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# Excitation of "quantized" oscillations and principle of reversibility of modulation — parametric interactions[1]

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## Introduction

The phenomenon of "quantized" oscillation excitation will be analyzed on the basis of a general model — a pendulum under the action of external HF periodic force, nonlinear as regards the pendulum coordinates.

The fundamental problem of pendulum investigation, the analytical study of which goes back to Huygens [1, 2], is isomorphic to a variety of physical phenomena, particularly such as radio-frequency driven Josephson junction and charge-density wave transport [3, 4]. This correspondence, recognized over a quarter of a century ago has led to various studies of phenomena related to the Josephson effect by means of mechanical analogs. The different types of steady-states, or attractors, of the driven pendulum and of rf-driven Josephson junction and the ranges of control parameters for which they occur have been studied extensively in numerical and analog simulations.

The pendulum is a well-known phenomenon intensively studied for over 300 years. At present, the pendulum is quite rightly considered to be one of the most general models in nonlinear dynamics [5-8]. Biased, we could say that any phenomenon that can be observed in the pendulum is of considerable generality. In systems of the "pendulum type" phenomena like "resonance","frequency pulling, synchronization and stabilization", etc. have been discovered. In the early 50-ies N. N. Bogolyubov and P. L. Kapitza discovered the possibility for stabilization of the upper instable equilibrium pendulum point using weak high-frequency modulation applied to the point of suspension — a phenomenon that is applied for example in heated plasma stabilization in experiments for termonuclear reaction utilization [5, 9]. It is not a mere coinsidence that the quantum-mechanic radio-frequency driven Josephson

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junction discovered recently, as well as the charge-density wave transport process are completely analogous to the pendulum with its strong sinusoidal nonlinearity. The inexhaustibility of the pendulum as a general model is corroborate once again by the presented herewith phenomenon of continuous oscillation excitation with an amplitude belonging to a discrete value set of possible stable amplitudes. We believe that the general problem of excitation of oscillations in different systems under an inhomogeneous action of a nonlinear external force (nonlinear with respect to the coordinate of the excited system) should be most beneficially analyzed on the example of the pendulum.

Principle is stated for modulation-parametric interactions as reversible processes, representing fundamental issue of effective control of electric systems equivalent impedances [10, 11]. Heuristic viewpoint of the principle understanding and application is presented, analysing the effect of unifrequency non-degenerative parametric regeneration [10, 11] and — herewith — the phenomenon of excitation of "quantized" oscillations.

Energy parametric input, transformation and transmission is described as a rule with reactive parameter variation. Such processes are characterized by generation and interconversion of combination frequencies. Two opposite in character yet simultaneous in manifestation processes take part — this of combination frequency generation accompanied by input signal — parametric element interaction and that of "reverse" combination frequencies conversion with the same parametric element. Thus is determined system reaction to input signal or signals with other combination frequencies and system influence as either regenerative or degenerative.

Within this context is formulated the principle of modulation-parametric reversible interactions, connected with mutually reversible transformation and mixing of signals in the presence of parametric elements. Variation of equivalent (effective) impedance parameters accompanies the process.

Analysing those simultaneous inseparable signal frequency transformation processes, analytical differentation into "forward" and "back" sub-processes is made. First stage of analysis represents equation set, referring to "forward" signal transformation and combination spectrum generation. Second analysis stage includes equation solution and gives system reaction in presence of combination components as result of "back" transformation.

The paper presents a picture of the internal formation of oscillations in the pendulum system and the mechanism of a modulation-parametric energy input channel for their maintenance.

## Analysis of the pendulum oscillations under inhomogeneous external driving in the proximity of small amplitudes: proof for the presence of a modulation-parametric channel for input of energy into the oscillation process

The inhomogeneous driving of an external high-frequency source of power supply causes a characteristic mixing of its frequency (or its spectrum of frequencies) with the frequency of the oscillations of the excited system. The characteristic argument of the system is an adaptively adjusted phase that provides a most favourable, from energetic point of view, interaction between the driven pendulum and the high-frequency source of power supply. In the language of frequency, the process of nonlinear interaction of the pendulum with a high-frequency external source is expressed by generating an infinite spectrum of combination frequencies. At that, the adaptive automatic adjustment to a most favourable phase is the necessary condition of such mixing of the two oscillation processes, at which spectral components with a frequency close to the equivalent resonance frequency of the pendulum appear. Those spectral components maintain the pendulum oscillations constant and steady and the phase conditions of excitation of one or another amplitude of oscillation have a discrete nature and differ at the respective initial conditions (initial kinematic parameters of the pendulum).

