \gamma_0^{\pm} \end{pmatrix}.$$

$$\alpha_0^{+} = \max_t \alpha(t), \quad \gamma_0^{+} = \max_t \gamma_1(t),$$

$$\alpha_0^{-} = \min_t \alpha(t), \quad \gamma_0^{-} = \min_t \gamma_1(t),$$

The constant matrix $C^{\pm}$ is referred to the n-th stability zone, i.e. $C^{\pm} \in O_n$ if $\frac{n^2 \pi^2}{T^2} < \operatorname{Det} C^{\pm} < \frac{(n+1)^2 \pi^2}{T^2}$ (see [1]).

On this basis the stability criterion can be formulated in the following manner.

If the inequalities

$$(8) \quad \frac{n^2 \pi^2}{T^2} \le \alpha_0^{\pm} \gamma_0^{\pm} - \beta_0^2 \le \frac{(n+1)^2 \pi^2}{T^2}$$

are satisfied and $\alpha_0^{+} + \gamma_0^{+} > 0$, $\alpha_0^{-} + \gamma_0^{-} > 0$, equation (1) will be stable and $H(t) \in O_n$, $n > 0$.

Provided that inequalities (8) are satisfied, but $\alpha_0^{+} + \gamma_0^{+} < 0$ and $\alpha_0^{-} + \gamma_0^{-} < 0$, equation (1) will also be stable and $H(t) \in O_{-n-1} (n > 0)$.

In the case of $\alpha_0^{+} \gamma_0^{+} - \beta_0^2 < 0$ and $\alpha_0^{-} \gamma_0^{-} - \beta_0^2 < 0$, $H(t)$ will belong to the zero area of instability.

Finally we shall dwell on the following practical issue. Let us assume that there is a particular cannonical system of the type of (1). How should we answer the question which area of stability or instability it belongs to? In order to come up with an answer, we should approximate the matrix of system $H(t)$ with two piece-wise constant matrices $H^-(t)$ and $H^+(t)$, so that $H^-(t) \leq H(t) \leq H^+(t)$. As the approximation accuracy increases, one of the following two conclusions is ultimately arrived at: (a) Matrices $H^-(t)$ and $H^+(t)$ fall in one area of stability or instability, which will also encompass matrix $H(t)$; (b) These matrices never fall in the same area: one of them is situated in the stable area, while the other one is located in the adjacent unstable zone. Then obviously matrix $H(t)$ lies on the boundary between these two areas.

The outlined method requires an ability to determine which area of stability or instability the piece-wise constant matrices belongs to. We divide up the interval $[0, T]$ into smaller sub-intervals like this: $0=t_0<t_1<t_2<...<t_n=T$, $t_i-t_{i-1}=\tau_i$. We set $A(t)=K_i$ for $t_{i-1}<t<t_i$. Then $x(t_i)=e^{K_i\tau_i}...e^{K_2\tau_2}e^{K_1\tau_1}$ and in particular

(9) $$x(T)=e^{K_n\tau_n}...e^{K_2\tau_2}e^{K_1\tau_1}.$$

It is convenient to calculate the matrix exponent e^K according to the formula:

$$e^K=\mathrm{ch}\,\mu\, I+\frac{1}{\mu}\mathrm{sh}\,\mu\, K,$$

$\pm\mu$ are characteristic figures of K. In the particular case of $\mu=0$, one can determine $e^K=I+K$.

The characteristic equation regarding $x(T)$ is of the form

$$\lambda^2-2a\lambda+1=0,$$

where $2a=\mathrm{Sp}\,x(T)$. Provided that $|a|<1$, it follows that $A(t)\in O$, (all solutions are bounded), and when $|a|>1$, $A(t)\in H$ is valid (there are unbounded solutions as well). With a view to identifying the number of the stable or unstable range, the rotation angle of the solution should be computed.

Let us take $x_0=\begin{pmatrix}1\\0\end{pmatrix}$ as an initial vector. Then for $t=t_1$ the solution will take the form $x_1=e^{K_1\tau_1}\begin{pmatrix}1\\0\end{pmatrix}$. If K_1 is unstable (i.e. $\mathrm{Det}\,K_1\leq 0$), the rotation angle for a period of time τ_1 will fall within the range $-\pi\leq\tau_1<\pi$. If K_1 is stable (i.e. $\mathrm{Det}\,K_1>0$), the matrix column can be used to determine the rotation angle only within an allowance of the term $m\pi$ (m is an integer). For the purpose of establishing the rotation angle accurately, it is necessary to set temporarily $T=\tau_1$ and to determine the stable range to which K_1 belongs. Let us assume that m is the number of this stable range, while φ_1 is the rotation angle over a period of time τ. Then $m\pi<\varphi_1<(m+1)\pi$ and the angle can be determined accurately.

