Българска академия на науките • Bulgarian Academy of Sciences
Аерокосмически изследвания в България • 12 • Aerospace Research in Bulgaria
София • 1996 • Sofia

Kick-excitation of "quantized" oscillations[1]

*Vladimir Damgov, Arun Holden**

Space Research Institute, Bulgarian Academy of Sciences
**Centre for Nonlinear Studies, Leeds LS2 9JT, U. K.*

Introduction

The phenomenon of continuous oscillation excitation with amplitude from discrete value set of possible stationary amplitudes [1] will be analyzed numerically on the basis of a general model — a pendulum under the inhomogeneous action of external HF periodic force.

In fact, we will discuss a class of systems with specific excitation — adaptive kick-excited systems. The kick-excitation can be represented by a short, as compared with the main period of oscillations, action of an external sine force.

The case discussed in the paper is rather a self-affined and quantitatively similar to the well-known problem examined by Fermi [2-4]. As an explanation for the origin of cosmic rays, Fermi proposed a mechanism for charged particles to accelerate by collisions with moving magnetic field structures. A great number of papers deals with the simplest model case — the so-called model of Fermi-Past-Ulam [5-12]. In the setup of Fermi-Past-Ulam scattering problem a ball is made to fly and impact dissipatively on a signal sinusoidally vibrating surface under the influence of the gravitational acceleration, which hence reverses the flight. The amplitude of the surface vibration of the cosine type and the coefficient of restitution between the ball and the surface control the ball dynamics.

In the recent years, the principle ability of using the Fermi mechanism for boosting space rockets in the gravitation field of the planets and stars has been discussed in the literature. This is the model of a so-called "gravitational engine", accelerating particles or bodies. The part of the vibrating plate may be played, for instance, by the field of a rotating binary star.

[1] An investigation supported by the Bulgarian National Foundation "Scientific Research" under Contract No TH-549/95.

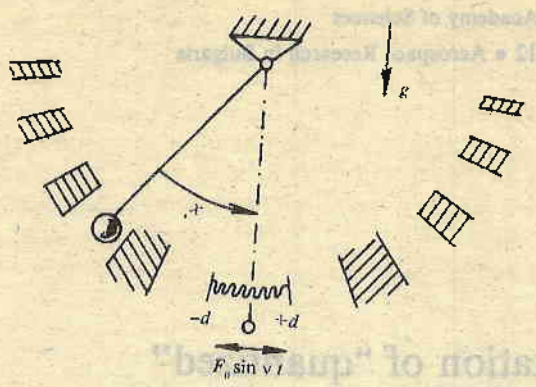


Fig. 1. Illustration of the system under consideration — a pendulum under the inhomogeneous action of an external periodic force

Similar phenomena occur in other subclasses of the class of kick-excited systems, e. g. in periodically kicked hard oscillators, ice-structure interaction model, kicked rotators, driven impact oscillators [3, 13-20].

At present the pendulum is also a widely used basic paradigm for analysis, both theoretical and experimental, of phenomena of excitation of complex, irregular and chaotic oscillations [3-20]. Extensive numerical and analog simulations have shown that this simple, low dimensional system exhibits complex behaviour including frequency and phase locking, intermittency and fractal basin boundaries.

The paper deals with common features in the behaviour of pendulum with invariable parameters in new conditions, namely the pendulum undergoes the action of continuous periodical external constrained force, which is inhomogeneous with respect to the coordinates of its motion.

Fig. 1 presents a schematic diagram of the pendulum system under consideration. The deviation of the pendulum from the lower equilibrium position is denoted by x. The external harmonic high-frequency force $F=F_0 \sin\nu t$, where F_0=const, acts in a limit zone $[-d, d]$ of the trajectory of motion of the pendulum, which is symmetrically located around the lower equilibrium point. This is the meaning of the notion "inhomogeneous action" related to the trajectory of motion of the pendulum, or the same expressed by the notion "nonlinear harmonic force" which should be understood as a nonlinear dependence of its amplitude on the coordinate of motion of the driving system — the pendulum. The direction of action of the external force is parallel to the direction of motion of the pendulum and is periodically reversed. When, initially, the pendulum is turned aside from the equilibrium position outside the zone $[-d, d]$ and is released to oscillate, it periodically passes through the zone $[-d, d]$ and is subject to the action of the external force $F=F_0 \sin\nu t$. At these conditions, a stationary mode of pendulum oscillation can be established with a quasi-constant amplitude, within one of the hatched areas of attraction in Fig. 1. The particular stationary amplitude of pendulum motion is determined by the initial deviation and the initial speed (i. e. by the initial conditions). Different modes of motions are possible for the pendulum, depending on the initial conditions: it either catches up with one of the possible stationary orbits, or its motion is quickly damped. This is the heuristic value of the phenomenon — the presence of a possible discrete set of stationary amplitudes, i. e. a specific "quantization" of the pendulum motion by intensity as a parameter. At the same time, there exist "forbidden" zones of initial conditions, for which the motion is only a damped one. Obviously, there is a phenomenon of "quantized" oscillation excitation, a "quantization" of the dynamic states in a macro system. The excitation of one amplitude or another depends on the initial conditions, at constant other parameters and conditions. We consider that the pendulum in this case is a self-oscillating system with high-frequency source of power supply (in contrast to the common perception that the self-oscillating systems should have a d. c. source of energy [21]). In quantum mechanics, the quantization (the notion of quanta, photons, phonons, gravitons) is postulated, and in the Theory of Relativity quantization is not derived by geometric considerations. At the phenomenon found, the "quantization" of transition of energy in portions directly follows from the mechanism of the processes and is formally mathematically defined. The quasiharmonic oscillator obeys the classical laws to a greater extent than any other systems. A number of problems, related to quasiharmonic oscillators, have the same solution in classical and quantum mechanics.

This paper presents a general picture of the motion of the pendulum under inhomogeneous excitation at different conditions. It is demonstrated that, due to the character of excitation (adaptive external kicking action), maintaining quasi-periodical and quasi-regular oscillations is possible. Bifurcation characteristics are presented and problems of excitation in the system of irregular and chaotic oscillations are discussed.

Numerical experiment of exciting "quantized" pendulum oscillations

A fascinating problem in modern dynamics is the origin of qualitative changes in the behaviour of nonlinear physical systems on very long time scales and the resulting low-frequency noise.

In this section we report a study of "quantized" oscillations excitation and bifurcation to irregular regimes in numerical simulations of the damped driven pendulum emphasizing the role of the phases of attraction for different stable states of the dynamical system.

Generally, almost periodic oscillations are excited in the system under consideration, due to the nature of excitation (the external force acts inhomogeneously: kicking excitation).

The inhomogeneously a. c. driven, damped pendulum system is given by the following set of three first order autonomous differential equasions

$$(1) \qquad \left| \begin{aligned} &\dot{x} = y, \\ &\dot{y} = -2\beta\dot{x} - \sin x + \varepsilon(x) F_0 \sin z, \\ &\dot{z} = \nu, \end{aligned} \right.$$

where x is the pendulum's angle of elevation, $y = \frac{dx}{d\tau}$ its angular velocity; the driven torque is a sinusoidal torque with amplitude F_0, frequency ν, and phase $z = \nu\tau + \varphi$, φ is the initial phase; β is the decrement of damping in the system; the dot denotes an operation of differentiation by the dimensionless time $\tau = \omega_0 t$, where ω_0 is the natural resonance frequency of the pendulum for oscillation with a disappearing small amplitude, the frequency of the external periodic source is in units of ω_0; the $\nu \gg 1$. case is considered.

The function $\varepsilon(x)$, which derermined the nonlinearity of the external periodic force related to the coordinate of the excited system is accepted to be expressed as

(2) $$\varepsilon(x) = \begin{cases} 1, \text{ at } |x| \le d, \\ 0, \text{ at } |x| > d, \end{cases}$$

where the parameter d thus defines a symmetrical zone of action in the area of the lower equilibrium position, d=<<1.

The Equations (1) and (2) imply that an almost symmetric solution is an almost periodic solution with a period T which is an odd-integer multiple of the driven period $\frac{2\pi}{\nu}$:

$$T = (2n+1)\frac{2\pi}{\nu},\ n = 1, 2, 3, ...$$

A fourth-order Runge — Kutta routine was employed to compute numerical solutions of Eqs. (1). All calculations were carried out in double precision arithmetic. The integration time step generaly was chosen to be 0,001 of the natural period. For each cycle of computations the discarding points were determined to be 500 thousand and calculating points to be 250 thousand. Comparison of the analytic and computed solutions to the linearized form of Eqs. (1) indicated that this technique gave numerical precision of seven decimal digits over one natural cycle.

Equations (1) constitute a flow in a three-dimensional phase space with dynamical variables x, $y=\dot{x}$ and the drive phase z. The control parameters F_0 and β and the initial conditions x_0 and $y_0 = \frac{dx_0}{d\tau}$ determine the pendulum's motion. Based on the physical mechanism of excitation, which will be described in greater detail below and which is associated with a frequency lock and phase synchronization, the frequency ν of the external driving force at the experiment should be constant. As it will be made clear below, the initial phase φ plays a significant role at the adaptive maintenance of the pendulum oscillations. At the same time, we take into consideration that at the initial start of the pendulum the phase φ has an equally probable value in the range from 0 to 2π, which means that the pendulum enters the action zone $[-d, d]$ of the external force at an equally probable (arbitrary) value of the initial phase φ. Once again, the meaning of the initial phase φ (its role will be explained below) should be pointed out. The phase φ determines the state of the external driving force at the time when the pendulum enters the action zone $[-d, d]$. Therefore, the phase φ is a varying value from period to period and it plays a dominating role at the adaptive self-maintenance of the pendulum oscillations. In all calculations the initial value of the initial phase φ is chosen to be zero; $\varphi=\varphi_0=0$. After the transition process is completed, a regime of automatic adaptive self-adjustment of the initial phase is established, around a value of φ_{st}, which is characteristic for any regime and the corresponding set of parameters.

