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A method for mathematical modelling of linear systems for automatic control

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Identification of system, i. e. determining their structure and parameters by observation, is one of the major problems of modern theory and technique of automatic control. In order to determine the unknown dynamic characteristics of a particular object or system, the relation between the input and output quantities must be represented in mathematical terms. At present, no common classification of identification problems and methods for their solution exists. The identification methods developed differ in types of identifiable objects and tuned models, and, partly, in criteria of identification quality, and, especially, in identification algorithms [1].

This work deals with a method for mathematical modelling of linear control systems, intended for linear dynamic object identification. This method also helps solve the problem of defining an optimum model whose output response approaches closely the output response of the identifiable object.

The method allows determination of the response of a dynamic object or system to the input stimulus, using mathematical modelling on a microcomputer system [2]. The investigated signals are subjected to linear filtering. The results are estimated by a criterion of identification quality. On the basis of the optimum model parameters, a physical model of a linear tracking system as a separate module of a microcomputer system is developed, using electronic components.

Let us consider a linear stationary tracking system with one input and one output, the external stimulus being a random stationary process, independent of the input signal [5]. The structural configuration of such a system with a model of a human operator is shown in Fig. 1.

The mathematical modelling of the system in Fig. 1 is accomplished on the basis of the observed quantities — input action $r(t)$ and output signal $\hat{y}(t)$. Applying the model developed, the parameters of the models of the human operator (HO) and the controlled object are determined such that the condition of optimum tracking be fulfilled, namely the difference $y_e(t)$ between the output signal of the physical system $y(t)$ and the output signal of its model $\hat{y}(t)$ tends to zero at any moment $t>0$ of the system operation:

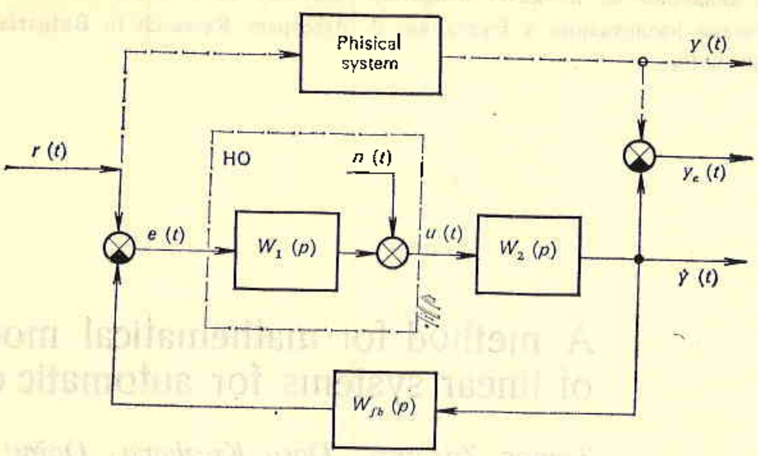

Fig. 1

(1) $$y_c(t) = y(t) - \hat{y}(t) \to 0.$$

The models of the human operator and the control object are described by the transfer functions $W_1(p)$ and $W_2(p)$, respectively. $W_{fb}(p)$ is the feedback transfer function. The input signal to the human operator model is the error or mismatch $e(t)$ between the input and output signal of the system:

(2) $$e(t) = r(t) - \hat{y}(t)\ W_{fb}(p).$$

The mathematical form accepted for representing the models of the human operator and control object is a linear differential equation with constant coefficients, since typical dynamic circuits are implemented in the physical modelling of the tracking system in Fig. 1. In fact, even for a very narrow frequency range of the input action, the human response is not completely linear and includes a linear part and residue or noise $n(t)$. The reaction of the human operator model is a sum of the signal $e(t)$ at its input, transformed by the operator $W_1\ (p)$, and the noise $n(t)$:

(3) $$u(t) = e(t)\, W_1(p) + n(t).$$

The output signal of the system model is defined as

(4) $$\hat{y}(t) = r(t)\, W(p),$$

where $W(p)$ is the equivalent transfer function of the whole system.

The output response of the system in Fig. 1 to an arbitrary input signal, taken as a sequence of unit pulses with an amplitude r and duration dt, can be calculated by Duhamel integral or convolution:

(5) $$y(t) = \int_0^T r(\tau) g(t-\tau) d\tau,$$

where $g(t-\tau)$ is a set of system responses to unit pulses with a weight coefficient $r(\tau)$, starting at moment τ and measured at moment $(t-\tau)$ from the beginning of the process. T is the period of the input signal $r(t)$.

All variables, noise included, are subjected to Fourier transformation.

The spectral noise component can have a magnitude, commensurate to the magnitude of the linear reaction of the human operator model. Expression (3) can be rewritten in the form

$$U(j\omega)=W_1(j\omega)\ E(j\omega)+N(j\omega). \tag{6}$$

The basic procedure employed in the mathematical modelling is linear filtering. The dynamic process of a given system is modelled by a set of filters with linear operators F_i and weight coefficients $r_i(t)$. The output response $\hat{y}(t)$ of the model is determined as a sum of the output responses of the filters:

$$\hat{y}(t)=\Sigma[F_i*r(t_i)], \tag{7}$$

where the symbol (*) denotes the linear operation F over r.

The class of linear filters is described with the convolution integral (3). A discreet analog of the convolution is the expression

$$\hat{y}(t_i)=\Sigma\, g(t_{i-\tau})\, r(t_i), \quad i=1,\ 2,\dots,n, \tag{8}$$

in which the pulse transition function $g(t)$ can be replaced by the frequency response of a linear filter F_i. This follows from the properties of the Fourier transformation.