The analysis is carried out on the basis of the following equation describing the pendulum oscillations under the effect of an external periodic force, nonlinear along the coordinates:

(1) $$\ddot{x} + 2\beta\dot{x} + \omega_0^2 \sin x = \varepsilon(x) F \sin \nu t,$$

where $x$ — angle of deviation of the pendulum from the vertical, $\omega_0$ — frequency of the small natural oscillations, $2\beta\dot{x} + \omega_0^2 \sin x$ — function accounting for the friction and the unharmonicity of the oscillations, $F>0$ — amplitude of the external force with high frequency $\nu >> \omega_0$, $\varepsilon(x)$ — function determining the position of the force in space.

The solution of Eq. (1) is presented in the form

(2) $$x - X_0 + \sum_n x_n,$$

where $X_0$ corresponds to the main oscillations of the pendulum, $\sum_n x_n$ — sum of combination components generated at the effect of the external force of the pendulum.

By substituting (2) into (1) and bearing in mind that $\sum_n x_n$ is small as compared to $X_0$, we may write down:

— Equation about the main oscillations

(3) $$\ddot{X}_0 + 2\beta\dot{X}_0 + \omega_0^2 \sin X_0 + \left[\omega_0^2 \cos X_0 \sum_n x_n\right]_{X_0} = \left[\varepsilon(X_0) F \sin \nu t\right]_{X_0} + \left[\frac{d\varepsilon(X_0)}{dx}\sum_n x_n F \sin \nu t\right]_{X_0};$$

— Equation about any $l$-th combination component out of the possible $n$-set

(4) $$\ddot{x}_l + 2\beta\dot{x}_l + \left[\omega_0^2 \cos X_0 \sum_n x_n\right]_l = \left[\varepsilon(X_0) F \sin \nu t\right]_l + \left[\frac{d\varepsilon(X_0)}{dx}\sum_n x_n F \sin \nu t\right]_l.$$

The subscripts "$X_0$" and "$\Gamma$" in (3) and (4) mean that only the components with the frequency of the main oscillations or the corresponding combination frequency are selected from the respective members of the Equations.

(5) $$X_0 = a\cos(\omega t + \varphi_0) = a\cos\theta,$$

(6) $$\sum_n x_n = \sum_{n=-\infty}^{\infty} A_n \cos[(\nu + n\omega)t + \varphi_n]$$

where $a$, $\omega$ and $\varphi_0$ — amplitude, frequency and phase of the main oscillation of the pendulum; $A_n$, $(\nu+n\omega)$ and $\varphi_n$ — amplitude, frequency and phase of the $n$-th combination frequency. The frequency $\omega$ is close to the resonance frequency of the small oscillations $\omega_0$.

The function $\varepsilon(x)$ may be analytically represented in a different way, such as

(7 *a*) $$\varepsilon(x) = \begin{cases} 1 \text{ at } |x| \le d \\ 0 \text{ at } |x| > d \end{cases}$$

(7 *b, c*) $$\varepsilon(x) = e^{-\frac{x^2}{2d^2}}, \quad \varepsilon(x) = \begin{cases} \frac{1}{2}\left[1 + \cos\left(\frac{\pi}{d}x\right)\right] & \text{at } |x| \le d \\ 0 & \text{at } |x| > d \end{cases}$$

where $d<<1$ and $d<<a$. At conditions (5) we use the expansion of the functions (7) in Fourier series:

(8) $$\varepsilon(X_0) = \varepsilon(a\cos\theta) = a_0 + 2\sum_{n=1}^{\infty} a_{2n}\cos 2n\theta,$$

(9) $$\frac{d\varepsilon(X_0)}{dx} = \frac{d\varepsilon(a\cos\theta)}{dx} = 2\sum_{m=1}^{\infty} b_{2m}\sin 2m\theta.$$

The following expressions are obtained for the coefficients in (8) and (9) for the function of the form of (7*a*):

(10) $$a_0 = \frac{2\theta_0}{\pi},\ a_{2n} = \frac{1}{\pi n}\sin 2n\theta_0,\ b_{2m} = \frac{2}{\pi a}\frac{\sin 2m\theta_0}{\sin\theta_0},\ \theta_0 = \arccos\frac{d}{a}.$$

Substituting (5), (6) and (8), (9) into (3) we obtain the shortened equations of establishing the amplitude $a$ and phase $\varphi_0$ of the pendulum oscillations accounting for an infinite spectrum of combination frequency:

(11) $$\dot{a} = -\beta a + \frac{G_1}{2\omega} - \frac{F}{2\omega}(a_{N-1} - a_{N+1})\cos N\varphi_0,$$

(12) $$\dot{\varphi}_0 = -\frac{\omega^2 - \tilde{\omega}_0^2}{2\omega} + \frac{G_2}{2\omega a} + \frac{F}{2\omega a}(a_{N-1} + a_{N+1})\sin N\varphi_0$$

where

$$(13)\quad \begin{Bmatrix} G_1 \\ G_2 \end{Bmatrix} = \omega_0^2 \frac{J_0(a)}{a}\left[ A_{1-N}\begin{Bmatrix}\sin\\ \cos\end{Bmatrix}(\varphi_0-\varphi_{1-N}) + A_{-1-N}\begin{Bmatrix}\sin\\ \cos\end{Bmatrix}(\varphi_0-\varphi_{-1-N})\right]$$

$$+\omega_0^2\sum_{k=1}^{\infty}(-1)^k J_{2k}(a)\left[ + \sum_{n=\pm 1-2k-N}^{*} A_n \begin{Bmatrix}\sin\\ \cos\end{Bmatrix}(\varphi_0 \mp 2k\varphi_0 \mp \varphi_n) + \sum_{n=\mp 1+2k-N}^{*} A_n \begin{Bmatrix}\sin\\ \cos\end{Bmatrix}(\varphi_0 \mp 2k\varphi_0 \pm \varphi_n)\right]$$

$$-F\sum_{m=1}^{\infty} b_{2m}\left[ - \sum_{n=\mp 1+2m}^{*} A_n \begin{Bmatrix}\sin\\ \cos\end{Bmatrix}(\varphi_0 \mp 2m\varphi_0 \pm \varphi_n) + \sum_{n=\pm 1-2m}^{*} A_n \begin{Bmatrix}\sin\\ \cos\end{Bmatrix}(\varphi_0 \mp 2m\varphi_0 \mp \varphi_n) \right.$$

$$\left. + \sum_{n=\pm 1+2m-2N}^{*} A_n \begin{Bmatrix}\sin\\ \cos\end{Bmatrix}(\varphi_0 \pm 2m\varphi_0 \mp \varphi_n) - \sum_{n=\pm 1-2m-2N}^{*} A_n \begin{Bmatrix}\sin\\ \cos\end{Bmatrix}(\varphi_0 \mp 2m\varphi_0 \mp \varphi_n)\right],$$

$J$ — Bessel function of the first order, $\tilde{\omega}_0^2 = \omega_0^2 \frac{2J_1(a)}{a}$ — change of the resonance frequency of the synchronous oscillations with the change of their amplitude, $N = \frac{\nu}{\omega}$ — multiplicity of the frequency division in the system, $\sum^{*}$ — means that any term of the sum consists of two addends — for the upper and lower sign of $n$, respectively.

In the regime of stationary oscillations $a$=0 and $\varphi_0$=0. By denoting $\delta_0 = 2\beta\frac{\omega}{\omega_0^2}$, $\eta_1 = \frac{G_1}{\omega_0^2 a}$, $\eta_2 = \frac{G_2}{\omega_0^2 a}$, $\xi = \frac{\omega^2 - \tilde{\omega}_0^2}{\omega_0^2}$, from (11) and (12) we obtain expressions for the amplitude and phase of the stationary oscillations of the pendulum:

$$(14)\qquad a = F\left\{\omega_0\sqrt{\left(\frac{\delta_0-\eta_1}{a_{N-1}-a_{N+1}}\right)^2+\left(\frac{\xi-\eta_2}{a_{N-1}+a_{N+1}}\right)^2}\right\}^{-1},$$

$$(15)\qquad \varphi_0 = \frac{1}{N}\operatorname{arctg}\left(\frac{\xi-\eta_2}{\delta_0-\eta_1}\frac{a_{N-1}-a_{N+1}}{a_{N-1}+a_{N+1}}\right).$$