We have obtained computer solutions of Eqs. (1) and analyzed the resulting data using three diagnostic tools: time series of the angular x and the angular velocity $\dot{x}$, phase-plane plots ($\dot{x}$ vs. x) and bifurcation characteristics (the oscillations amplitude vs. the controlled parameters). The numerical solutions are obtained for F_0 values in the range $0,1 \le F_0 \le 50,0$, for β values of the damping in the range $0,0001 \le \beta \le 0,5$, for d values in the range $0,001 \le d \le 0,05$, for fixed values of the driving frequency $\nu=51,0$; $97,0$; ... and always starting from the initial conditions $y_0 = \frac{dx_0}{d\tau} = 0$, x=vary. In all cases in order to eliminate transients, the solution were

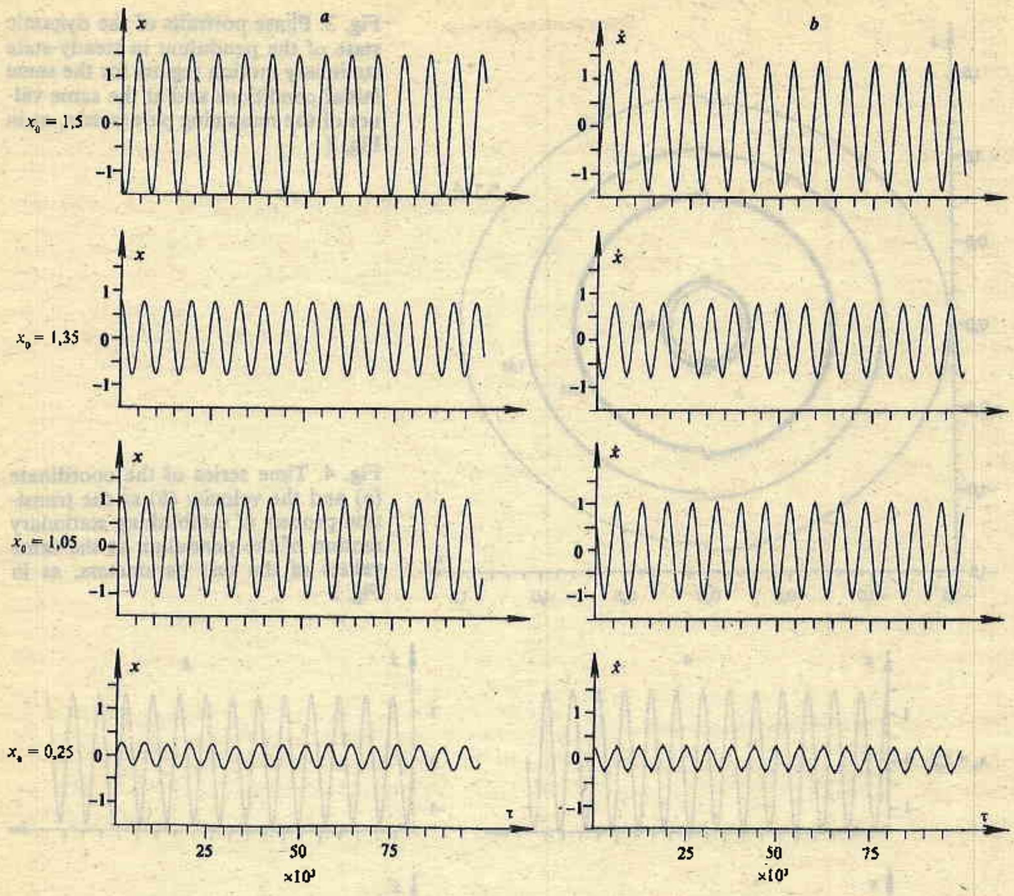


Fig. 2. Time series of the coordinates $x(a)$ and the velocity $\dot{x}(b)$ in steady-state stationary regime of motion of the pendulum at the following initial conditions: x_0=1,5, 1,35, 1,05 and 0,25. The values of the rest unchanging parameters are: y_0=0, F_0=2, ν=51, β=0,01, d=0,025

run through at least 1000 periods of the driving force before actual data taking was started.

Below, we present the main results of the numerical experiment at the following values of the parameters: β=0,01, ν=51,0, F_0=2,0, d=0,025, y_0=0, x_0=vary.

Figs. 2 and 3 show the time series and the combined phase portraits of a stationary steady-state pendulum motion at four different initial conditions: x_0=0,25; 1,05; 1,35; 1,5. Figure 2*a* shows the time series of the coordinate x and Fig. 2*b* shows the time series of the angular velocity $\dot{x}$. Both in Fig. 2*b* and Fig. 3 the abrupt changes of the velocity of pendulum motion in the narrow driving zone of the external force are clearly distinguished. By Fig. 2 and Fig. 3 we have sought to illustrate different possible regimes and cases. At an initial condition of x_0=1,5, periodic oscillations are excited, very close to the harmonic ones, with a stationary amplitude of ~ 1,45. The initial condition of x_0=1,05 determines a stationary amplitude of ~ 1,1. In both cases the value of the initial condition is chosen very close to the possible ("allowed") amplitude values. The area of attracting related to the initial conditions for any one of the possible stationary amplitudes (see the hatched areas in Fig. 1) varies from 15%

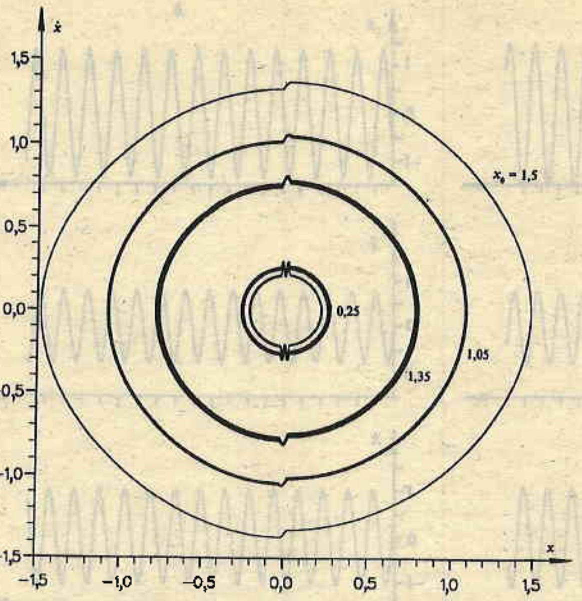
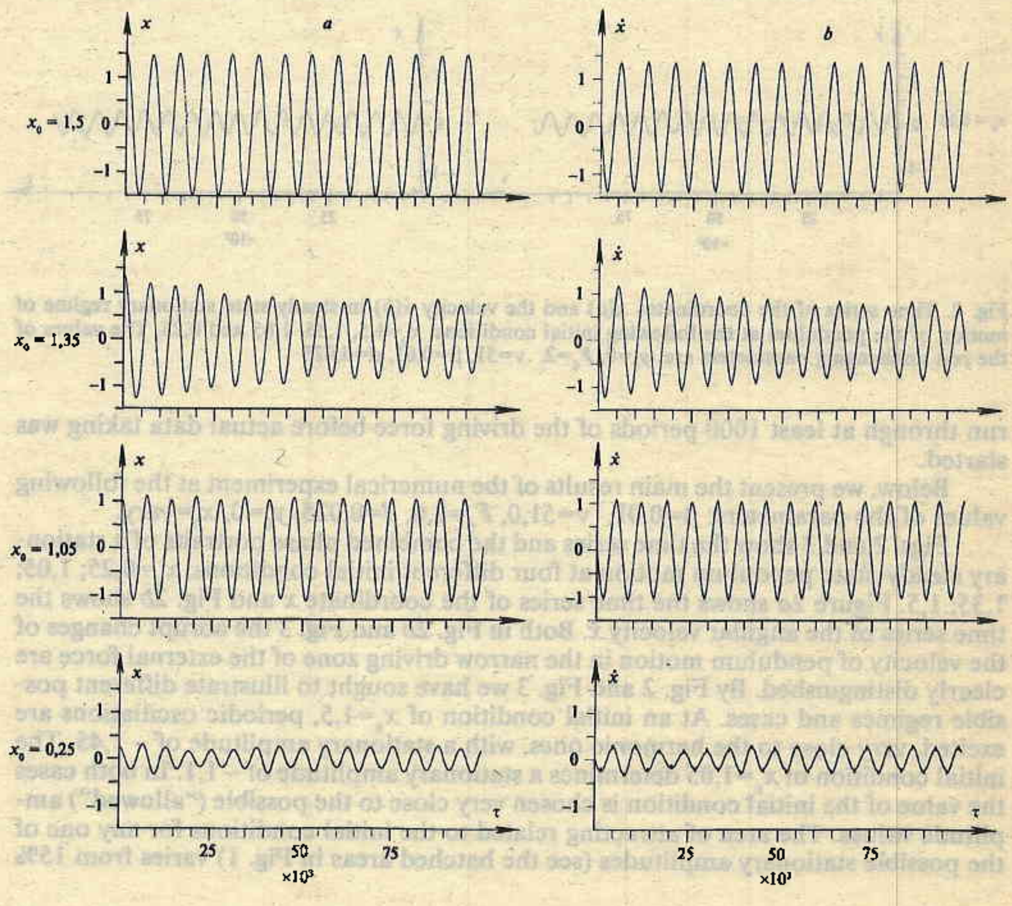


Fig. 3. Phase portraits of the dynamic state of the pendulum in steady-state stationary motion regime for the same initial conditions and at the same values of the remaining parameters, as in Fig. 2