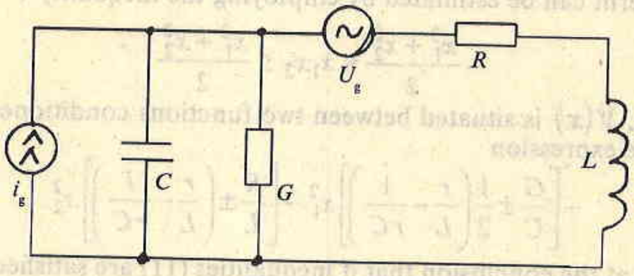
For $t=t_2$ the solution will be $x_2=e^{K_2\tau_2}e^{K_1\tau_1}x_0$; once again the rotation angle φ_2 is determined and the total resultant rotation angle is summed up: $\varphi=\varphi_1+\varphi_2$. The complete rotation angle is: $\varphi=\varphi_1+\varphi_2+...+\varphi_n$. If $|a|<1$ and $m\pi<\varphi<(m+1)\pi$, $A(t)\in O_m$ (see [1]). And if $|a|>1$, the solution will be unstable: $A(t)=H_m$. We should determine m. As it was shown above, there is only one matrix K, satisfying the equation $e^{KT}=\pm B$, $B=e^{K_n\tau_n}.e^{K_{n-1}\tau_{n-1}}....e^{K_2\tau_2}.e^{K_1\tau_1}$, (out of the two possible signs we select the one for which K is real). Let us assume that a_+ and a_- are the natural vectors of matrix K: $Ka_{\pm} = \pm\mu a_{\pm}$ ($\mu>0$). We have already shown that all vectors lying in quadrants I and II (this refers to the natural vectors) turn at an angle φ over time T: $m\pi<\varphi<(m+1)\pi$, and the vectors situated in quadrants II and IV turn at an angle φ: $(m-1)\pi<\varphi<m\pi$. The problem of determining the rotation angle boils down to plotting vectors a_+ and a_-, clarifying which quadrant vector $x_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ lies in, and determining the angle by using one of the inequalities given above.

Thus, by employing simple algebraic operations one can always determine the stable or unstable area to which the interval–constant matrices $H^-(t)$ and $H^+(t)$ belong.

Stability criteria of a generalized linear resonance circuit

We shall consider a generalized linear oscillating circuit of the type shown in Fig. 1, assuming that for $t>0$ its parameters change in accordance with an arbitrary continuous law, yet they remain positive:

$$C(t), G(t), L(t), R(t)>0$$



Фиг. 1.

The free process is described by the following system of differential equations concerning the charge q of the capacitor and the magnetic flux Φ of the inductance:

$$\text{(10)} \quad \left| \begin{aligned} \frac{dx_1}{dt} &= -\frac{G}{C}x_1 - \frac{r}{L}x_2, \\ \frac{dx_2}{dt} &= \frac{1}{rC}x_1 - \frac{R}{L}x_2, \end{aligned} \right.$$

where $x_1 = \dfrac{q}{q_{00}}, x_2 = \dfrac{\Phi}{\Phi_{00}}, r = \dfrac{\Phi_{00}}{q_{00}}$; q_{00}, Φ_{00}, r - constants.

The linear system (10) is a particular case of the system $\dfrac{d}{dt}x = f(x,t)$ (see [1]), where x = colon (x_1, x_2), $f(x,t) = A(t)x$, $A(t) = \begin{pmatrix} -\dfrac{G(t)}{C(t)} & -\dfrac{r}{L(t)} \\ \dfrac{1}{rC(t)} & -\dfrac{R(t)}{L(t)} \end{pmatrix}$

Two criteria (sufficient conditions) for oscillating circuit stability will be recounted.