A complex spectrum of frequencies is generated in the system, induced by the strongly inhomogeneous effect of the external force. At that, as a result of the reversibility of the modulation-parametric interactions (see [10-12]) the combination components perform input of energy both in the main oscillations of the pendulum and in the oscillations of any combination components of that spectrum. In this way, maintaining the stationary oscillations in the system is an integral effect of summing up the actions produced and mediated by an infinite spectrum of combination components. Since the excitation of the components as a result of the action of an external high-frequency source is of synchronous nature, the system under consideration is marked by strong phase selectivity. Here, similarly to the cyclic accelerators in wich phase selection and acceleration is only performed on the "equilibrium" particles, a substantial meaning have those combination components $x_n$ (see (6) and (12)), the phase $\varphi$ of which has the most appropriate value from the point of view of the optimum input of energy in the zone of driving of the external high-frequency source. This represents the adaptivity of the system and its self-adjustment to a steady-state stationary regime. In other words, the total action of the combination frequencies which contribute at the quasi-stationary frequency of oscillation of the pendulum, performs phase adjustment of those oscillations to the most favourable phase from energy point of view of the establishment of stationary regim. This transition process was analyzed by writing down shortened equations similar to those in (11) and (12), about a certain volum of combination components (4) and their joint solving with (11) and (12) by numerical methods. Establishing the stationary amplitude $a$ and phase $\varphi_0$ of the pendulum oscillations has itself oscillating and rapidly damped nature. Processes of establishing of stationary regimes are observed, at which the amplitude and phase continue to "wander" in a certain area around the equilibrium vlues of $a$ and $\varphi_0$.

It is interesting to note that there exist $l$-th combination frequencies that satisfy the equation

$$\nu + l\omega = \pm\omega \tag{16}$$

and fall within the bandwidth of the pendulum as an oscillating link of the system. They "directly" affect the main pendulum oscillations, without skipping the parametric effect due to the reversibility of the modulation-parametric interactions. Indeed, if only those $l$-th components satisfying the condition (16), i. e. $l=\pm 1 - N$, are taken into account out of the entire spectrum of combination frequencies, the infinite sums disappear in the expression (13) and it takes on the following form:

$$\begin{aligned}
\begin{Bmatrix} G_1 \\ G_2 \end{Bmatrix} = \omega_0^2 \Bigg\{ & \left[J_0(a) \pm J_2(a)\right] A_{1-N} \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} (\varphi_0 - \varphi_{1-N}) \\
& + \left[J_0(a) \mp J_2(a)\right] A_{1-N} \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} (\varphi_0 - \varphi_{-1-N}) \Bigg\} \\
& - \frac{F}{2} \Bigg\{ (b_N - b_{N+2}) \Bigg[ A_{1-N} \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} \left[(N+1)\varphi_0 - \varphi_{1-N}\right] \\
& + A_{-1-N} \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} \left[(N+1)\varphi_0 + \varphi_{-1-N}\right] \Bigg] + (b_{N-2} - b_N) \Bigg[ A_{1-N} \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} \left[(N-1)\varphi_0 + \varphi_{1-N}\right] \\
& + A_{-1-N} \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} \left[(N-1)\varphi_0 - \varphi_{-1-N}\right] \Bigg] \Bigg\}.
\end{aligned} \tag{17}$$

In order to determine the stationary amplitude and phase of the two components accounted for: $x_{1-N} = A_{1-N}\cos(\omega t + \varphi_{1-N})$ and $x_{-1-N} = A_{-1-N}\cos(-\omega t + \varphi_{-1-N})$

Eq. (41) may be written in complex matrix form (see, for example, [12] for the method of complex notation):

$$
(18)\qquad \begin{bmatrix} -\omega^2 & 0 \\ 0 & -\omega^2 \end{bmatrix} \begin{bmatrix} x^c_{1-N} \\ x^c_{-1-N} \end{bmatrix} + \begin{bmatrix} j\beta\omega & 0 \\ 0 & -j\beta\omega \end{bmatrix} \begin{bmatrix} x^c_{1-N} \\ x^c_{-1-N} \end{bmatrix}
$$

$$
+ \begin{bmatrix} \omega_0^2 J_0(a) & -\omega_0^2 J_2(a) e^{-j2\varphi} \\ -\omega_0^2 J_2(a) e^{j2\varphi} & \omega_0^2 J_0(a) \end{bmatrix} \begin{bmatrix} x^c_{1-N} \\ x^c_{-1-N} \end{bmatrix}
$$

$$
- \frac{F}{2} \begin{bmatrix} b_N\left(e^{jN\varphi_0} + e^{-jN\varphi_0}\right) & \left[b_{N-2} e^{j(N-2)\varphi_0} + b_{N+2} e^{-j(N+2)\varphi_0}\right] \\ \left[b_{N+2} e^{j(N+2)\varphi_0} + b_{N-2} e^{-j(N-2)\varphi_0}\right] & b_N\left(e^{jN\varphi_0} + e^{-jN\varphi_0}\right) \end{bmatrix} \begin{bmatrix} x^c_{1-N} \\ x^c_{-1-N} \end{bmatrix}
$$