Fig. 4. Time series of the coordinate (*a*) and the velocity (*b*) at the transition process of establishing stationary motion of the pendulum at the same values of the rest parameters, as in Fig. 2



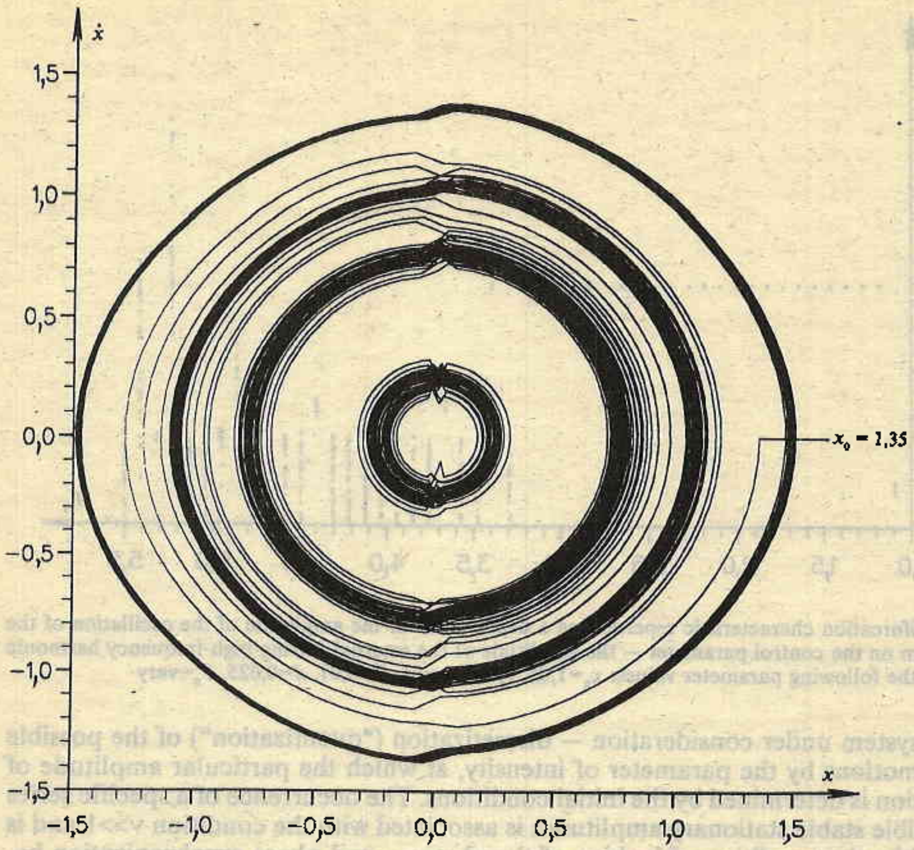


Fig. 5. Phase portraits of the dynamic state of the pendulum at the transition process of establishing stationary motion of the pendulum at the same values of the rest parameters, as in Fig. 2

to 7%, with an increase of the absolute values of x_0 from 0,25 to 1,5, respectively. When the initial condition is set to be between these areas of attracting, different modes are possible – either the oscillations are quickly damped, or the motion is "trapped" and stabilized on one of the possible lower ("allowed") orbits. The latter possibility is illustrated in Fig. 4 and Fig. 5, which present the transition processes of establishing a stationary motion for the same 4 initial conditions as those specified related to Fig. 2 and Fig. 3. At an initial condition of x_0=1,35 (clearly seen in Fig. 5) – a value located between the "allowed" values of the stationary amplitudes ~ 1,1 and ~ 1,45, the pendulum becomes "heavier" and passes through the possible stationary orbit with an amplitude of ~ 1,1, then is "trapped" on an orbit with amplitude ~ 0,75. Another feature of the presented data of the pendulum behaviour is that while its motion around one of the orbits, with an amplitude of ~ 1,45 in this case, is sufficiently close by its nature to the harmoni principle of motion, at the motion around other orbits (with lower values in this case) an amplitude – two and amplitude – three modulated motion may be observed. This is especially characteristic in the case of the orbit with amplitude ~0,25, which is an amplitude – three motion (see Fig. 3 and Fig. 4).

As a whole, Figs. 2, 3, 4, and 5 illustrate the most important common features

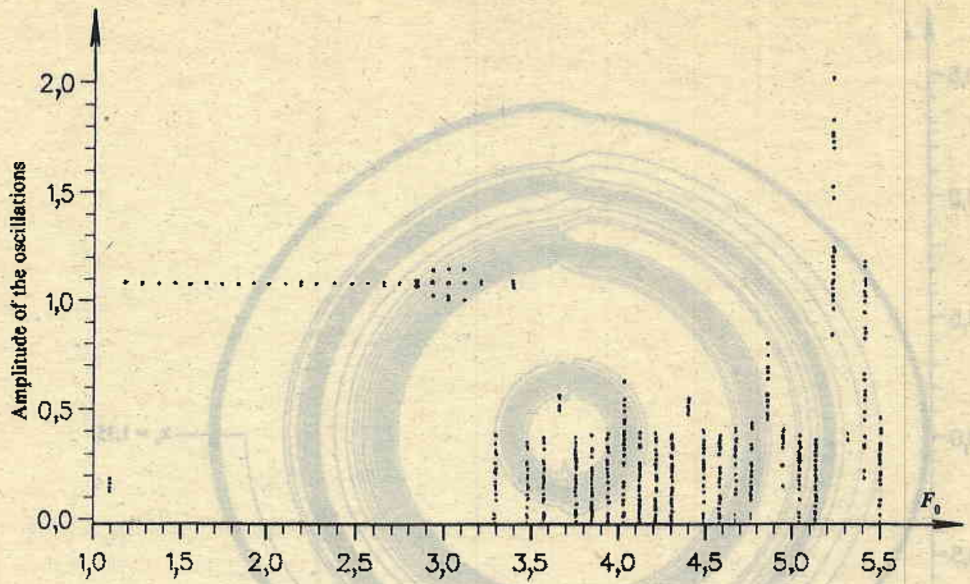


Fig. 6. Bifurcation characteristic repersenting a dependence of the amplitude of the oscillation of the pendulum on the control parameter — the amplitude of the external driving high-frequency harmonic force at the following parameter values: x_0=1,05, y_0=0, ν=51, β=0,01, d=0,025, F_0=vary

of the system under consideration — discretization ("quantization") of the possible stable motions by the parameter of intensity, at which the particular amplitude of oscillation is determined by the initial conditions. The occurrence of a specific series of possible stable stationary amplitudes is associated with the condition ν>>1 and is defined by the condition of locking of the phase φ and phase synchronization between the motion of the pendulum and the external periodic force. The physical mechanism of phase adaptivity and its role for the maintenance of unchanging oscillations of the pendulum at a considerable change of a number of parameters and conditions will be clarified below and in the subsequent Sections.

Fig. 6 shows a bifurcation characteristic that presents, in this case, a dependence of one of the possible steady-state amplitudes of pendulum oscillations (~1,1) on a control parameter which in the case is the value of the amplitude of the external driving high-frequency harmonic force. The presence of a threshold value for the amplitude of the driving force (~1,1) is seen, and for values above this threshold a steady-state stationary regime of pendulum oscillations with amplitude ~1,1 is realized. In the range of values of F_0~[1,1, 2,8], i. e. when the amplitude of the external excitation force is changed by almost 200%, the amplitude of pendulum oscillations remains practically unchanged and the motion is period — 1.

This property is the second very important principle of the system under consideration — the independence of the steady-state stationary amplitude of pendulum oscillation of the change of the amplitude of the external high-frequency driving force in a wide range.

At a value of the excitation amplitude of F_0~2,8, a bifurcation of tripling the period occurs. Amplitude — three oscillations exist up to values of F_0~3,26, when, as a result of a new complicated bifurcation, complex irregular oscillations occur. This bifurcation is preceded by a return to a quasi-periodic determinate regime, followed by a sharp transition from quasi-periodic regime to an irregular one (such sudden

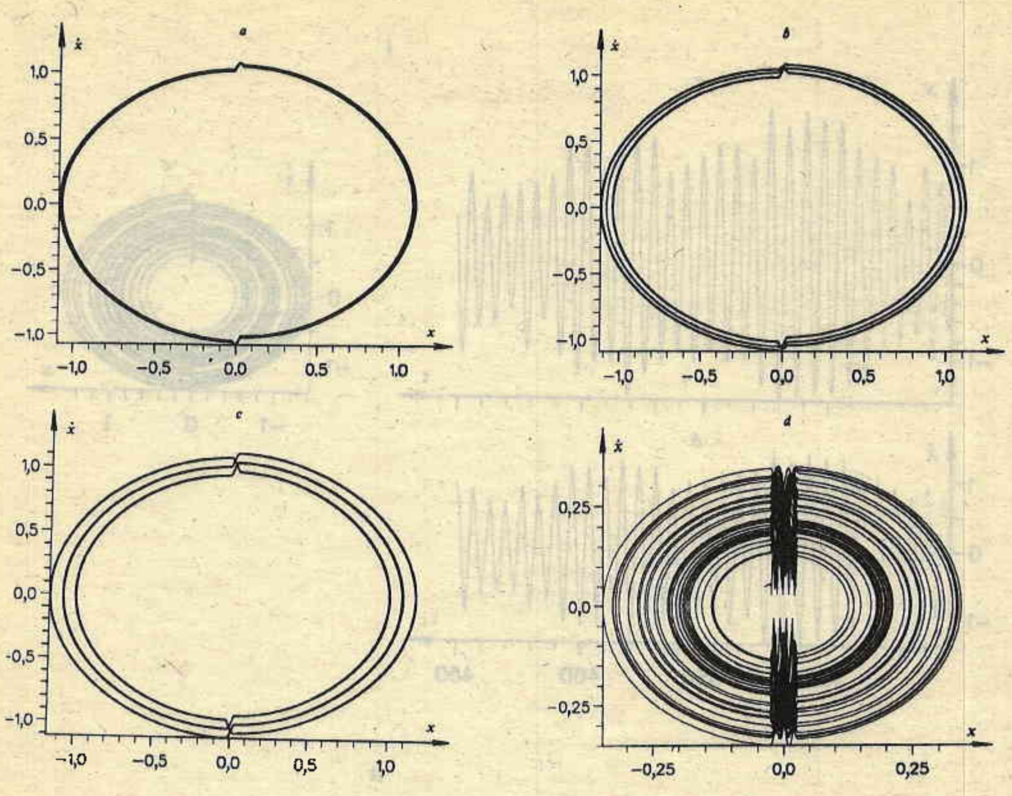


Fig. 7. Phase portraits of the motion in the system of Fig. 1 at (*a*) F_0=2,775, (*b*) F_0=2,8, (*c*) F_0=3,2595, (*d*) F_0=3,26 and the same values for the rest parameters, as for Fig. 2; x_0=1,05