C r i t e r i o n 1. The oscillating circuit will be stable according to Lyapunov's definition, if a positive constant r can be selected, for which, given $t \geq 0$, the following inequalities will be satisfied:

(11)
$$\left|\frac{G}{C} \pm \frac{1}{2}\left(\frac{r}{L} - \frac{1}{rC}\right)\right| \geq 0,$$
$$\left|\frac{R}{L} \pm \frac{1}{2}\left(\frac{r}{L} - \frac{1}{rC}\right)\right| \geq 0.$$

In order to prove this we select a definitely positive Lyapunov function [2] in the form

(12)
$$V(x) = \frac{1}{2}(x, x) = \frac{1}{2}\left(x_1^2 + x_2^2\right)$$

Its derivative in compliance with system (11) is

$$\dot{V}(x) = x_1\dot{x}_1 + x_2\dot{x}_2 = -\frac{G}{C}x_1^2 - \frac{R}{L}x_2^2 - \left(\frac{r}{L} - \frac{1}{rC}\right)x_1x_2.$$

The last term can be estimated by employing the inequality

$$-\frac{x_1^2 + x_2^2}{2} \leq x_1x_2 \leq \frac{x_1^2 + x_2^2}{2}.$$

Therefore, $\dot{V}(x)$ is situated between two functions conditioned by the different signs of the expression

$$-\left[\frac{G}{C} \pm \frac{1}{2}\left(\frac{r}{L} - \frac{1}{rC}\right)\right]x_1^2 - \left[\frac{R}{L} \pm \left(\frac{r}{L} - \frac{1}{rC}\right)\right]x_2^2.$$

We arrive at the conclusion that if inequalities (11) are satisfied, the derivative of Lyapunov's function is non-positive, which, in this case, is the condition of oscillating circuit stability according to Lyapunov's definition.

Given a satisfaction of the strict inequalities (11), i.e.

(13)
$$\left|\frac{G}{C} \pm \frac{1}{2}\left(\frac{r}{L} - \frac{1}{rC}\right)\right| > 0,$$
$$\left|\frac{R}{L} \pm \frac{1}{2}\left(\frac{r}{L} - \frac{1}{rC}\right)\right| > 0.$$

Lyapunov's conditions for asymptotic stability are met.

C o n s e q u e n c e: The oscillating circuit with positive parameters, where $G(t)$ and $R(t)$ are arbitrary time functions, while C and L are constants, is asymptotically stable according to Lyapunov's definition. Indeed, in this case parameter r can be selected so that $r = \sqrt{\frac{L}{C}}$. Then the bracketed expression in (13) will be nullified. The same consequence can be arrived at by using the energy conservation law as a starting point. Then we have to take into account that in the case of constant reactances there is no energy input in the circuit. The energy is continuously dissipated in the active elements of the circuit at varying speed.

Criterion 2. The oscillating circuit will be stable, if, for an arbitrary t, the following system of inequalities is satisfied:

$$(14)\qquad \begin{vmatrix} R-\dfrac{k-1}{2}\dfrac{dL}{dt}-\dfrac{l}{2}\dfrac{L}{C}\dfrac{dC}{dt}\geq 0, \\ G-\dfrac{l-1}{2}\dfrac{dC}{dt}-\dfrac{k}{2}\dfrac{C}{L}\dfrac{dL}{dt}\geq 0, \end{vmatrix}$$

where k and l are an arbitrary couple of integers taken from the set: $0, \pm 1, \pm 2, ..., \pm\infty$.

In order to prove this we set $r=1$ ($q_{00}=1$ and $\Phi_{00}=1$) in (1) and choose a definitely positive Lyapunov's function in the form

$$(15)\qquad V=L^{k-1}C^{l}\Phi^{2}+L^{k}C^{l-1}q^{2}.$$

Then, provided that inequality (14) is satisfied, its total derivative

$$\frac{dV}{dt}=-2L^{k-2}C^{l}\left[R-\frac{k-1}{2}\frac{dL}{dt}-\frac{l}{2}\frac{L}{C}\frac{dC}{dt}\right]\Phi^{2}$$
$$-2L^{k}C^{l-2}\left[G-\frac{l-1}{2}\frac{dC}{dt}-\frac{k}{2}\frac{C}{L}\frac{dL}{dt}\right]q^{2},$$

will be non-positive, i.e. once again Lyapunov's condition concerning the stability of the specific oscillating circuit under consideration is met. Analogously, provided that the strict inequalities (14) are satisfied, Lyapunov's criteria of asypmtotic stability of the oscillating circuit are met.