$$
= \begin{bmatrix} \left\{ \omega_0^2 J_1(a) e^{j\varphi_0} + j\frac{F}{2}\left[a_{N+1} e^{j(N+1)\varphi_0} - a_{N-1} e^{-j(N-1)\varphi_0}\right] \right\} \\ \left\{ \omega_0^2 J_1(a) e^{-j\varphi_0} + j\frac{F}{2}\left[a_{N-1} e^{j(N-1)\varphi_0} - a_{N+1} e^{-j(N+1)\varphi_0}\right] \right\} \end{bmatrix},
$$

where $x^c_{1-N}$ and $x^c_{-1-N}$ — complex quantities. Substituting the parameters $A_{\pm 1\text{-}N}$ and $\varphi_{\pm 1\text{-}N}$ obtained from (18) into (17), the possible discrete spectrum of stationary amplitudes and phases of pendulum oscillation can be determined from (14) and (15).

Thus, the analysis demonstrates that except the main modulation channel there is a parametric channel of input of energy into the system. The quantitative analysis carried out with numerical methods on the basis of Eqs. (7*a*) and (10)-(15) showed the following. Accounting for the presence of a modulation-parametric channel of input of energy into the system makes the mechanism of phase selfadjustment, underlying the energy exchange and the maintenance of continuous oscillations in the system, considerably more flexible. At that, from the one hand, the discrete series of possible steady-state amplitudes is substantially specified and, on the other hand, the oscillations in the system demonstrate greater independence of the random changes of the external factors, as well as of the quality factor of the oscillating link and the amplitude of the driving force in a wide range.

## Conclusion: Final discussion on the problem under consideration and differences between two basic cases

In the interaction between the excited system and the power supply (external HF source) a constrain force is formed, which is frequency of phase (in general-argument) modulated force. As it has been shown above, a characteristic argument of the system can be some adaptively tuning phase, providing the most advantageous interaction between the excited oscillating system and the high-frequency power supply. These facts are caused to call the considered method of oscillation excitation "argument method".

From the point of view of the single — frequency problem, the principles of interaction in the superhigh frequency devices with an electronic mechanism of generation, cyclic accelerators of charged particles, the Fermi's mechanism of acceleration of cosmic particles, etc. may be considered as partial manifestations of the found more general "argument mechanism" of excitation of periodic motions in macroscopic oscillating systems.

The existence of such unusual properties of functioning of the considered class of oscillating systems with finite degrees of freedom allows us to speak about certain synergetic principles of grouping into stable formations, which are inherent in the simplest oscillating systems and processes. In the case considered the system does not just get energy from the external source in the forced mode, i. e. in the regime requiring conditions for the functioning of the external action and itself begines to act over the source, changing and adapting the appearing force of the action under the proper regime of functioning.

It is known that normally the frequency-phase modulation is considered beyond the relation with its energy interpretation; to obtain it, additional sources of energy and special modulator devices are always used in the radiophysical systems. In the case under consideration, the principle of modulation acts as a mechanism providing the interaction of the excited oscillation system with the external high frequency source.

If there were not an inverse adaptive action of the system on the exciting source, then, for example, it would be possible in linear oscillating systems to excite forced oscillations only with the frequency of the external force. The argument method of input of energy from a high frequency source allows to excite in linear systems continuous oscillations with their natural resonance frequency.

The discreteness of the amplitudes of the excited oscillations is also retained in the cases of a zero or negative coefficient of friction, and in this case the phase adaptivity provides nullifying or removing the "excessive energy".

Returning to the problem of the mechanism of excitation of the pendulum in the light of the stated above, the following should be additionally noted.

A phenomenon of J. Bethenod is known [13], which is explained by Rocar through a parametric change of the reactive parameter. We will show that it is possible in a similar system to excite asynchronous oscillations or rotating motions of the pendulum by using the hysteresis section of the resonance curve of the "dynamically" nonlinear electric resonance circuit formed in this way. The useful acceleration is created at high resonance amplitudes and the unchanging reaction against pendulum motion (stopping) occurs at low amplitudes as a result of an appropriate jump in the hysteresis section of the resonance curve.

