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Discretization of the radius of the electric-charge cyclotron motion in the field of electromagnetic wave[1]

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Introduction

As the history of research development shows, revealing of generation mechanisms in planetary magnetosphere radio sources is connected with the level of understanding of physical phenomena and the concept development in modern radiophysics. The most vivid example is the investigation of cyclotron maser processes and the subsequent discovery of similar processes in the nature of all magnetized planets [1-4].

It has been shown [4-7] that a discrete spectrum of stable amplitudes of an oscillatory system exists when the system is subjected to an inhomogeneous force at a frequency which is much higher than the resonant frequency of the oscillatory system. In the case of pendulum considered in [5], the interaction nonhomogeneity has been especially arranged — by restriction of external harmonic force action over small part of the trajectory.

When electromagnetic wave interacts with resonators, the effect of "quantization" of possible stationary stable oscillating amplitudes arises without satisfying any especially organized conditions (like the inhomogeneous action of external harmonic force).

An electric charge, moving on a circular orbit in a homogeneous permanent magnetic field is considered. When the charge is irradiated by a flat electromagnetic wave having a length commensurable with the orbit radius, an effect of discretization of the possible stable orbit radii has been observed.

A recurrent expression for the possible stable radius values (correspondingly, for the possible rotation speed values) is derived. It is shown, that a radius threshold values exists that for the values above it, a discretization of the possible stable radius values arises.

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A stability general investigation is carried out.

Analysis

Let us consider electric charge q in magnetic field B and electric field E. The equation of motion in three-dimensional Euclid's space is,

$$m\frac{d^2r}{dt^2}=F=q(E+V\times B)-2m\beta V, \tag{1}$$

where m is the mass of the moving charge q, $V=\frac{dr}{dt}$ is the velocity, β is coefficient of dissipation.

Considering Eq. (1) and assuming that the motion is in the plane $z=0$, we can write:

$$\left| \begin{aligned} & m\frac{dV_x}{dt}=q(E_x+V_yB_0)-2m\beta V_x, \\ & m\frac{qV_y}{dt}=q(E_y-V_xB_0)-2m\beta V_y. \end{aligned} \right. \tag{2}$$

For constant magnetic field $B=e_zB_0=\text{const}$ the cyclotron frequency

$$\omega_0=-\frac{qB_0}{m}. \tag{3}$$

Taking into account Eq. (3), Eqs. (2) take the form

$$\left| \begin{aligned} & \frac{dV_x}{dt}=-\frac{\omega_0}{B_0}E_x-\omega_0V_y-2\beta V_x, \\ & \frac{dV_y}{dt}=-\frac{\omega_0}{B_0}E_y+\omega_0V_x-2\beta V_y. \end{aligned} \right. \tag{4}$$

A solution, corresponding to rotation plus drift is sought in the form

$$x=R\cos\Psi+at,\quad y=R\sin\Psi+bt,\quad \Psi=\omega t+\varphi, \tag{5}$$

where R, φ, a, b are constants in the stationary regime, $\omega=\text{const}$.

Let us introduce the sign $\langle\ \rangle$, denoting the averaging by time t. Then the values a, b can be found from the next equations:

$$\left| \begin{aligned} & -\frac{\omega_0}{B_0}\langle E_x\rangle-\omega_0 b-2\beta a=0, \\ & -\frac{\omega_0}{B_0}\langle E_y\rangle+\omega_0 a-2\beta b=0. \end{aligned} \right. \tag{6}$$

Integrating (4), we can write

$$\left| \begin{aligned} & V_x=-\frac{\omega_0}{B_0}\int E_x dt-\omega_0 y-2\beta x+\text{const}_1, \\ & V_y=-\frac{\omega_0}{B_0}\int E_y dt+\omega_0 x-2\beta y+\text{const}_2. \end{aligned} \right. \tag{7}$$

Neglecting $\frac{da}{dt}$ and $\frac{db}{dt}$ from (5) we obtain

$$
(8)\quad \begin{aligned}
V_x &= \frac{dx}{dt} = -\omega R \sin\Psi + \frac{dR}{dt}\cos\Psi - \frac{d\varphi}{dt} R \sin\Psi + a,\\
V_y &= \frac{dy}{dt} = \omega R \cos\Psi + \frac{dR}{dt}\sin\Psi + \frac{d\varphi}{dt} R \cos\Psi + b.
\end{aligned}
$$

