

## Statistical Relationship between Aeronomic and Geophysical Parameters for the South Atlantic Geomagnetic Anomaly Region

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The examination of the results from the systematic observations on aeronomic and geophysical parameters in the South Atlantic Region results in the conclusion that the region westward from the mid-Atlantic ridge is of particular interest to meteorology and geophysics. This is based on the analysis of 35 parameters for which data were provided by meteorological, geophysical and space observations: (1) average atmospheric temperature of the contact layer—summer; (2) average surface water temperature; (3) deficit of irrigation point more than 850 mb; (4) deficit of irrigation point more than 500 mb; (5) average air pressure of the contact layer—summer; (6) mean air temperature  $h=10$  km—summer; (7) mean air pressure  $h=10$  km—summer; (8) average annual air temperature  $h=10$  km; (9) average annual air pressure  $h=10$  km; (10) total cloudiness; (11) northern component of the geomagnetic field  $X$   $h=0$  km; (12) eastern component of the geomagnetic field  $Y$   $h=0$  km; (13) vertical component of the geomagnetic field  $Z$   $h=0$  km; (14) geomagnetic field intensity  $H$   $h=0$  km; (15) northern component of the geomagnetic field  $X$   $h=10$  km; (16) eastern component of the geomagnetic field  $Y$   $h=10$  km; (17) vertical component of the geomagnetic field  $Z$   $h=10$  km; (18) geomagnetic field intensity  $H$   $h=10$  km; (19) northern component of the geomagnetic field  $X$   $h=350$  km; (20) eastern component of the geomagnetic field  $Z$   $h=350$  km; (21) vertical component of the geomagnetic field  $Z$   $h=350$  km; (22) geomagnetic field intensity  $H$   $h=350$  km; (23) velocity of charged particles precipitation from space; (24) northern component of nondipole geomagnetic field  $X'$   $h=0$  km; (25) eastern component of nondipole geomagnetic field  $Y'$   $h=0$  km; (26) vertical component of the nondipole geomagnetic field  $Z'$   $h=0$  km; (27) nondipole field intensity  $H'$   $h=0$  km; (28) northern component of nondipole geomagnetic field  $X'$   $h=10$  km; (29) eastern component of nondipole geomagnetic field  $Y'$   $h=10$  km; (30) vertical component of the nondipole geomagnetic field  $Z'$   $h=10$  km; (31) nondipole geomagnetic field intensity  $H'$   $h=10$  km; (32) northern component of nondipole geomagnetic field  $X'$   $h=350$  km; (33) eastern component of the nondipole geomagnetic field  $Y'$   $h=350$  km; (34) vertical

component of nondipole geomagnetic field  $Z' h=350$  km; (35) nondipole geomagnetic field intensity  $H' h=350$  km.

In this paper relationships characterizing various in nature processes are studied. In general, these are meteorological elements (temperature, pressure) components of the geomagnetic field — observed and nondipole, characteristics of phenomena resulting from the solar activity.

The data on these parameters are statistically processed. This comprises the determination of correlative and regressive dependences between topographic changes of the geophysical parameters studied. The procedure is performed with statistical models and essentially represents a model problem (regressive model) but in this case the model is formally mathematical. Its parameters do not have direct physical interpretation. The efforts when applying the model were to obtain the intervals within which the average values of the regression line, which expresses the functional relationship between the studied parameters, are defined with given confidence level. The selection of the parameters aimed at better representativeness of the analysed relationships, i. e. better characterization of the processes and phenomena observed. Nevertheless, data on single parameters do not refer to one and the same above sea level height. Therefore, changes of the temperature and the dry air pressure, as well as of the components of the geomagnetic field are taken into account at heights of:  $h=0$  km (sea level),  $h=10$  km (average bottom boundary of the troposphere), and  $h=350$  km (spacecraft trajectory height with reference to measurements for parameter No 23).

Based on the results obtained in [6, 7] and revealing some interesting dependences between the components of the geomagnetic field, on the one hand, and the meteorological elements — on the other, the emphasis of the statistical analysis performed here is placed on the determination of similar relationships and the specification of the ones already defined. As far as the analysis of some dependences between meteorological and geomagnetic field parameters made in [6] shows that some of the relationships obtained are not of particular importance and do not contain qualitatively new information on the processes observed, further discussions would not refer to them. This would be replaced by data which may help to specify and confirm the relationships defined.