Consequence: The oscillating circuit will be stable, if, for $t\geq 0$, the following system of inequalities is satisfied:

$$R+\frac{1}{2}\frac{dL}{dt}\geq 0,\quad G+\frac{1}{2}\frac{dC}{dt}\geq 0.$$

These inequalities are yielded by (11) at $k=l=0$. In this case Lyapunov's function (15) acquires a clear physical meaning, since it represents the instantaneous energy stored in the circuit reactances.

Let us now consider the case when $R(t), G(t)>0$ for $t\geq 0$, and let us assume that the continuously changing reactances of the generalized oscillating circuit (Fig. 1) can take both positive and negative values. In reality a similar situation occurs in the case of Josephson superconducting junctions, whose equivalent inductance takes negative values during a part of the changing period. In a more general treatment, this is a system, where the effect of a single–frequency non–degenerate parametric regeneration is manifested [3].

The free process in the generalized oscillating circuit (Fig. 1), excluding sources i_g and U_g, can be described by the following system of differential equations concerning the voltage U at the capacitor and the current i flowing through the inductance:

$$(16)\qquad \begin{vmatrix} \dfrac{dx_1}{dt}=-\dfrac{1}{C}\left(G+\dfrac{dC}{dt}\right)x_1-\dfrac{1}{rC}x_2, \\ \dfrac{dx_2}{dt}=\dfrac{r}{L}x_1-\dfrac{1}{L}\left(R+\dfrac{dL}{dt}\right)x_2, \end{vmatrix}$$

where $x_1 = \frac{U}{U_{00}}, x_2 = \frac{i}{i_{00}}, r = \frac{U_{00}}{i_{00}}; U_{00}, i_{00}, r$ are constants.

Lyapunov's function is set in the form

(17) $$V = \operatorname{sign} C \frac{rC}{2} x_1^2 + \operatorname{sign} L \frac{L}{2r} x_2^2,$$

where, for example,

$$\operatorname{sign} C = \begin{cases} 1, \text{for } C > 0 \\ 0, \text{for } C = 0 \\ 1, \text{for } C < 0 \end{cases}$$

Hence function (17) is positively defined. Its derivative by virtue of (16) is

$$\frac{dV}{dt} = -\operatorname{sign} C\, r\left(G + \frac{1}{2}\frac{dC}{dt}\right)x_1^2 - \operatorname{sign} L \frac{1}{r}\left(R + \frac{1}{2}\frac{dL}{dt}\right)x_2^2 - (\operatorname{sign} C - \operatorname{sign} L)\, x_1 x_2.$$

The bracketed expression in the last term can take the following values: -2, -1, 0, 1, 2. If the extreme values are considered, it becomes obvious that $\frac{dV}{dt}$ is always located between the following two functions:

(18) $$-\operatorname{sign} C\, r\left(G + \frac{1}{2}\frac{dC}{dt}\right)x_1^2 - \operatorname{sign} L \frac{1}{R}\left(R + \frac{1}{2}\frac{dL}{dt}\right) \pm 2 x_1 x_2.$$

Since evidently $-\left(x_1^2 + x_2^2\right) \le \pm 2 x_1 x_2 \le \left(x_1^2 + x_2^2\right)$ it can be seen that the two functions (18) are located between the functions

(19) $$-\left[\operatorname{sign} C\, r\left(G + \frac{1}{2}\frac{dC}{dt}\right) \pm 1\right] x_1^2 - \left[\operatorname{sign} L \frac{1}{r}\left(R + \frac{1}{2}\frac{dL}{dt}\right) \pm 1\right] x_2^2.$$

Lyapunov's theorems (from Lyapunov's second method) and expression (19) allows of obtaining criteria of the stability or instability of the oscillating circuit in this case.

S t a b i l i t y c r i t e r i o n . The oscillating circuit will be stable, if the following system of non-strict inequalities is satisfied in the interval $[t_0, \infty)$ and given a positive r

(20) $$\left|\begin{array}{l} \operatorname{sign} C\, r\left(G + \frac{1}{2}\frac{dC}{dt}\right) - 1 \ge 0, \\ \operatorname{sign} L \frac{1}{r}\left(R + \frac{1}{2}\frac{dL}{dt}\right) - 1 \ge 0. \end{array}\right.$$

The oscillating circuit will be asymptotically stable, if the system of strict inequalities (20) is satisfied under the same conditions.