The substitution of (8) into (7) gives:

$$
(9)\quad \begin{aligned}
\frac{dR}{dt}\cos\Psi - \frac{d\varphi}{dt} R\sin\Psi &= -\frac{\omega_0}{B_0}\int E_x dt - a - \omega_0 bt\\
&\quad -2\beta at + (\omega-\omega_0)R\sin\Psi - 2\beta R\cos\Psi + \text{const}_1,\\
\frac{dR}{dt}\sin\Psi + \frac{d\varphi}{dt} R\cos\Psi &= -\frac{\omega_0}{B_0}\int E_y dt - b + \omega_0 at\\
&\quad -2\beta bt - (\omega-\omega_0)R\cos\Psi - 2\beta R\sin\Psi + \text{const}_2.
\end{aligned}
$$

Considering (9) it gets clear how const_1 and const_2 have to be determined for the constant part of the Eqs. (7) to be fallen out.

Considering also (6) we can write

$$
(10\,a)\quad \frac{dR}{dt}\cos\Psi - \frac{d\varphi}{dt} R\sin\Psi = -\frac{\omega_0}{B_0}\left(\text{periodical part of} \int E_x dt\right) + (\omega-\omega_0) R\sin\Psi - 2\beta R\cos\Psi,
$$

$$
(10\,b)\quad \frac{dR}{dt}\sin\Psi + \frac{d\varphi}{dt} R\cos\Psi = -\frac{\omega_0}{B_0}\left(\text{periodical part of} \int E_y dt\right) - (\omega-\omega_0) R\cos\Psi - 2\beta R\sin\Psi.
$$

From Eqs. (10) we have

$$
(11\,a)\quad \frac{dR}{dt} = \frac{\omega_0}{B_0}\left[-\cos\Psi\left(\text{periodical part of} \int E_x dt\right) - \sin\Psi\left(\text{periodical part of} \int E_y dt\right)\right] - 2\beta R,
$$

$$
(11\,b)\quad \frac{d\varphi}{dt} = \frac{1}{R}\,\frac{\omega_0}{B_0}\left[\sin\Psi\left(\text{periodical part of} \int E_x dt\right) - \cos\Psi\left(\text{periodical part of} \int E_y dt\right)\right] - (\omega-\omega_0).
$$

We consider a plane electromagnetic wave (i. e. $\boldsymbol{E}.\boldsymbol{k}=0$, where $\boldsymbol{k}$ is wave vector), $E \sim \cos(\tilde{\nu}t - k_x x - k_y y - k_z z + \alpha)$.

Let us assume that $k_x = k_z = 0$ and $k_y = k$. Then $E_y = E_z = 0$ and

$$
(12)\quad E_x = -k\tilde{E}_0 \cos\left[(\tilde{\nu} - kb)\,t - kR\sin\Psi + \alpha\right].
$$

Assuming that $\nu = \tilde{\nu} - kb$ and $-k\tilde{E}_0 = E_0$, Eq. (12) can be rewritten in the form

$$
(13)\quad E_x = E_0 \cos(\nu t + \alpha - kR\sin\Psi).
$$

We assume

(14) $$\nu = N\omega,\ N = 1, 2, 3, \ldots$$

For the sake of solving Eqs. (11) and considering Eq. (13), we derive the following expansions:

(15) $$\cos\Psi\left[\text{periodical part of} \int \cos(\nu t - kR\sin\Psi + \alpha)\,dt\right]$$

$$= \text{finite part}\left\{\frac{J_0(kR)}{2N\omega}\{\sin[(N-1)\omega t + \alpha - \varphi] + \sin[(N+1)\omega t + \alpha + \varphi]\}\right.$$

$$+ \sum_{j=1}^{\infty} J_{2j}(kR)\left\{\frac{\sin[(2j+N-1)\omega t + (2j-1)\varphi + \alpha] + \sin[(2j+N+1)\omega t + (2j+1)\varphi + \alpha]}{2(2j+N)\omega}\right.$$

$$\left. + \frac{\sin[(2j-N-1)\omega t + (2j-1)\varphi - \alpha] + \sin[(2j-N+1)\omega t + (2j+1)\varphi - \alpha]}{2(2j-N)\omega}\right\}$$

$$+ \sum_{j=1}^{\infty} J_{2j-1}(kR)\left\{\frac{\sin[(2j-2-N)\omega t + (2j-2)\varphi - \alpha] + \sin[(2j-N)\omega t + 2j\varphi - \alpha]}{2(2j-1-N)\omega}\right.$$

$$\left.\left. - \frac{\sin[(2j-2+N)\omega t + (2j-2)\varphi + \alpha] + \sin[(2j+N)\omega t + 2j\varphi + \alpha]}{2(2j-1+N)\omega}\right\}\right\},$$