In order to collect data on the mentioned parameters, we use the results which in one form or another are already published or at the given initial conditions are computed with reference to the purpose of their application.

The data are calculated and located in the nodes of a  $5^{\circ}$  network. Thus, the source difference does not affect the calculations of the initial data.

The region in consideration is located at the southwestern part of the Atlantic Ocean from  $20$  to  $70^{\circ}$  west longitude and from  $0^{\circ}$  to  $50^{\circ}$  south latitude. The cartographic calculation of the parametric values is effected as the value of the respective isoline is taken in the nodes of the  $5^{\circ}$  network, and for the case where this is not available, the value is linearly approximated. The succession of the value calculations is selected arbitrarily, but once selected, it is kept the same for the parameters considered. The same procedure is applied for the cases where the data are taken by the respective catalogues or are computed. In this case the calculation is performed with the method of sweep. The parametric values are calculated in 70 points describing the region from the Atlantic Ocean with the above-given coordinates.

Values both for observed and for nondipole geomagnetic fields and used in the analysis of the geomagnetic field component data. The data are obtained as the geomagnetic potential is expanded to a series of spherical harmonic fun-

tions with accuracy up to the 100th term (for the observed geomagnetic field) and the first term of the development representing the dipole geomagnetic field is subtracted for the nondipole field.

The parametric data are statistically processed and are studied by linear correlative analysis and correlative ratio.

The results from the linear correlative analysis are elements of the normalized correlative matrix. Each matrix element is a coefficient of the mutual correlation. The degrees of fitness of the experimental data for the studied parameters with respect to the straight line is evaluated through the coefficient of the common linear correlation

$$(1) \quad r_{ij} = \frac{1}{n} \frac{\sum_{l=1}^n (a_{il} - \bar{a}_i)(a_{jl} - \bar{a}_j)}{\sigma_{a_i} \sigma_{a_j}},$$

$$\bar{a}_i = \frac{\sum_{l=1}^n a_{il}}{n}, \quad \sigma_{a_i} = \sqrt{\frac{\sum_{l=1}^n (a_{il} - \bar{a}_i)^2}{n}},$$

where  $r_{ij}$  is the coefficient of mutual correlation between the  $i$ -th and the  $j$ -th variables,  $\sigma_a$ — standard deviation,  $\sigma_{a_i}$ ,  $a_{jl}$ — values observed for the  $i$ -th and  $j$ -th variables,  $n$ — number of observation (number of values observed for  $a_i$  and  $a_j$  for the case  $h=70$ ).

The existence of functional dependence between a given couple of variables without revealing the type of this dependence is evaluated by the correlative ratio given in expression

$$(2) \quad \eta_{ij} = \frac{\sum_{l=1}^{m_i} (a_{il} - \bar{a}_i)^2 n_i}{\sum_{i=1}^m \sum_{l=1}^{n_i} (a_{il} - \bar{a}_i)^2},$$

where  $a_i = \frac{\sum_{l=1}^{n_i} a_{il}}{n_i}$ ,  $\bar{a}_i = \frac{\sum_{i=1}^{m_j} \sum_{l=1}^{n_i} (a_{il})}{n_i}$ ,  $n = \sum_{i=1}^m n_i$ ,  $n$  is the number of

experimental data for  $a_i$  and  $a_j$ ,  $m_j$  is the number of intervals of  $a_j$  (along the  $X$ -axis);

$$m_j = \frac{\Delta_j}{\delta_j},$$

where

$\Delta_j$  is the complete interval of  $a_j$  variation observed,  $\delta_j$  is the  $a_j$  measurement accuracy (indeterminacy interval of  $a_j$ ).

For the purpose of this work a polynomial regression model is used, linear with respect to the constants  $\alpha$  of the assumed regression between  $a_i$  and  $a_j$ .

$$(3) \quad \hat{a}_i = \sum_{l=1}^Q a_{il} a_j^{l, \text{opt}}$$

The set  $\{a_j\}$  incorporates components of observed and nondipole geomagnetic fields for heights of  $h=10$  km and  $h=350$  km, and the set  $\{a_i\}$  represents certain meteorological elements as average month and annual temperature and pressure for dry air at  $h=10$  km and velocity of space charged particle precipitation. The accuracy with which the model  $\hat{a}_i$  describes the available experimental data has been determined through Fischer's criterion

$$(4) \quad F = \frac{\sum_{l=1}^Q (\hat{a}_{il} - \bar{a}_i)^2 (n-k-1)}{\sum_{l=1}^Q (a_{il} - \hat{a}_i)^2 k}$$