qualitative changes are usually called crises). The three specific portions of the bifurcation characteristic are also illustrated in Figs. 7, 8, and 9. At a value of F_0=2,775 the oscillations are almost harmonic (see Fig. 7*a*). At a minor change of the value of F_0, amplitude — three oscillations are established as a result of bifurcation (see Fig. 7*b*). These oscillations undergo some increase, without changing their nature, up to a value of F_0=3,2595 (see Fig. 7*c*). At further minor increase of the value of the control parameter F_0, a new bifurcation occurs and the oscillations in the system become strongly irregular (see Fig. 7*d*).

The bifurcation characteristics are of a similar nature for the remaining possible stationary amplitudes of pendulum motion in the "allowed" spectrum of Amplitudes of oscillating motion.

Figs. 8 and 9 represent an illustration of the irregular oscillations of the pendulum at different values of the parameter F_0 and the parameter β, which represent the decrement of damping of the pendulum. Fig. 9 gives an idea of pendulum behaviour in extreme conditions — hyperloading and strong external driving.

Fig. 10 is another bifurcation characteristic, when the value of the damping decrement β is chosen as a control parameter. At small values of β the motion in the system has a strongly irregular nature. With the increase of the value of β, as a result of bifurcations, steady-state stationary quasi-harmonic pendulum oscillations are established, which exist over the range of values of β~[0,007, 0,02]. At the set value of

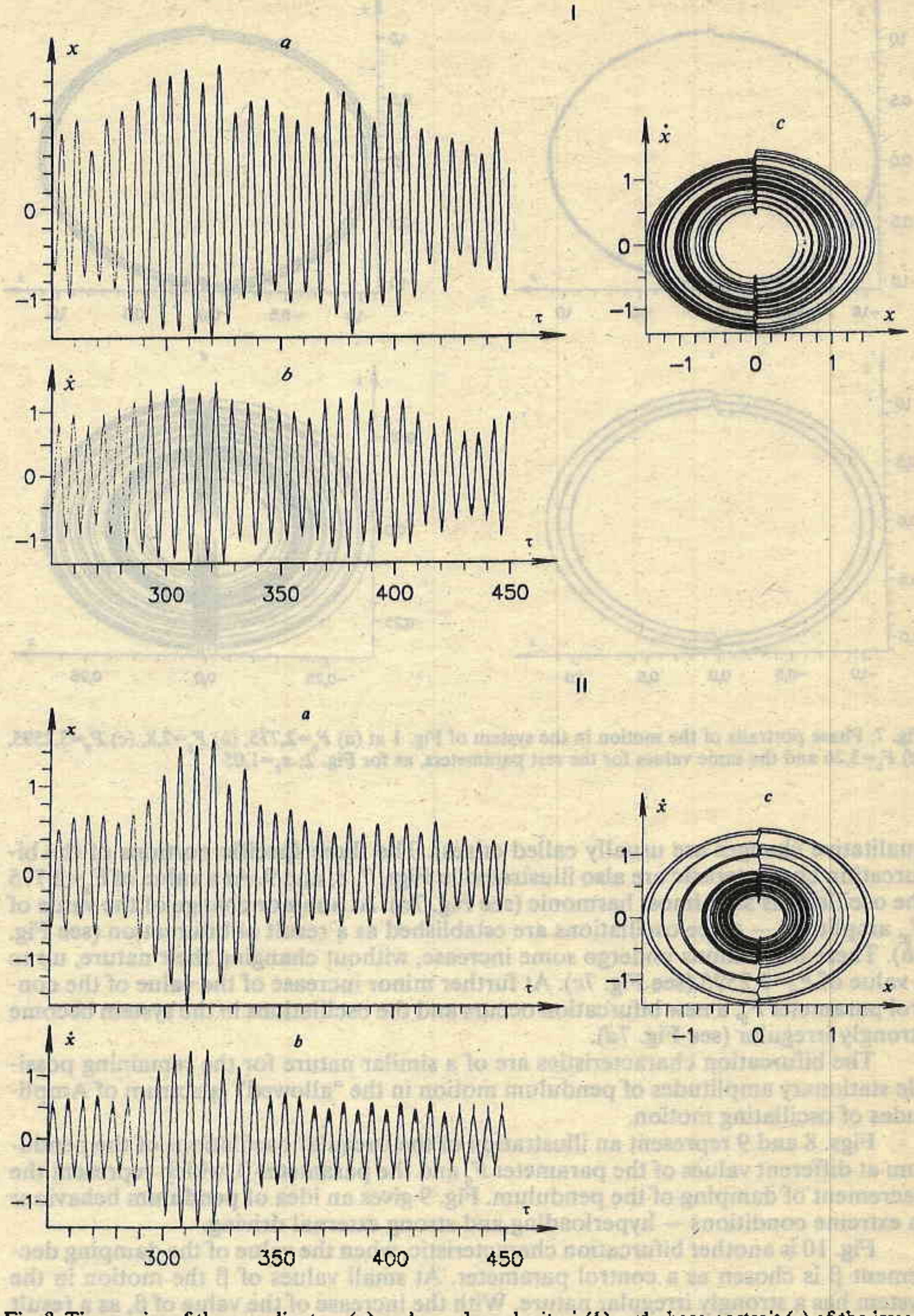
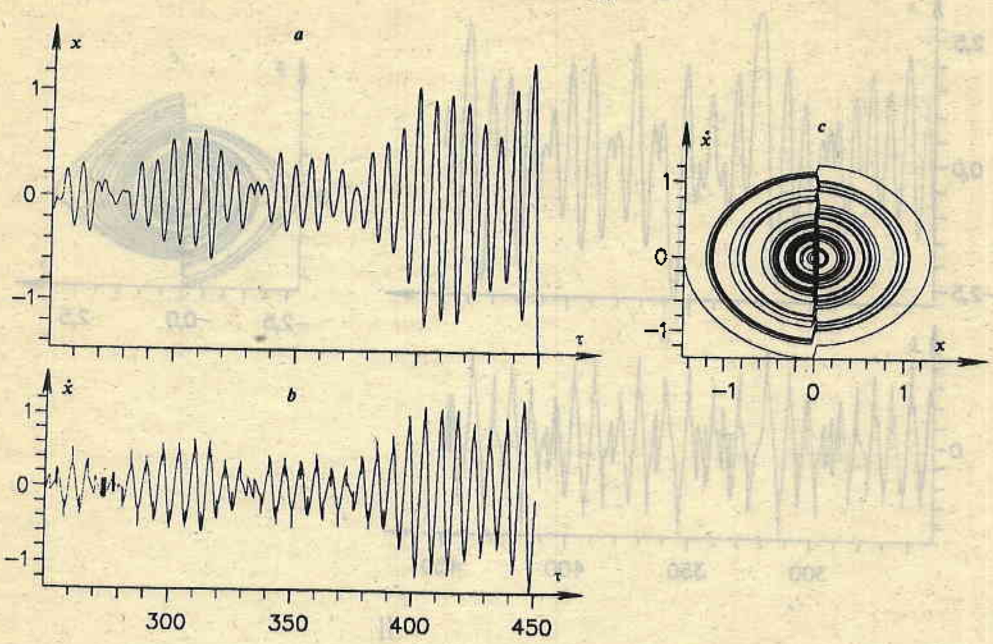
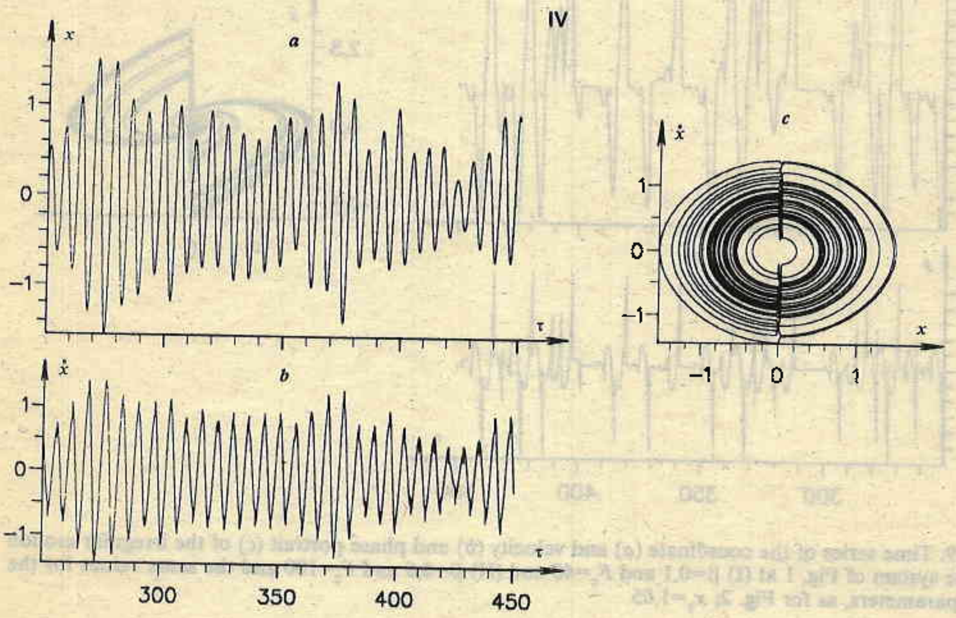