(16) $$\sin\Psi\left[\text{periodic part of} \int \cos(\nu t - kR\sin\Psi + \alpha)dt\right]$$

$$= \text{finite part}\left\{\frac{J_0(kR)}{2N\omega}\{\cos[(N-1)\omega t - \varphi + \alpha] - \cos[(N+1)\omega t + \varphi + \alpha]\}\right.$$

$$+ \sum_{j=1}^{\infty} J_{2j}(kR)\left\{\frac{\cos[(2j+N-1)\omega t + (2j-1)\varphi + \alpha] - \cos[(2j+N+1)\omega t + (2j+1)\varphi + \alpha]}{2(2j+N)\omega}\right.$$

$$\left. + \frac{\cos[(2j-N-1)\omega t + (2j-1)\varphi - \alpha] - \cos[(2j-N+1)\omega t + (2j+1)\varphi + \alpha]}{2(2j-N)\omega}\right\}$$

$$+ \sum_{j=1}^{\infty} J_{2j-1}(kR)\left\{\frac{\cos[(2j-N-2)\omega t + (2j-2)\varphi - \alpha] - \cos[(2j-N)\omega t + 2j\varphi - \alpha]}{2(2j-1-N)\omega}\right.$$

$$\left.\left. - \frac{\cos[(2j+N-2)\omega t + (2j-2)\varphi + \alpha] - \cos[(2j+N)\omega t + 2j\varphi + \alpha]}{2(2j-1+N)\omega}\right\}\right\},$$

where $J(.)$ are Bessel functions of first kind.

Using (11) we can write the shortened (averaged) equations:

(17 a) $$\left\langle\frac{dR}{dt}\right\rangle = -\frac{\omega_0}{B_0} E\left\langle\cos\Psi\left\{\text{periodical part of}\left[\int \cos(\nu t - kR\sin\Psi + \alpha)\,dt\right]\right\}\right\rangle - 2\beta R,$$

(17 b) $$\left\langle\frac{d\varphi}{dt}\right\rangle = \frac{1}{R}\,\frac{\omega_0 B_0}{B_0}\left\langle\sin\Psi\left\{\text{periodical part of}\left[\int \cos(\nu t - kR\sin\Psi + \alpha)\,dt\right]\right\}\right\rangle - (\omega - \omega_0).$$

From (15), (16) and (17) we obtain

(18) $$\langle \cos\Psi \left\{ \text{periodical part of } \left[\int \cos(\nu t - kR\sin\Psi + \alpha)\, dt \right] \right\} \rangle = -\frac{1}{\omega} J'_N(kR) \sin(N\varphi - \alpha);$$

(19) $$\langle \sin\Psi \left\{ \text{periodical part of } \left[\int \cos(\nu t - kR\sin\Psi + \alpha)\, dt \right] \right\} \rangle = \frac{N}{\omega kR} J_N(kR) \cos(N\varphi - \alpha),$$

where $J'_N(.)$ is the first derivative of the Bessel function of the first kind.

Taking into account (18) and (19) Eqs. (17) can be rewritten as

(20 a) $$\langle \frac{dR}{dt} \rangle = f(R, \varphi),$$

(20 b) $$\langle \frac{d\varphi}{dt} \rangle = g(R, \varphi),$$

where

(21 a) $$f(R, \varphi) = \frac{\omega_0}{\omega} \frac{E_0}{B_0} J'_N(kR) \sin(N\varphi - \alpha) - 2\beta R,$$

(21 b) $$g(R, \varphi) = \frac{\omega_0}{\omega} \frac{E_0}{B_0} \frac{N}{kR^2} J_N(kR) \cos(N\varphi - \alpha) - (\omega - \omega_0).$$