The resonance circuit (Fig. 1) is power-supplied by a source of AC voltage $E$ with frequency much higher than and incomparable to the natural frequency of the pendulum. The latter forms a freel hung ferromagnetic plate acted upon by the magnetic field of the solenoid $L$. At the same time, there is an "inverse" action expressed by periodic modulation of the equivalent inductance of the solenoid.

Fig. 2 qualitatively shows the dependence of the solenoid inductance on the angle of deviation $y$ of the pendulum from its equilibrium position $y=0$. The periodic change of the solenoid inductance at the pendulum motion leads to the resonance characteristic of the electric resonance circuit becoming substantionally nonlinear. Fig. 3 qualitatively shows the dependence of the voltage $U$ across the electric resonance circuit on the said angle of deviation $y$. The pendulum motion results in characteristics with a clearly expressed hysteresis zone that are typical for the nonlinear resonance (the so-called "beak-like" characteristics) which are symmetrical about

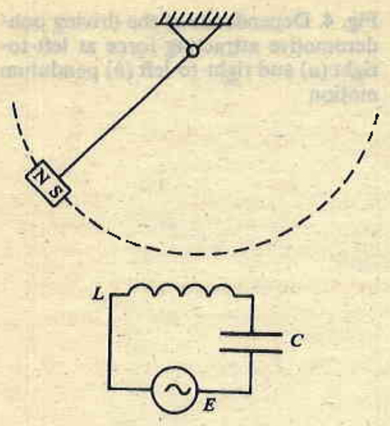
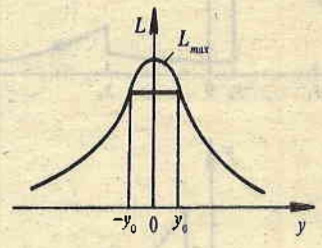



Fig. 1. Electrical resonance circuit periodically interaction with an oscillating pendulum

Fig. 2. Dependence of the solenoid inductance on the position of the ferromagnetic plate of the pendulum




the point $y=0$. Since a variable current leads through the solenoid, a ponderomotive attracting force $F$, which is always directed to the equilibrium position of the pendulum $y=0$, drives the ferromagnetic plate of the pendulum. A very specific interaction is obtained as a result.

The solenoid attracts the ferromagnetic plate, which when approaching it changes its inductance in such a way that the electric resonance circuit "goes" to a resonance. The voltage across the resonance circuit is increased as a result of this approach to the resonance, which causes an increase of the current in the solenoid and even stronger attraction of the ferromagnetic plate. When the plate approaches the resonance position of $y=0$, the system "skips" the resonance and, due to the nonlinearity, the amplitude on the resonance circuit is changed with a jump and its value becomes many times smaller. When the plate leaves the range $y=0$, the solenoid already has a stopping action on it, but it is done at much lower amplitudes of voltage across the resonance circuit. In this way the input of energy is positive per period. The stated process is also illustrated in Fig. 4. For example, at left - to - right pendulum motion (Fig. 4*a*) up to position $y=0$ the force $F$ will have speeding action, and later, at $y>0$ — stopping action. It is seen from Fig. 3 that the pendulum is speeded up at high voltage amplitudes (up to $U_{max}$) across the solenoid and is stopped at relatively small amplitudes, which do not exceed $U_1$. The correspondence between the voltage amplitudes $U$ and the driving force $F$ can be established by the comparison of Fig. 3 and Fig. 4*a*. At right - to - left pendulum motion (Fig. 4*b*) the process is quite similar — the acceleration substantially exceeds the stopping action since, due to the symmetry, on the one hand, and the dynamic duality of values, on the other hand, the dependence of the force on the coordinate is mirror-reversed. The oscillating pendulum motion results in useful speeding up once per period. In this way an effec-

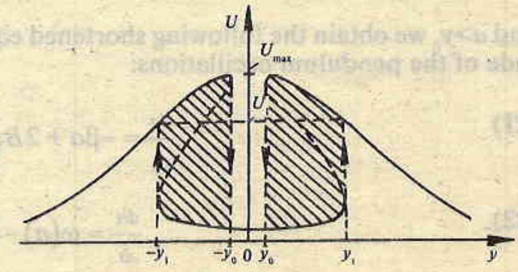



Fig. 3. Resonance characteristic of an oscillating circuit at periodic motion of the ferromagnetic plate over a solenoid