Table 1

Dependence between parameters Nos	Polynomial power	Fischer's criterion $F$
6-15	4	24.562
7-15	3	17.217
8-15	4	33.205
9-15	4	16.043
6-16	3	128.984
7-16	3	84.021
8-16	3	138.683
9-16	3	97.467
6-17	3	169.284
7-17	4	194.753
8-17	3	182.982
9-17	4	158.005
6-18	4	41.963
7-18	4	58.854
8-18	4	35.886
9-18	4	56.233
6-28	4	6.312
7-28	3	8.749
8-28	4	3.971
9-28	4	11.025
6-29	2	32.112
7-29	2	26.349
8-29	2	35.652
9-29	1	23.650
6-30	4	457.025
7-30	4	211.722
8-30	1	503.402
9-30	1	204.835
6-31	2	716.24
7-31	2	470.012
8-31	2	813.287
9-31	2	392.186
23-19	1	39.813
23-20	2	21.872
23-21	3	8.212
23-22	4	4.417
23-32	1	18.667
23-33	2	7.688
23-34	3	14.062
23-35	4	14.863



where:  $\hat{a}_i$  is the calculated  $a_i$  values according to the model,  $\bar{a}$  is the average  $a_i$  value.

The evaluation of the regression model accuracy is performed under confidence level of  $p=0.05$  as  $n < 100$  and powers of freedom for the numerator equal to the power  $k$  of the polynomial and for the denominator  $(n-k-1)$ .

Resulting from the application of the regression analysis through a step regression from first to fourth power in agreement with equation (3), the form of the best regression models describing the experimental data is obtained. The statistical characteristics of some of them, as polynomial power, Fischer's criterion and regression coefficients are listed in Tables 1 and 2.

As already shown, the major characteristic of the studied regression fitness is the  $F$ -ratio. From a theoretical point of view, at  $n < 100$  the confidence level  $p$  cannot be greater than 0.05, regardless of the fact that most of the

Table 2

Dependence between parameters Nos	Regression coefficients				
	$a_0$	$a_1$	$a_2$	$a_3$	$a_4$
6-15	-42.7513	—	—	—	1896.4082
7-15	264.1711	—	—	1082.054	—
8-15	-46.7699	—	—	—	2645.8073
9-15	270.0776	—	—	—	3172.8139
6-16	-44.0839	—	—	-21451.13	—
7-16	263.4516	—	—	-43705.367	—
8-16	-48.10348	—	—	-27277.75	—
9-16	267.0065	—	—	-40133.273	—
6-17	-33.7062	—	—	959.1721	—
7-17	285.0156	—	—	—	-8899.355
8-17	-34.9268	—	—	1216.5942	—
9-17	285.2434	—	—	—	-7688.215
6-18	-29.8447	—	—	—	-2100.906
7-18	296.5337	—	—	—	-5136.988
8-18	-30.7741	—	—	—	-2507.963
9-18	296.417	—	—	—	-4513.433
6-28	-41.3221	—	—	—	14656.716
7-28	267.4776	—	—	-5083.813	—
8-28	-44.203	—	—	—	14821.469
9-28	271.1931	—	—	—	37053.063
6-29	-48.7036	-191.4451	4785.293	—	—
7-29	253.5461	-404.1636	9922.5461	—	—
8-29	-54.1898	-244.3384	6297.5352	—	—
9-29	268.156	-284.437	—	—	—
6-30	-35.2721	-44.1034	-457.1682	—	12419.199
7-30	283.4253	—	—	—	94.8064
8-30	-40.091	-57.8803	—	—	—
9-30	278.7434	-84.0031	—	—	—
6-31	-32.2859	—	-481.9697	—	—
7-31	228.4802	—	-1045.3175	—	—
8-31	-33.1792	—	-607.8804	—	—
9-31	289.3406	—	-918.3919	—	—
23-19	104.2394	-526.9827	—	—	—
23-20	-1.8724	—	22216.097	—	-3975742
23-21	2.75185	161.1247	—	-171341.81	-39591.88
23-22	28.0485	—	—	—	-4422.418
23-32	-7.5785	-390.6569	—	—	—
23-33	8.9018	—	30352.238	—	-18628128
23-34	18.43375	-447.606	-1955.16	34244.188	144292.13
23-35	18.6511	-582.01953	—	—	-119720.7

regression models obtained and shown in Table 1 should have a confidence level of  $p < 0.01$ . As it can be seen from this Table, almost all values of  $F$  are greater than the numbers corresponding to a confidence level of  $p = 0.01$ , but conclusions on the significance of the model can be valid only for  $p = 0.05$  due to  $n < 100$ . This is not contradictory to the comparative model analysis, but we have to keep in mind that all models are at confidence level  $p = 0.05$  (or 95% reliability from a statistical point of view).