Fig. 8. Time series of the coordinate x (*a*) and angular velocity $\dot{x}$ (*b*) and phase portrait (*c*) of the irregular motion in the system of Fig. 1 at (I) F_0=5,190, (II), F_0=5,195, (III) F_0=6,480 and (IV) F_0=6,51 and the same values for the rest parameters, as for Fig. 2; x_0=1,05

III



IV



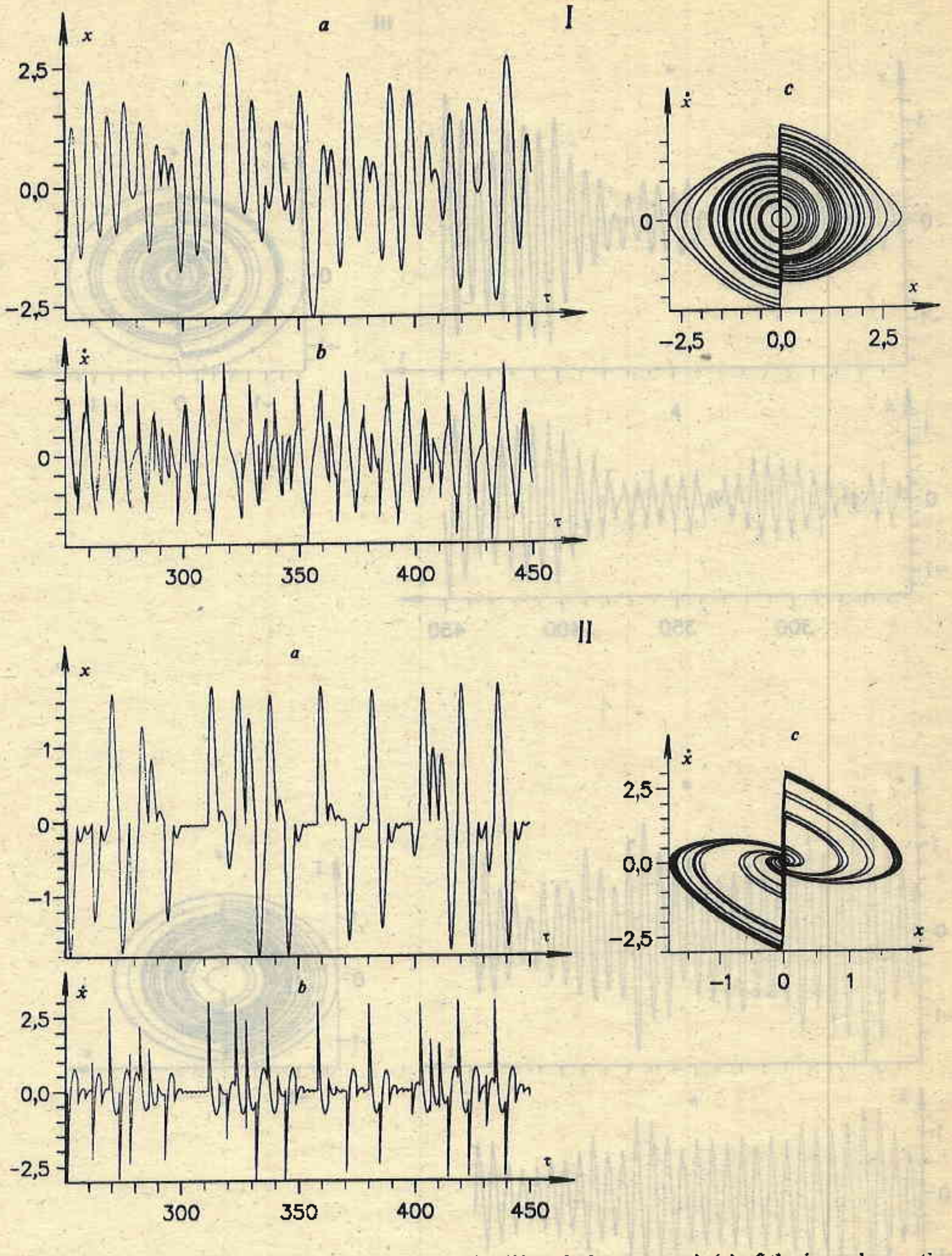


Fig. 9. Time series of the coordinate (*a*) and velocity (*b*) and phase portrait (*c*) of the irregular motion in the system of Fig. 1 at (I) $\beta=0{,}1$ and $F_0=40$ and (II) $\beta=0{,}5$ and $F_0=100$ and the same values for the rest parameters, as for Fig. 2; $x_0=1{,}05$

the amplitude $F_0=2{,}0$, for values of $\beta>0{,}02$, the pendulum oscillations mainly degenerate into faster or slower damped ones.

Fig. 11 gives a general idea of the nature of the pendulum oscillations on the

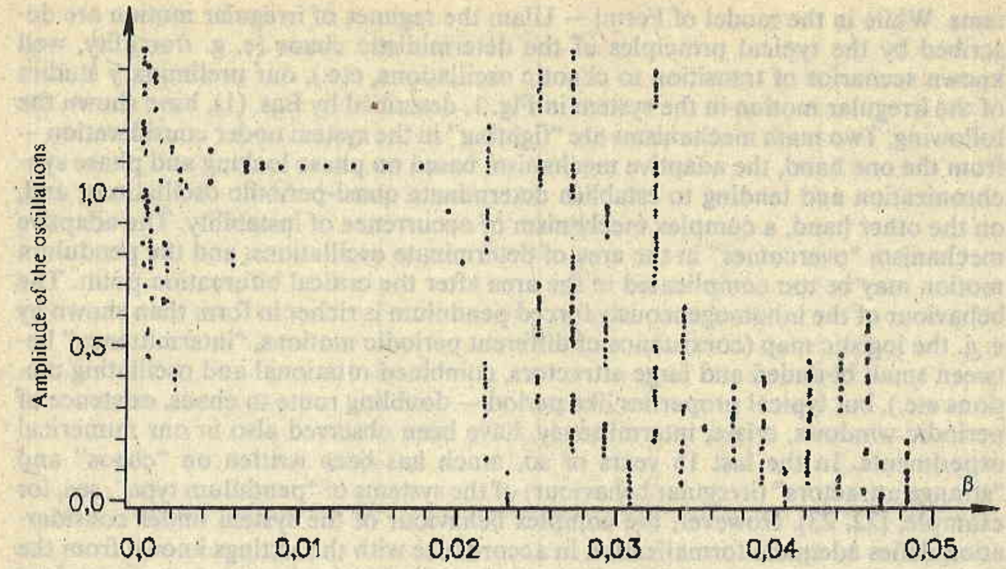


Fig. 10. Bifurcation characteristic representing the dependence of the amplitude of the pendulum oscillations on the value of the damping decrement β at the following parameter values: x_0=1,05, y_0=0, ν=51, d=0,025, F_0=2, β=vary

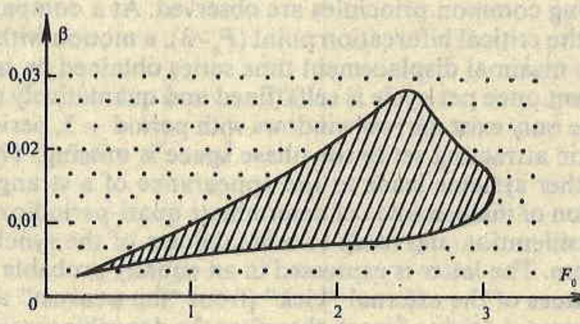


Fig. 11. Illustration on the nature of motion in the system of Fig. 1 in the plane of main parameters — damping decrement β vrs amplitude of the external driving force F_0. The hatched area corresponds to steady-state stationary oscillations; the dots denote the area of complicated irregular pendulum oscillations

plane of basic parameters — the state diagram is given as a function of both the external driving amplitude F_0 and the damping parameter β. The continuous line and the hatching define the area of parameters that ensure stationary oscillations. For values of parameters outside this area, the oscillations have strongly irregular nature.

The areas of parameter values corresponding to an irregular motion of the pendulum (see Fig. 6 at F_0>3,26 and Fig. 11 — the space filled with dots) require a very extensive and detailed investigation. As it was already pointed out, the system under consideration in Fig. 1 is relative by a number of attributes to the Fermi's problem in the setup of Fermi — Ulam, represented by a vibrating surface, from which an elastic ball is bouncing freely and is falling back by the gravitational effect. In spite of this, there are substantial differences in the nature of the irregular motion in the two systems. While in the model of Fermi — Ulam the regimes of irregular motion are described by the typical principles of the deterministic chaos (e. g. fractality, well known scenarios of transition to chaotic oscillations, etc.), our preliminary studies of the irregular motion in the system in Fig. 1, described by Eqs. (1), have shown the following. Two main mechanisms are "fighting" in the system under consideration — from the one hand, the adaptive mechanism based on phase locking and phase synchronization and tending to establish determinate quasi-periodic oscillations, and, on the other hand, a complex mechanism of occurrence of instability. The adaptive mechanism "overcomes" in the area of determinate oscillations, and the pendulum motion may be too complicated in the area after the critical bifurcation point. The behaviour of the inhomogeneously forced pendulum is richer in form than shown by e. g. the logistic map (coexistance of different periodic motions, "intermittency" between small bounded and large attractors, combined rotational and oscillating motions etc.), but typical properties like period — doubling route to chaos, existence of periodic windows, crises, intermittency, have been observed also in our numerical experiments. In the last 15 years or so, much has been written on "chaos" and "strange attractors" (irregular behaviour) of the systems of "pendulum type", see, for example, [22, 23]. However, the complex behaviour of the system under consideration defies adequate formalization in accordance with the settings known from the literature. The pendulum trajectories can be bounded and unbounded, the pendulum can have steady-state behaviour that is non an equilibrium point, not periodic, and not quasiperiodic.