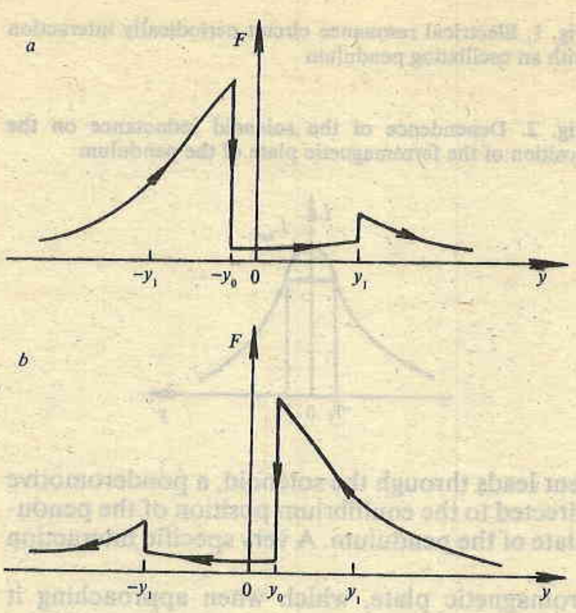



Fig. 4. Dependence of the driving ponderomotive attracting force at left-to-right (*a*) and right-to-left (*b*) pendulum motion

tive input of energy into the pendulum is obtained, which compensates for its natural losses due to friction. The input energy is proportional to the area of the hysteresis section of the resonance curve (the hatched area in Fig. 3). An synchronous excitation of stationary oscillations or rotating pendulum motion is realized.

The analytical study of the represented effect may be performed on the basis of the following equation describing the mechanical motion in the system:

(19) $$\ddot{y} + 2\beta\dot{y} + \omega_0^2 \sin y = \frac{kL_0 J_0^2}{2ml^2}\left\{\frac{\dot{y}+|\dot{y}|}{2}\delta(y+y_0) + \frac{\dot{y}-|\dot{y}|}{2}\delta(y-y_0)\right\},$$

where $\beta$ is a coefficient accounting for the energy losses of the pendulum due to friction, $\omega_0$ is the natural frequency of small oscillations of the pendulum, $ml^2$ is the inertial torque pendulum, $l$ is the pendulum length, $m$ is the ferromagnetic plate mass. $L_0$ and $y_0$ are explained by Figs. 2, 3 and 4, $J_0$ is the averaged effective value of the electric current, $k$ is a coefficient accounting for the width of the ponderomotive pulse of the force, $\delta(y \pm y_0)$ is Dirac's delta-function.

As a first approximation, the solution of Eq. (19) may be represented as:

(20) $$y = a\sin\theta,\ \theta = \omega t + \alpha.$$

By substituting (19) into (20) and assuming $\dot{x} = a\omega\cos\theta,\ \frac{da}{dt}\sin\theta + a\frac{d\alpha}{dt}\cos\theta = 0$ and $a > y_0$ we obtain the following shortened equation related to the phase and amplitude of the pendulum oscillations:

(21) $$\frac{da}{dt} = -\beta a + 2B\sqrt{1 - \frac{y_0^2}{a^2}},$$

(22) $$\frac{d\alpha}{dt} = \omega(a) - 2B\frac{y_0}{a^2},$$

where $\omega(a)$ is the frequency of pendulum oscillations, depending on its amplitude, $B = \frac{kI_0 J_0^2}{2ml^2}$.

For the estabilshed oscillations ($\frac{da}{dt} = 0$ and $\frac{d\alpha}{dt} = 0$) from (21) the following solutions are obtained:

(23) $$a_1^2 = \frac{2B^2}{\beta^2}\left[1 - \sqrt{1 - \frac{\beta^2}{B^2} y_0^2}\right],$$

(24) $$a_2^2 = \frac{2B^2}{\beta^2}\left[1 + \sqrt{1 - \frac{\beta^2}{B^2} y_0^2}\right].$$

The steadiness study shows that solution (23) is unsteady and solution (24) is steady. Therefore, in order to excite continuous stationary oscillations the pendulum should receive such an initial puls that its amplitude should be greater than $a_1$.