Under these conditions we can see that between all the 40 regression models there is a group of 15 models for which the  $F$ -value is significantly higher (about one or two orders more) than the value of the remaining ones.

Figures 1-4 show the type of some model regressive curves, describing the correspondent experimental data.

The study of the experimental data with the multiple regressive analysis applying a step regression from first to fourth power shows that between indices Nos 6, 17; 7, 17; 7, 18; 6, 30; 7, 30; 6, 31; 7, 31 there is a clearly expressed curvilinear dependence described by curves from first to fourth power. This is confirmed by the high values of the  $F$ -ratio (Table 1). But in some cases the general grouping of experimental data considerably differs from the model curve obtained with step regression. This is due to the fact that the program used for the polynomial step regression provides for the examination of dependences only by polynomials from first to fourth power. As a regular sequence the models defined by the program as best cannot be considered best in general. There are cases when although the program has shown a given curve of the same power as the best regression model for two different dependences, this curve does not describe the scatter of experimental data in a unified manner, for example for dependences Nos 23, 19 and 23, 32. The comparison between the  $F$ -ratio values for them shows that although the model curve for both models to be of first power, the model selected for the first is formally better, due to the twice greater value of  $F$  for 23, 19 compared to the one for 23, 32.

Table 3

Dependence between parameters Nos.	Correlative coefficients $r$	Confidence intervals	
		$r_d$	$r_u$
5-32	-0.9013	-0.909	-0.8957
6-27	0.9411	0.9366	0.9447
6-30	-0.9231	-0.9275	-0.9186
6-35	0.9503	0.9478	0.9536
7-35	0.9127	0.909	0.9154
8-27	0.9536	0.9508	0.9562
8-30	-0.9386	-0.9425	-0.9354
8-35	0.963	0.9611	0.9661

Table 4

Dependence between parameters Nos.	Correlative ratio	Fischer's criterion $F$
8-17	0.8182	182.982
8-30	0.9797	503.402
8-31	0.9672	813.287
9-17	0.7962	158.002
9-30	0.9386	204.853
9-31	0.8712	392.186

The presence in experimental data of one or two values much larger than the other ones or different in sign results in significant change of the model curve type. In consequence, the regression model becomes worse. This can be

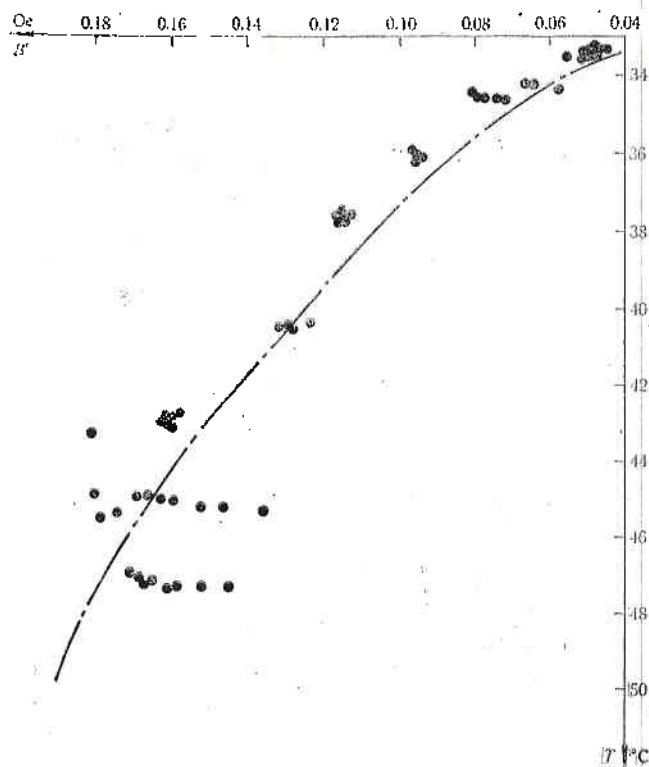


Fig. 1. Regression dependence between the intensity of the nondipole geomagnetic field ( $h=10$  km) and the average atmospheric temperature ( $h=10$  km)

eliminated by rejection of the single values because their presence may be of incidental nature with respect to the examined problem.