The stationary solution corresponds to the conditions

(22) $$\langle \frac{dR}{dt} \rangle = 0, \quad \langle \frac{d\varphi}{dt} \rangle = 0.$$

For the sake of stability analysis we vary

(23) $$\delta\frac{dR}{dt} = \frac{\partial f}{\partial R}\delta R + \frac{\partial f}{\partial \varphi}\delta\varphi, \qquad \delta\frac{d\varphi}{dt} = \frac{\partial g}{\partial R}\delta R + \frac{\partial g}{\partial \varphi}\delta\varphi.$$

Using f_R, f_φ, g_R and g_φ to denote the derivatives $\frac{\partial f}{\partial R}$, $\frac{\partial f}{\partial \varphi}$, $\frac{\partial g}{\partial R}$ and $\frac{\partial g}{\partial \varphi}$ in Eqs. (23) for constant (stationary) values of R and φ, corresponding to the stedy-state oscillations, the stability condition can be written

(24) $$Re(\lambda_{1,2}) < 0,$$

where

(25) $$\lambda_{1,2} = \frac{f_R + g_\varphi}{2} \pm \sqrt{\left(\frac{f_R - g_\varphi}{2}\right)^2 + f_\varphi g_R}$$

since the time dependence of the small deviations of R and φ from their steady-state values is governed by the equations $\delta R = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$ and $\delta\varphi = B_1 e^{\lambda_1 t} + B_2 e^{\lambda_2 t}$, where A_1, A_2, B_1 and B_2 are constants.

Considering Eq. (25) the condition (24) can be rewritten as

$$\begin{aligned}&(26\text{ a}) \qquad f_R+g_\varphi<0,\\ &(26\text{ b}) \qquad f_R g_\varphi - f_\varphi g_R>0.\end{aligned}$$

The partial derivatives can be expressed:

$$(27)\qquad \begin{cases} f_R=\dfrac{\omega_0}{\omega}\,\dfrac{E_0}{B_0}\,k\left[-\dfrac{1}{kR}J'_N(kR)+\left(\dfrac{N^2}{k^2R^2}-1\right)J_N(kR)\right],\\ f_\varphi=\dfrac{\omega_0}{\omega}\,\dfrac{E_0}{B_0}NJ'_N(kR)\cos(N\varphi-\alpha),\\ g_R=\dfrac{\omega_0}{\omega}\,\dfrac{E_0}{B_0}\,\dfrac{Nk^2}{k^2R^2}\left[-\dfrac{2}{kR}J_N(kR)+J'_N(kR)\right]\cos(N\varphi-\alpha),\\ g_\varphi=-\dfrac{\omega_0}{\omega}\,\dfrac{E_0}{B_0}\,k\,\dfrac{N^2}{k^2R^2}\,J_N(kR)\sin(N\varphi-\alpha).\end{cases}$$

Considering Eqs. (22) and (21), Eqs. (27) become

$$(28)\qquad \begin{cases} f_R=-4\beta+\dfrac{\omega_0}{\omega}\,\dfrac{E_0}{B_0}\,k\left(\dfrac{N^2}{k^2R^2}-1\right)J_N(kR)\sin(N\varphi-\alpha),\\ f_\varphi=\dfrac{\omega_0}{\omega}\,\dfrac{E_0}{B_0}NJ'_N(kR)\cos(N\varphi-\alpha),\\ g_R=-\dfrac{2}{R}(\omega-\omega_0)+\dfrac{\omega_0}{\omega}\,\dfrac{E_0}{B_0}Nk^2\dfrac{1}{k^2R^2}J'_N(kR)\cos(N\varphi-\alpha),\\ g_\varphi=-\dfrac{\omega_0}{\omega}\,\dfrac{E_0}{B_0}k\,\dfrac{N^2}{k^2R^2}J_N(kR)\sin(N\varphi-\alpha).\end{cases}$$

Combining, from (28) we can write

$$(29)\qquad f_R+g_\varphi=-4\beta-F_0J_N(\rho)\sin\gamma,$$

$$(30)\qquad f_Rg_\varphi-f_\varphi g_R=F_0^2\frac{N^2}{\rho^2}\left[\left(1-\frac{N^2}{\rho^2}\right)J_N^2(\rho)-J_N'^2(\rho)\right]$$

$$+F_0\left[4\beta\frac{N^2}{\rho^2}J_N(\rho)\sin\gamma+2(\omega-\omega_0)\frac{N}{\rho}J'_N(\rho)\cos\gamma\right]+(N^2-\rho^2)(\omega-\omega_0)^2+4\beta^2N^2,$$

where the following designations are introduced:

$$\frac{\omega_0}{\omega}\,\frac{E_0}{B_0}k=F_0,\quad kR=\rho,\quad N\varphi-\alpha=\gamma.$$