When the amplitude of the external driving force F_0 is a control parameter (see Fig. 6) the following common principles are observed. At a comparatively small increase of F_0 after the critical bifurcation point ($F_0>3$), a motion with stochastic character appears (the maximal displacement time series obtained by sampling the pendulum displacement once per cycle is selfaffined and quantatively similar to brownian motion). There only exist narrow windows with period — 3, period — 7, period — 14. A characteristic attracting set in the phase space is missing. The eruptive instability, which in other systems leads to the appearance of a strange attractor, only breaks the condition of maintenance of determinate quasi-periodic oscillations in the system under consideration and leads to a breakdown of the synchronous input of energy in the system. The latter is expressed in an equally probable manifestation of wide spectra of values of the external "kick" (from "the weakest" to "the strongest" influence of the external driving force), therefore the describing point can with equal probability be located in any point of the phase space. The character of the motion substantially depends on the value of the damping decrement β. At small values of β there is a mature random process. When the value of β is increased, noticeable fractal structures are possible to appear in the phase space. In some cases, the chaotic motion abruptly terminates, only to resume after some "laminar time". At the same time, both for values of the amplitude F_0 slightly above the critical bifurcation value ($F_0>3$) and at significant values of F_0 ($F_0>10$), there exist areas in which the pendulum oscillations generally have a damping character. The time constant of damping depends on the values of the parameters β and F_0, its value significantly increasing with the value of F_0.

Theoretical studies and experiments on the externally forced pendulum [24] showed that chaotic oscillations of the pendulum are obtained after the breaking of symmetry of oscillations. At the same time, in the system under consideration of a pendulum under an inhomogeneous external action breaking the symmetry oscillations is, on the contrary, a necessary component of the adaptive mechanism for maintaining determinate oscillations. With the increase of the value of the parame-

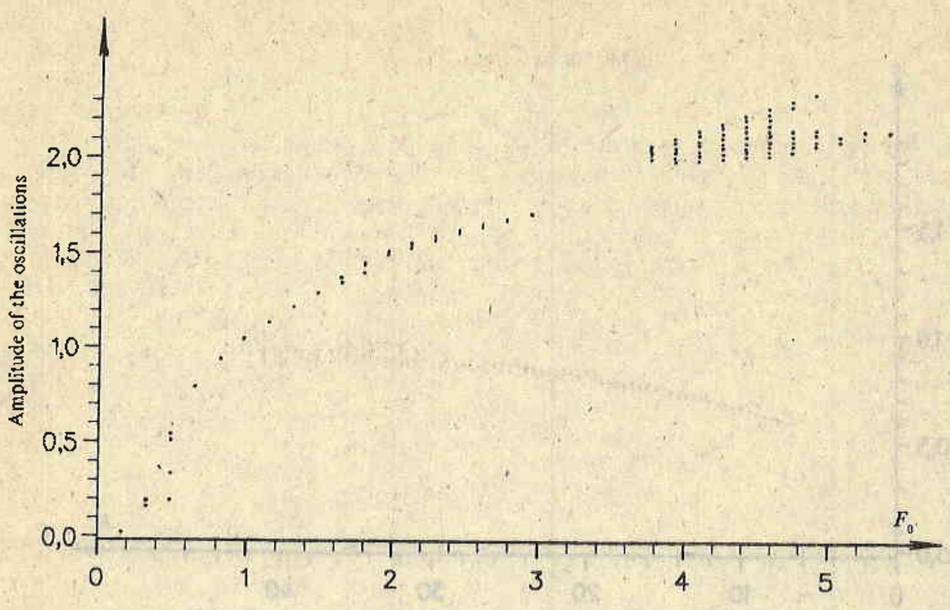


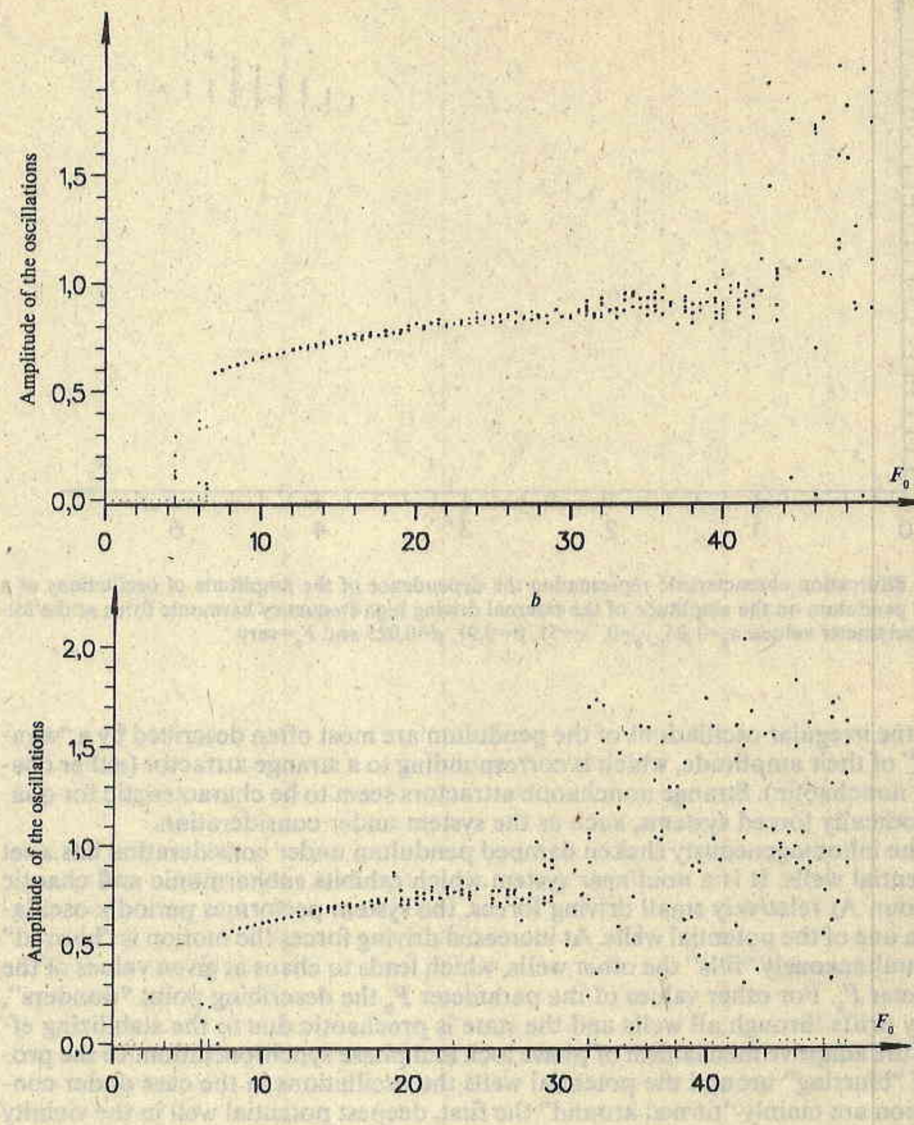
Fig. 12. Bifurcation characteristic representing the dependence of the amplitude of oscillations of a "linear" pendulum on the amplitude of the external driving high-frequency harmonic force at the following parameter values: x_0=1,05, y_0=0, ν=51, β=0,01, d=0,025 and F_0=vary

ter F_0, the irregular oscillations of the pendulum are most often described by a "wandering" of their amplitude, which is corresponding to a strange attractor (either chaotic or nonchaotic). Strange nonchaotic attractors seem to be characteristic for quasi-periodically forced systems, such as the system under consideration.

The inhomogeneously shaken damped pendulum under consideration has a set of potential wells. It is a nonlinear system which exhibits subharmonic and chaotic behaviour. At relatively small driving forces, the system performes periodic oscillations in one of the potential wells. At increased driving forces the motion is "blurred" and simultaneously "fills" the other wells, which leads to chaos at given values of the parameter F_0. For other values of the parameter F_0 the describing point "wanders", vaguely drifts through all wells and the state is prechaotic due to the stabilizing effect of the adaptive mechanism of phase lock and phase synchronization. In the process of "blurring" around the potential wells the oscillations in the case under consideration are mainly "turned around" the first, deepest potential well in the vicinity of x~0,25, which has stronger attracting adaptive properties due to the presence of conditions for nonsymmetric amplitude — three modulated oscillations.

It is interesting to compare the data stated above with the case of a "linear" pendulum. In this case, the function sin x is substituted by x in the system of equations (1).