I n s t a b i l i t y c r i t e r i o n . The oscillating circuit will be unstable, if the following system of strict inequalities is satisfied at $t \to \infty$ and a positive r:

(21) $$\begin{array}{l} \operatorname{sign} C\, r\left(G + \frac{1}{2}\frac{dC}{dt}\right) + 1 < 0, \\ \operatorname{sign} L \frac{1}{r}\left(R + \frac{1}{2}\frac{dL}{dt}\right) + 1 < 0. \end{array}$$

It follows from system (16) that if the signs of all oscillating circuit parameters are changed to their opposites, the result will be a system of the same type. This implies that if all the parameters of the oscillating circuit change in time according to arbitrary laws, yet in such a way that their signs change to their opposits simultaneously, from the viewpoint of stability the oscillating circuit will be equivalent to another oscillating circuit, whose parameters change in time in compliance with laws equal to the modules of the respective laws governing the changes of the initial oscillating circuit.

Conclusion

Systems described by differential equations of the second and higher order with periodic coefficients have been tackled by a lot of works: beginning with the classical ones of Lyapunov and ending up with the numerous publications by modern researchers. In spite of the considerable number of instructive mathematical publications, the problem of analyzing qualitatively the free processes in a parametric oscillating circuit cannot be regarded as solved. There is an essential difference between the analysis of the abstract mathematical equation and the particular engineering–and–physical system. As a rule the engineering–and–physical problem is made up of three parts. The first one allows of using the physical properties of the system as a starting point for obtaining its schematic and analytical description as well as the respective mathematical equation. The second part consists in solving and exploring the equation obtained. The third part provides a physical-and-engineering interpretation of the results. The mathematical problem is a component of the engineering-and-physical one and constitutes the latter's second part. The powerful mathematical means used in its solution often allow of obtaining thoroughgoing results. Thus, in a certain sense, the engineering-and-physical approach is broader than the mathematical one, but the latter is more profound. When solving the engineering-and-physical problem, it is important to adapt and use adequately a relevant mathematical technique. The paper has seeked to combine the general formulation of the engineering–and–physical problem concerning the processes in a generalized periodical oscillating circuit with the profundity of the mathematical exploration.

References

1. Birjuk, N.D., V.N. Damgov. Qualitative Analysis of the Free Processes in a Generalized Linear Oscillating Circuit With Periodic Parameters. Part 1. Structure of the Differential Equations and Classification of the Free Processes in Hamiltonian Oscillating Circuits. – Aerospace Research in Bulgaria, 13 (in print).

2. Yakubovich, V.A, V.M. Starginskii. Linear Differential Equations With Periodic Coefficients and Applications. Moscow, Nauka Publ., 1987

3. Damgov, V.N. Principle of Modulation–Parametric Reversible Interactions. – Comptes Rendus de l'Academie Bulgare des Sciences, 46, 1993, № 2, 57 – 60.

Received 6. XI. 1996

Качествен анализ на свободните процеси в обобщен линеен трептящ кръг с периодични параметри
Част 2. Стабилност на каноничните системи и на обобщена линейна резонансна верига

Николай Д. Бирюк, Владимир Н. Дамгов

(Резюме)

В част първа на статията [1] бяха дискутирани възможните преобразувания на уравненията, описващи трептящ кръг с периодични и почти периодични във времето параметри. Бяха анализирани целесъобразните области на приложение на една или друга форма на системата диференциални уравнения и бе приложена класификация на трептящите кръгове по най-важния признак – устойчивостта и неустойчивостта на Ляпунов.

Основното внимание тук е фокусирано върху проблема за устойчивостта на каноничната система диференциални уравнения в общ вид. Формирани са критерии за устойчивостта и стабилността на обобщена линейна резонансна система с периодични във времето параметри. Доказва се важността за практиката на общия качествен анализ, на "разумния" баланс между общото абстрактно математично изследване и конкретния инженерен анализ на резонансните системи с периодични или почти периодични параметри. Само такъв подход позволява, от една страна, да се получи представа за множеството закономерности и свойствата на изследваната система, а от друга страна – да се избере най-подходящият математичен апарат за провеждане на набелязания анализ с необходимата дълбочина. В работата е направен опит да се съчетае възможно най-общото формулиране на физико-техническата задача за анализ на сложните процеси в периодична или почти периодична трептяща система с възможната задълбоченост на математическото изследване. Разработеният аналитичен подход е приложен конкретно към обобщена линейна трептяща система с периодични параметри, като са получени съвсем ясни и практически удобни критерии за устойчивостта и неустойчивостта на системата.