First we consider the case of small amplitudes, i. e. $|\rho|\ll 1$. In this case we can use the following assimptotical expressions for the Bessel functions [8]

$$(31)\qquad J_N(\rho)=\left(\frac{\rho}{2}\right)^N\sum_{l=0}^{\infty}\frac{(-1)^l}{l\,!\,(N+l)\,!}\left(\frac{\rho}{2}\right)^{2l}\simeq\frac{1}{N\,!}\left(\frac{\rho}{2}\right)^N+\cdots$$

$$(32)\qquad J'_N(\rho)=\frac{1}{2(N-1)\,!}\left(\frac{\rho}{2}\right)^{N-1}+\cdots$$

From (20), (21) and (22) we find

(33 a) $$F_0 J'_N(\rho)\sin\gamma = 2\beta\rho,$$

(33 b) $$F_0 \frac{N}{\rho^2} J_N(\rho)\cos\gamma = \omega - \omega_0.$$

Substituting (31) and (32) into (33) we determine

(34) $$\begin{cases} \operatorname{tg}\gamma \simeq \dfrac{2\beta}{\omega-\omega_0}, \\ |\rho|^{N-2} \simeq \dfrac{2^N (N-1)!}{|F_0|} [(2\beta)^2 + (\omega-\omega_0)^2]^{\frac{1}{2}}. \end{cases}$$

From (34) it is evident that the spectrum of the possible amplitudes is uninterrupted and that there are no conditions for the amplitude discretization in this case.

When $|\rho| \ll 1$ and $N \gg 1$, from (29) and (30) we find

(35) $$f_R + g_\varphi = -4\beta < 0,$$

(36) $$f_R g_\varphi - f_\varphi g_R \simeq N^2(\omega-\omega_0)^2 + 4\beta^2 N^2 > 0,$$

i. e. the condition (26) is satisfied and in this case the system motion is stable.

Let us now consider the resonance case, which means

(37) $$\omega - \omega_0 \simeq 0,$$

or considering Eq. (33 b) this is equivelent to

(38) $$|F_0| \frac{N}{\rho^2} \gg |\omega - \omega_0|,$$

Two possibilities follow from Eqs. (33):

a) $J_N(\rho) = 0$ and $\cos\gamma \neq 0$ or b) $\cos\gamma = 0$.

We show that when the amplitudes are large ($\rho \gg 1$) the motion in the case *a*) is unstable as long as in the case *b*) motion is stable.

Case *a*),

(39) $$J_N(\rho) = 0 \text{ and } \cos\gamma \neq 0.$$

Then $J'_N(\rho) \neq 0$. In Eq. (30) we neglect $(\omega - \omega_0)$ and $J_N(\rho)$ in correspondance with (37) and (39). We find: $f_R g_\varphi - f_\varphi g_R \simeq -\frac{F_0^2 N^2}{2} J'^2_N(\rho) + 4\beta^2 N^2$. However from (33 a) it follows

(40) $$F_0 \frac{J'_N(\rho)}{\rho} = \frac{2\beta}{\sin\gamma}, \text{ i. e. } f_R g_\varphi - f_\varphi g_R \simeq 4\beta^2 N^2 \left(1 - \frac{1}{\sin^2\gamma}\right) < 0$$

and apparently in this case the motion is unstable.

Case *b*),

(41) $$\cos\gamma = 0, \text{ or}$$

(42) $$\sin\gamma = (-1)^m, \quad m = 0;\ 1$$

(the two cases are possible, e. g. with adding π to γ).

As here β is not of essential significance, for the sake of simplifying we put

(43) $$\beta \to 0.$$

From Eq. (33 a) it follows

(44) $$J_N'(\rho) = 0.$$

The condition (44) determine the possible discrete spectrum of amplitudes ρ. These amplitudes do not depend on the force E_0 (or F_0).