Fig. 12 shows a bifurcation characteristic, which in this case is a dependence of one of the steady-state stationary amplitudes of oscillation of the "linear" pendulum on the value of the amplitude of the external driving high-frequency harmonic force F_0. By comparing it with the bifurcation characteristic in Fig. 6, we can note the following. While in the nonlinear case there is a clearly expressed threshold by value of the amplitude F_0 (F_0~1,1 in Fig. 6), over which stationary steady-state oscillations with unchanging amplitude are excited, in the linear case (Fig. 12) there is a portion a



of values of the control parameter $0,5<F_0<1,8$, for which a smooth increase of the amplitude of oscillation in the system is characteristic. For values $F_0>1,8$ the characteristic reaches a relatively flat section, but nevertheless the amplitude of oscillation of the "linear" pendulum obviously depends on the value of F_0. At the nonlinear pendulum (Fig. 6), in a wide area of values of F_0 $(1,1<F_0<2,8)$ the amplitude of its oscillations is practically fully independent of the amplitude of the external force (the external driving amplitude may vary up to 200%, at which the oscillating process remains unchanging). The adaptive mechanism of self-adjustment acts less strongly in the case of a "linear" pendulum. This is obvious, since the absence of nonlinearity precludes the action of the modulation-parametric mechanism of energy input into the oscillation process, characteristic for the nonlinear case. Continuing with the comparison, it is seen that in the case of a "linear" pendulum the multi-

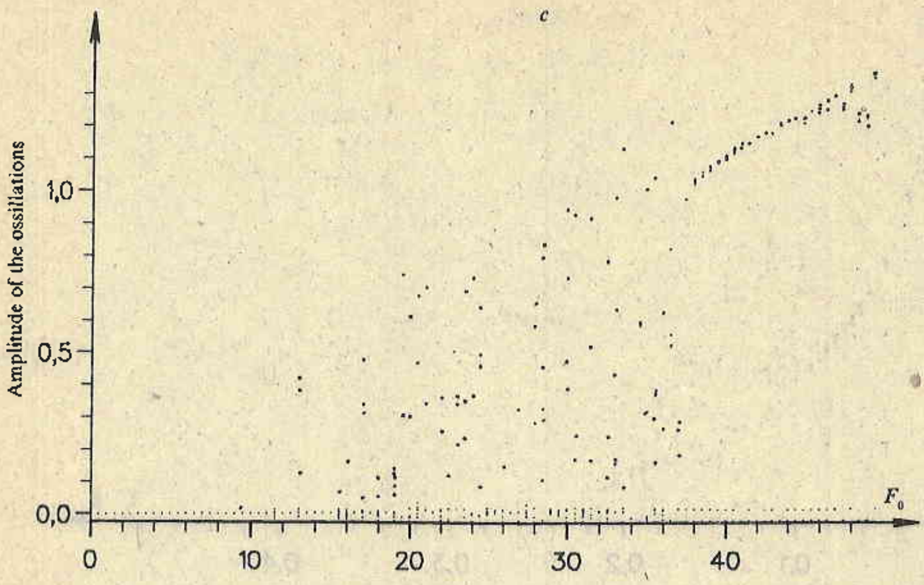
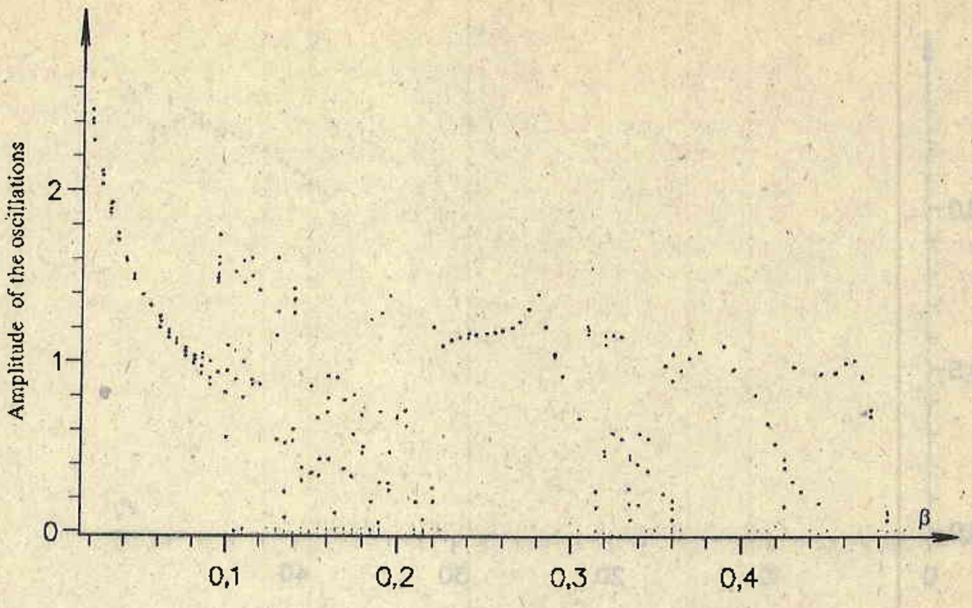


Fig. 13. Bifurcation characteristic representing the dependence of the amplitude of oscillations of a "linear" pendulum on the amplitude of the external driving high-frequency harmonic force at β=0,09 (*a*), β=0,1 (*b*) and β=0,225 (*c*) and the following values of the rest parameters: x_0=0,75 (*a* and *b*) and x_0=1,05 (*c*), y_0=0, ν=51, *d*=0,025, F_0=vary

amplitude regimes at increased values of F_0 (F_0>3,6) are much better expressed than in the nonlinear case.

The increase of the value of the decrement β substantially softend the section of the bifurcation characteristic described by an increase of the amplitudes of oscillation. This is illustrated in Fig. 13. It is seen that in this case the threshold value of the amplitude F_0 and the area of stable stationary steady-state oscillations are much better expressed. The case of a very high value of the parameter β shown in Fig. 13*c* is described by a wide area of irregular oscillations and the steady-state stationary oscillations are realized at high values of the driving amplitude F_0 (F_0>37). In order to get a more full idea, fig. 14 shows a bifurcation characteristic of the "linear" pendulum, the control parameter is the decrement of damping β, and the value of the amplitude of the external driving force has been chosen to be high, F_0=40. The areas of stationary steady-state oscillations and irregular oscillations are clearly distinguished. At small values of the oscillations are of quasi-determinate nature, as opposed to the nonlinear case, where for the same values of β the oscillations are of clearly expressed irregular nature.

Now it is time to give a more detailed explanation and illustrate more evidently the adaptive mechanism of maintaining unchanging the pendulum oscillations, which was mentioned several times in the above text. Fig. 15 shows conditionally a period of the sinusoid of the external driving harmonic force at F_0=1,1 (Fig. 15*a*) and F_0=2,8 (Fig. 15*b*). The time of interaction of the pendulum with the external high-frequency source is determined by the phase φ, corresponding to the time when the pendulum flies into the zone of driving [-*d*, *d*] and the phase φ_{out}, when it leaves the zone. The pendulum is speeded up during the positive half period and is stopped during the negative half period. The resulting energy absorbed by the pendulum is



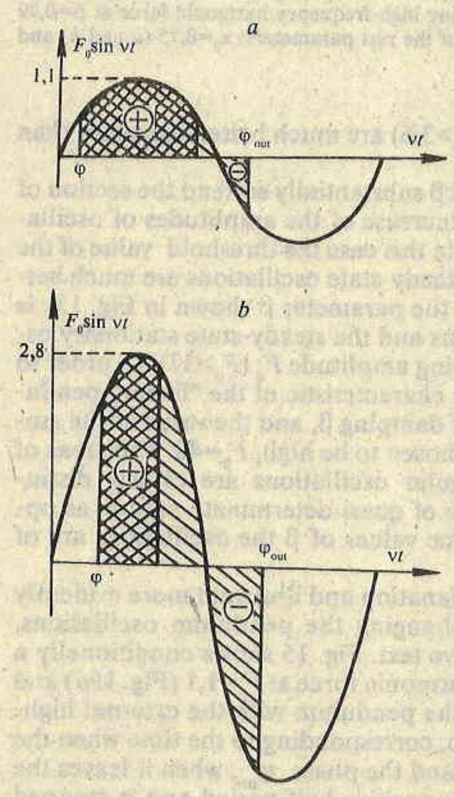


Fig. 14. Bifurcation characteristic representing the dependence of the amplitude of oscillations of a "linear" pendulum on the value of the damping decrement β at the following parameter values: x_0=1,05, y_0=0, F_0=40, ν=51, *d*=0,025, β=vary

Fig. 15. Illustration to the explanation of the adaptive mechanism of selfcontrol of the input portion of energy in the pendulum oscillations: one period of the external harmonic force at F_0=1,1 (*a*) and F_0=2,8 (*b*). The phase φ corresponds to the time when the pendulum enters the driving zone and $φ_{out}$ — to the time when it leaves the zone

proportional to the double hatched area marked with the sign ⊕ in Fig. 15. In a regime of stationary steady-state oscillations of the pendulum its amplitude remains unchanged at a drastic change of the amplitude of the external driving force, e. g. at a change of F_0 at almost 200% within the range $1,1<F_0<2,8$ (see Fig. 6). The portion of energy absorbed by the pendulum, delivered by the external source, remains unchanged at an arbitrary value within this range. This is automatically achieved due to the adaptive change and self-adjustment of the phase φ, where the pendulum flies into the driving zone [*-d, d*]. It is clearly seen from Fig. 15*a* that changing the amplitude F_0 from 1,1 to 2,8 causes the phase φ to be changed in such a way that the double hatched area with the sign ⊕ remains unchanged (compare Fig. 15*a* and Fig. 15*b*), therefore the portion of energy input in the pendulum remains unchanged.

Consider the influence of the parameters ν and d about which almost nothing has been said so far.

The analysis shows that the condition $\nu>>1$ should be satisfied in order to obtain a discrete series of steady-state amplitudes of pendulum oscillation, i. e. the frequency of the external driving force should be much higher than the natural resonance frequency of the pendulum. For example, in the range of values $1<\nu<10$ and the rest unchanging parameters, the pendulum may only have one steady-state amplitude of oscillation. At increased values of ν a discrete series of possible steady-state amplitudes is realized. As it was always seen, at $\nu=51$ there exist 4 steady-state stationary amplitudes of motion of the pendulum: ~0,25, ~0,75, ~1,1 and ~1,45 (see Figs. 2 and 3). At further increase of the value of ν, the number of the stationary amplitudes of motion of the pendulum is also increased. E. g., at $\nu=97$, the possible discrete series of stationary amplitudes amounts to 9 values: ~0,25, ~0,43, ~0,60, ~0,75, ~0,93, ~1,1, ~1,20, ~1,33, ~1,45.