The filters employed can be of different types. In order to obtain the best correspondence between the output response of the physical system $y(t)$ and the output response $\hat{y}(t)$ (1) of the model with a small number of filters, the pulse characteristics must be similar to those of the system investigated. This match is estimated with the criterion of identification quality:

$$J(c)=M\{F[y_e(t),\ c]\}, \tag{9}$$

where F [,] is the loss function, and M {,} is a symbol of mathematical expectation.

In the method considered, a quadratic loss function is employed, since it leads to relatively simple linear estimation algorithms. So criterion (9) takes the form

$$J(c)=M\{F[y_e^2(t),\ c]\}. \tag{10}$$

The minimization of the quadratic criterion (10) is the condition for optimim tracking and corresponds to minimization of the mean square mismatch error

$$\varepsilon=\left|\left[\sum_{i=0}^{n} y(t_i)-\hat{y}(t_i)\right]^2\right|^{1/2}=\|y-\hat{y}\|_{L_2}, \tag{11}$$

where L_2 indicates that the norm is in Euclidean space.

The criterion, thus defined, is a function of parameters $c=\{c_1, c_2,\dots, c_k\}$ of the separate units, i. e. $\varepsilon=f(c)$.

An optimum model of the system is obtained at a set of parameters c for which the mean square mismatch error $y_e(t)$ reaches a minimum value, i. e.

$$\min|f(c)|=\min\left|\left[\sum_{i=0}^{n} y(t_i)-\hat{y}(t_i)\right]^2\right|^{1/2}. \tag{12}$$

Parameters c are determined by equating to zero the partial derivative of the function f with respect to c and solving the set of equations obtained

(13) $$\frac{\partial f}{\partial c_i}=0, \quad i=1,\ 2,\ \dots,\ n.$$

Usually, the set of equations (12) is non-linear and is solved by the gradient methods [4] or by modified Newton's method.

The first step of the method developed for mathematical modelling of linear systems involves determination of the structure of the investigated system, i. e. the number of elementary units and the scheme of their connection. Even the most complicated system for automatic control can be described by a combination of the three basic schemes of connecting elementary units — series, parallel and feedback.

Each unit in the program model of the linear systems is realized as linear digital filter with a transfer function $F_i(p)$. A syntactic description of the system is accomplished, reflecting the connections between the individual units:

(14) $$W(p)=S\{F_i(p)\},$$

where S is an operator, representing a mathematical equivalent of the system syntactic description, and $W(p)$ is the equivalent transfer function of the system.

Using the described method for mathematical modelling, the identification problem for a particular control system could be solved by one of the following approaches:

1. Fully known syntactic description of the system — number of elementary units, their kind and scheme of connection. The problem is reduced to determination of the parameters of each elementary unit $c_i=\{c_{i1}, c_{i2}, \dots, c_{ij}\}$ so that to meet the chosen criterion of identification quality f:

(15) $$f=\inf\|y-\hat{y}\|_{L_2}.$$

2. Partially known syntactic description — number and connections of elementary units known, but not their type. The problem is reduced to multiple solving of problem 1 within the framework of a certain set of elementary units $E\{F_i\}$ for the allowable k combinations of the units belonging to that set. The criterion of identification quality is

(16) $$F=\min\{f_1,\ f_2,\ \dots,\ f_k\}.$$

3. The syntactic description of the system is unknown. The problem is reduced to multiple solving of problem 2 for a certain set M of l possible connection schemes, including units belonging to the set $E\{F_i\}$. The criterion of identification quality is

(17) $$\mathscr{H}=\min\{F_1,\ F_2,\ \dots,\ F_l\}.$$

When solving these problems, unstable solutions can arise, and this demands a priori information for regularizing the solutions [3]. Then condition (8) is replaced by a new one of the kind

(18) $$\varepsilon=\|y-\hat{y}\|_{L_2}+\|\Omega\|,$$

where Ω is a regularizing functional, reflecting the a priori information. Depending on the kind of Ω, additional constraints on the vector solutions of the parameters c can be introduced, for example, by applying Chebishev's criterion or limiting the values of the parameters in reasonable limits.

The proposed method for mathematical modelling enables the investigation of arbitrary linear control systems in a wide frequency range. A significant advantage of this method is that simultaneously with the estimation of the identification quality by the criterion selected, the stability of the system in the specific frequency range is checked, too. At the same time, the suggested method of mathematical modelling makes possible the confinement of the possible realizations within the tolerable values of the technical units and the assessment of the parametric sensitivity to detuning of individual components.

This method is applicable in designing and investigating a wide class of complex technical systems under severe operating and economical limitations and, in particular, systems, related to space research.

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Метод за математическо моделиране на линейни системи за автоматично управление

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(Резюме)

С разработения метод за математическо моделиране се решават задачите за идентификация на широк клас системи за управление. Този метод дава възможност въз основа на програмно реализиран математичен модел на линейна динамична система да се определи реакцията ѝ на входното въздействие. Обработката на изследваните сигнали се извършва на принципа на линейната филтрация в честотната област. Въз основа на избран критерий за качество на идентификацията се оценява изходната реакция на модела на системата и чрез итеративен алгоритъм се определят оптималните параметри на системата. Методът за математическо моделиране позволява да се ограничат възможните реализации на моделите в рамките на допустимите стойности на техническите звена и да се оценява параметричната чувствителност на всеки модел към разстройка на отделните компоненти.