Taking into account Eqs. (37), (41), (43) and (44), from Eq. (30) we find

(45) $$f_R g_\varphi - f_\varphi g_R \simeq F_0^2 \frac{N^2}{\rho^2} \left(1 - \frac{N^2}{\rho^2}\right) J_N^2(\rho).$$

When

(46) $$\rho > N$$

from (45) it follows that the stability condition (26 b) is satisfied: $f_R g_\varphi - f_\varphi g_R > 0$.

From (29) and (42) we obtain

(47) $$f_R + g_\varphi \simeq -4\beta - (-1)^m F_0 J_N(\rho).$$

Selecting the values for m, it is possible to satisfy also the second stability condition (26 a): $f_R + g_\varphi < 0$, or

(48) $$F_0 \sin \gamma J_N(\rho) > 0.$$

Conclusions

The analysis shows the folowing two essential features of the system considered.

1. Discrete set of possible stationary stable amplitudes is existing, which can be approximately determined using the eq. (44) under the conditions expressed by Eqs. (38), (41), (46) and (48).

2. There exists a threshold for the amplitude, determined by the condition (46), that for the values above it the discrete states are stable.

The phenomenon of continuous oscillations excitation with amplitude from discrete value set of stationary amplitudes has been demonstrated on the basis of a common model — oscillator under wave action. It is shown that phenomenon manifestation conditions are realized in a natural way in an oscillator system interacting with a continuous electromagnetic wave.

Modelling system of oscillating charge under wave action has been considered. It has been shown that the continuous wave with spectral components, considerably higher than the oscillator charge natural frequency, excites charge oscillations with quasinatural frequency and amplitude belonging to discrete value set of possible stationary amplitudes, dependent only on the initial conditions. The considered model may be used for phenomenological investigation of plasma particles with electromagnetic waves interactions and waves in the Earth ionosphere and planetary magnetospheres. Hypothesis of adaptive non-linear parametric wave generation may be suggested for Solar wind control of Jovian heterometric radiations, Saturn modulated radio emissions and Uranian auroral kilometric radiations. The mechanism is connected with natural interaction inhomogeneity and its type can be defined as cyclotron instability in the generation processes.

There is general agreement between researchers that planetary radiation is emitted in extraordinary mode by maser cyclotron process and all celestial bodies with magnetic field and energetic electron source are strong radioemitters due to cyclotron maser instability. We hope that the effect, presented in our work, may throw a new light and enrich the concept of generation mechanisms.

We have presented a mechanism of cyclotron processes that might prove fundamental considering planetary magnetosphere radioemission. It can be shown that the mechanism may give rise to radioemission not only in narrow range of angles almost perpendicular to the magnetic field in source region, but any time when a wave packet falls upon the charged particle oscillator.

Here-with is shown the potential for excitation of low-frequency continuous oscillations with discrete amplitude set under the influence of wave with incompatibly higher frequency — in that number fall waves from the ultraviolet band, near and far IR range and the radioband. Possibly, this mechanism is combined with multiple re-emission with frequency downward transformation and collision mechanisms are accompanied by radioemission generation mechanisms due to plasma waves transformation into electromagnetic under the "wave-particle" and "wave — wave" interactions.

The mechanism may also be combined with maser cyclotron processes, giving initial excitation (initial conditions) in the presence of magnetic field, whereas later a wave pumping from electromagnetic background is added.

Radioemission spectrum characteristics might be determined by the properties of the discussed effect — on one hand, a wave with same (unchangeable) frequency parameter may excite oscillations in wide frequency band and different amplitudes; on the other hand — waves with different frequency parameters may excite oscillations with same frequency (e. g., in gyroresonance frequency area and local plasma frequency, due to the resonance effects).

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Дискретизация радиусов циклотронного движения электрического заряда в поле электромагнитной волны

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(Резюме)

Рассмотрено движение электрического заряда по круговой орбите в неоднородном магнитном поле. При облучении заряда плоской электромагнитной волной с длиной сравнимой с радиусом орбиты, наблюдается эффект дискретизации возможных устойчивых радиусов орбиты.

Выведено рекуррентное выражение для возможных устойчивых значений радиуса (соответственно, для возможных значений скорости вращения). Показано, что существует пороговое значение радиуса, выше которого возникает дискретизация величин возможных устойчивых радиусов.

Проведено общее исследование устойчивости.