Since the function $\varepsilon(x)$ was chosen to be even function of the type (2), then ν should take on odd values. Regardless that at $\nu>>1$ this condition is considerably softened, the numerical analysis shows a significant difference of the oscillation modes, e. g. at $\nu=51$ and $\nu=50$. At the numerical and theoretical analysis it is possible that the preset ν to be odd. At a natural system, built on the conditions of Fig. 1 and the Expr. (2) the condition ν to be odd is automatically achieved, since due to the adaptivity of the system and its non-isochronism (the frequency of pendulum oscillations depends on the amplitude of its oscillations) this condition corresponds to the regime that is most favourable as related to energy input. Indeed, in order to ensure the stationary oscillation of the pendulum, the latter should enter the driving zone [*-d, d*], both "from the left" and "from the right", each time at the same phase of the high-frequency driving force, differing from the preceding cycle with $l\pi$, where $l=1, 3, 5, ...$ Obviously the stationary oscillations of the pendulum will be excited, if the ratio of the external force frequency to the pendulum oscillation frequency is a multiple to an odd integer.

It should be particularly noted that only the precise odd integer ratio of frequencies and zero deviation from the respective resonance frequency ensure symmetrical almost harmonic oscillations of the pendulum. When these conditions cannot be simultaneously satisfied, nonsymmetrical regimes of oscillations of the pendulum are realized. A clear example of such nonsymmetrical regime at $\nu=51,0$ is amplitude — three oscillations of the pendulum around a value of ~0,25 (see Fig. 3).

The value $\nu=51,0$ gives the ratio between the external driving force frequency and the natural frequency of the pendulum at disappearingly small amplitudes of oscillation. At a finite amplitude (e. g.~0,25) the ratio between the external driving force frequency and the equivalent to that amplitude resonance frequency of the pendulum (this ratio will be denoted by N) is greater than 51, i. e. $N>51$ due to the non-isochronism of the oscillations. For example, the numerical experiment shows that frequency ratios N=53, 55, 57, and 59 correspond to the stationary oscillations with amplitudes ~0,25, ~0,75, ~1,1, and ~1,45 (see Fig. 3). Only at pendulum oscillations with amplitude ~1,45 there are conditions for oscillations with a precisely odd multiplicity of frequencies N=59. As seen from Figs. 2 and 3, in this case the pendulum oscillations are "purest" and closest to the harmonic ones. In the remaining cases there is some fluctuation of the resonance frequency around the precise multiple frequency. At higher values of ν it is possible the more complete fulfillment of the conditions; N to be an odd integer, which provides symmetrical oscillations of the pendulum. For example, at ν=97 the 9 steady-state amplitudes listed above are realized, to which 9 odd multiplicities of the frequencies correspond: from N=101 to N=117.

The numerical analysis of the influence of the parameter d, defining the zone of action of the external high-frequency harmonic force showed the following. There exist ranges of values for d, in which the possible discrete series of steady-state amplitudes of oscillations of the pendulum and their values remain unchanged. These ranges of values of d depend on the frequency of the external driving force. For example, in the case of ν=51, the main oscillation processes and regimes of the pendulum remain unchanged in the range of values for d=[0,01, 0,045] and the rest unchanging parameters. This is again possible due to the adaptivity of the system, when the phase φ, corresponding to the moment when the pendulum enters the zone of action, is so changed that the portion of energy of the external source that is input into the oscillating process should remain unchanged.

Conclusion

The paper presents the phenomenon of excitation of continuous oscillations with a possible discrete set of stable amplitudes. The discussion is performed on the basis of a model system representing a pendulum driven by an external harmonic force, which is nonlinear by the angle of its deviation. The inhomogeneous action of the external force is set by constraning the zone of its action on a certain small part of the trajectory of motion.

The basic properties characterizing the mechanism of "quantized" oscillation excitation are:

(1) Excitation of oscillations of the quasieigenfrequency of the system with a set of discrete stationary amplitudes, depending only on the initial conditions: i. e., discretion of the energy absorption processes, a specific "quantization" of the amplitude or intensities of the excited oscillations.

(2) The possibility for an effective division of the frequency with high rate frequency of the unary transformation. Principally new is the possibility to excite oscillations of the eigenfrequency under the action of external high frequency force upon the unperturbed linear and conservative linear and nonlinear oscillating systems.

(3) Adaptive self-control of the energy contribution in the oscillating process, revealed as a maintenance of the amplitude values and the oscillations frequency in the system in case of significant change of the amplitude of the external action, the quality factor (Q-factor, load, losses), and other external actions, i. e. this is a phenomenon of strong adaptive stabilization of regimes at a parameter change up to hundreds per cent. This effect of "dynamic stabilization" can play an important role in other, quite different physical phenomena such as quadrupole mass filters and various types of plasma confinement.

The simple pendulum is a very old device, yet it is a paradigm of contemporary nonlinear dynamics. The equation of motion for the driven, damped pendulum models a veriety of physical phenomena, e. g. such as radio-frequency driven Josephson junctions and charge-density-wave transport, etc. This fact, supported by the research of a great number of scientists for centuries, allows us to speak about the inexhaustibility of the pendulum as a basic paradigm of nonlinear dynamics and, on the basis of our research on the general model of a pendulum to move to generalization such as the class of kick-excited systems. The deterministic dynamical systems of "pendulum type", driven by external nonlinear to coordinates forces, exhibit large families of irregular non-periodic solutions in addition to the expected and studied harmonic and subharmonic motion. The physical significance of these and other irregular motions which appear to occur in pendulum systems apparently is to be yet more studied and discussed.

References

1. Damgov, V., P. Georgiev. — Aerospace Research in Bulgaria, **11**, 1994, 3-18.
2. Fermi, E. — Physical Review, **75**, 1949, 1169-1192.
3. Pike, E., L. Lungiato (Eds). Chaos, Noise and Fractals. Bristol, Adam Hilger, 1987.
4. Ford, J. — Physical Reports, **213**, 1992, No 5, 271-310.
5. Ulam, S. — Proceedings of the 4 th Berkeley Symp. on Math. Stat. and Probability. University of California Press, **3**, 1961, 31-47.
6. Holmes, P. — J. Sounds and Vibrations, **84**, 1982, No 2, 173-189.
7. Lichtenberg, A., M. Liberman. Regular and Stochastic Motion. New York, Springer — Verlag, 1983.
8. Everson, R. — Physica D, **19**, 1986, 355-383.
9. Tufillaro, N., A. Albano. — American J. Physics, **54**, 1986, No 10, 939-944.
10. Isomäki, H. — In: Nonlinear Dynamics in Engineering Systems, ed. W. Schiehlen. Berlin, Springer, 1990, 61-96.
11. Zimmerman, R., C. Celaschi, L. Neto. — American J. Physics, **60**, 1992, No 4, 370-375.
12. Jain, S. — Physical Review Letters, **70**, 1993, No 23, 3553-3556.
13. Shaw, S., P. Holmes. — J. Applied Mechanics, **50**, 1983, 849-857.
14. Tompson, J., R. Ghaffari. — Physical Review A, **27**, 1983, No 3, 1741-1743.
15. Schuster, H. Deterministic Chaos. New York, Physik — Verlag, 1984.
16. Isomäki, H., J. von Boehm, R. Rathy. — Physics Letters A, **107**, 1985, No 8, 343-346.
17. Tung, Pi-cheng. — JSME Int. J., Series III, **35**, 1992, No 3, 378-386.
18. Soliman, M., J. Thompson. — Int. J. Bifurcation and Chaos, **2**, 1992, No 1, 81-91.
19. Troesch, A., D. Karr, K. Beier. — Int. J. Bifurcation and Chaos, **2**, 1992, No 3, 607-620.
20. Cecchi, G. et al. — Chaos, **3**, 1993, No 1, 51-62.
21. Hayashi, Ch. Nonlinear Oscillations in Physical Systems. New Jersey, Prinston, Prinston University Press, 1985.
22. Hastings, S., J. McLeod. — American Mathematical Monthly, **100**, 1993, No 6, 563-572.
23. Christiansen, P., R. Parmentier. Structures, Coherence and Chaos in Dynamical Systems. Manchester University Press, 1989.
24. Bayly, Ph., L. Virgin. — Int. J. Bifurcation and Chaos, **2**, 1992, No 4, 983-988.

Received 28. XI. 1994

Кик-възбуждане на „квантовани“ трептения

Владимир Дамгов, Арън Холден

(Резюме)

Представен е числен анализ на явлението възбуждане на трептения с дискретен ред устойчиви амплитуди под въздействието на външна, нелинейна по координатата периодична сила. Численият експеримент е направен на основата на уравнение, описващо движението на махало. Дадени са времеви серии, съвместени фазови портрети, бифуркационни характеристики. Като управляващи параметри са взети амплитудата на външната въздействаща сила и коефициентът на демпфиране в системата. Детерминираната проява на явлението се характеризира с две важни закономерности: дискретизация („квантовост“) на възможните устойчиви амплитуди и силна адаптивна устойчивост при значителни изменения на амплитудата на външното въздействие, качествения фактор на трептящото звено и други външни влияния. Дадена е нагледна физическа интерпретация на самоадаптивните свойства, обусловени от характерен фазов параметър на системата. Нерегулярното поведение на системата се характеризира със сложна комплексна динамика: период — 3, 7, 14 осцилации; трептения, подобни на Брауновото движение; развити хаотични трептения (детерминистичен хаос); нерегулярни движения, описвани със странен, но не хаотичен атрактор и др. Формиран и предложен е клас самоадаптивни кик-възбудими системи.