The argument method of input of energy into oscillating processes may find wide application at solving important problems of creating new methods and systems of excitation and maintenance of continuous oscillations and transformations of signals and energy, which may be provisionally grouped in the following way:

1. Transformation of the signals by frequency with high efficiency at a single division of frequency in several ten, hundred and thousand times, which is represented by various radio-engineering devices with a discrete series of steady-state regimes of operation, e. g. transducers of SHF oscillations to lower frequencies, high-efficiency frequency dividers, oscillators in a wide frequency range, frequency modulators in the optical range, high-efficiency transformers of energy in the optical and near infrared range (there are powerful sources producing this energy) into energy of the submillimeter range (where there are still missing powerful sources of electromagnetic waves), which is needed to carry out the control processes such as the ones in a thermonuclear reaction;

2. Transforming the energy from one kind to another, e. g. electric energy to mechanical, and vise-versa, which is performed by electrical and electromechanical transformers, electrical signal generators, wave energy transducers, nontraditional methods of transformation of thermal energy to electrical, etc.

3. Stabilization of various parameters at their change in a wide range (such as 50 to 100 to 200%), including voltage stabilizers for microprocessor systems with a great range of permissible load changes, etc.

4. Development of new base components for computing devices, having a great number of discrete steady states.

5. Intensification of various processes through special organization of argument interaction of different oscillating systems or wave processes, e. g. manipulations such as cavitation destruction, cleaning, emulgating non-mixable liquids and liquid-phase materials, development of various wave technologies.

6. Modelling of micro and macro processes with the methods of the classic Theory of Oscillations, explanation and developing of models of the processes of interaction of electromagnetic waves in the ionosphere and magnetosphere of the Earth, the phenomena of generating powerful low-frequency waves in the near space in the presence of cosmic electromagnetic background, interaction of particles with elec-

tromagnetic waves in plasma medium, radio emission of the magnetospheres of the outer planets in the Solar System, at the creation of a mega-quantum resonance-wave model of the Solar System [14], etc.

## References


1. Huygens, Ch. The Pendulum Clock or Geometrical Demonstrations Concerning the Motion of Pendula as Applied to Clocks. Iowa State University Press, 1986.
2. Moles, J. — Ohysica D, 31, 1988, 252-268.
3. MacDonald, A., M. Plischke. — Physical Review B, 27, 1983, 201-211.
4. Pedersen, N. — Europhysics News, 19, 1988, 53-55.
5. Strijak, T. Method for Investigation of Dynamical Systems of "Pendulum Type". Alma Atta, Nauka Publisher, 1981.
6. Sagdeev, R., D. Usikov, G. Zaslawsky. Nonlinear Physics. From the Pendulum to Turbulence and Chaos. N. Y., London, Harwood Academic Publishers, 1988.
7. Morozov, A. — In: Research Reports in Physics. Nonlinear Waves. Eds. Gaponov-Grekov, A. et al. Berlin, Springer — Verlag, 1990, 276-282.
8. Mawhin, J. — Applied Mathematics and Mechanics (ZAMM), 73, 1993, No 4-5, T54-T62.
9. Yip, S., F. Dimaggio. — Computers and Structures, 46, 1993, No 4, 725-740.
10. Damgov, V. — Comptes Rendus de l'Académie Bulgare des Sciences, 46, 1993, No 2, 57-60.
11. Damgov, N. — Earth, Moon, and Planets, 61, 1993, 87-117.
12. Damgov, V. — IEE Proc., Part G: Electronic Circuits and Systems, 131, 1984, No 1, 1-9.
13. Minorsky, N. Nonlinear Oscillations. N. Y., D. van Nostrand Company Inc., 1962.
14. Damgov, V., D. Douboshinsky. — Earth, Moon, and Planets, 56, 1992, 233-242.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## Възбуждане на „квантовани" трептения и принцип на обратимост на модулационно-параметричните взаимодействия

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(Резюме)

Представено е аналитично изследване на явлението възбуждане на „квантовани" трептения от гледна точка на формиране на модулационно-параметричен канал за ефективно влагане на енергия в колебателния процес. Подчертана е широката проява на принципа на обратимост на модулационно-параметричните взаимодействия, формулиран и предложен от автора. Последният се формира на основата на „правото" и „обратното" преобразуване на спектър от изходни комбинационни компоненти в присъствието на параметричен или нелинеен елемент. На тази основа е изведена динамиката на формиране на вътрешната структура на трептенията в система, представена от махало под външно нехомогенно въздействие на периодичен източник. Показано е, че образуваният модулационно-параметричен канал за ефективно влагане на енергия придава съществена гъвкавост на самоадаптивния механизъм на възбуждане и поддържане на трептения с възможен дискретен ред устойчиви амплитуди. В заключителната дискусия аналитично е демонстрирано, че при взаимодействие на махало с електричен трептящ кръг е възможна проявата на друг (асинхронен) механизъм на възбуждане и поддържане на незатихващи трептения, но с една определена амплитуда.