As already mentioned, both the data for observed and nondipole geomagnetic field obtained by the exclusion of the dipole field effects are subject to statistical processing. The comparison of results obtained shows that when the nondipole field is processed, the relationships determined between the indices of various groups (different nature) are more strongly expressed and clear nonlinearity of the dependences is observed. This is confirmed by the significantly greater values of the  $F$ -ratio (estimating the model accuracy).

Resulting from the application of the correlative analysis between the examined 35 parameters, the dependences (165 in number) characterized by the high coefficient of the common correlation are defined within the boundaries  $\pm 0.7 \leq r \leq \pm 1$ . The correlative coefficients related with dependences between indices of various nature (Table 3) are of particular interest. Part of these coefficients refer to relationships between meteorological parameters and components of the nondipole geomagnetic field and the rest — between these parameters and the components of the observed geomagnetic field. It is seen tha

for many nonlinear dependences having high values for the  $F$ -ratio the coefficient of the linear correlation is also high, as this is valid for all the linear models from Table 1. The comparison performed confirms the expressed functional dependence between variables observed (Table 3). The value of the cor.

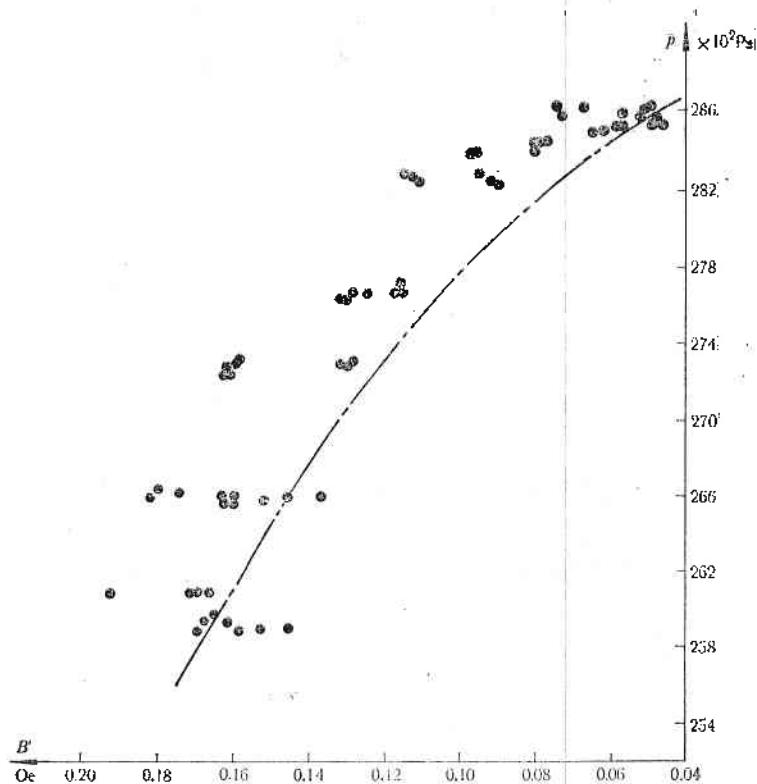


Fig. 2. Regression dependence between the intensity of the nondipole geomagnetic field ( $h=10$  km) and the average atmospheric pressure ( $h=10$  km)

relative ratios (Table 4) for some of the discussed relationships shows the presence of strong functional dependence which confirms the reliability of the relationship defined.

Results obtained permit to draw the following conclusions:

1. The application of data related with the nondipole part of the geomagnetic field results in stronger manifestation of the relationships observed and provides for relatively better interpretation from physical point of view.

2. Strongly expressed functional dependences are determined between the intensity of the nondipole geomagnetic field  $H$  (level  $h=10$  km) and the average air pressure and temperature (for the same level) (Figs. 1 and 2), as well as between the vertical component of the nondipole geomagnetic field and the same parameters (for  $h=10$  km, Figs. 3 and 4). The experimental data scatter is described for these dependencies with model curves of second or fourth power.

It can be seen from the comparison of the results from the statistical analysis that the functional dependences obtained, where components of the non-



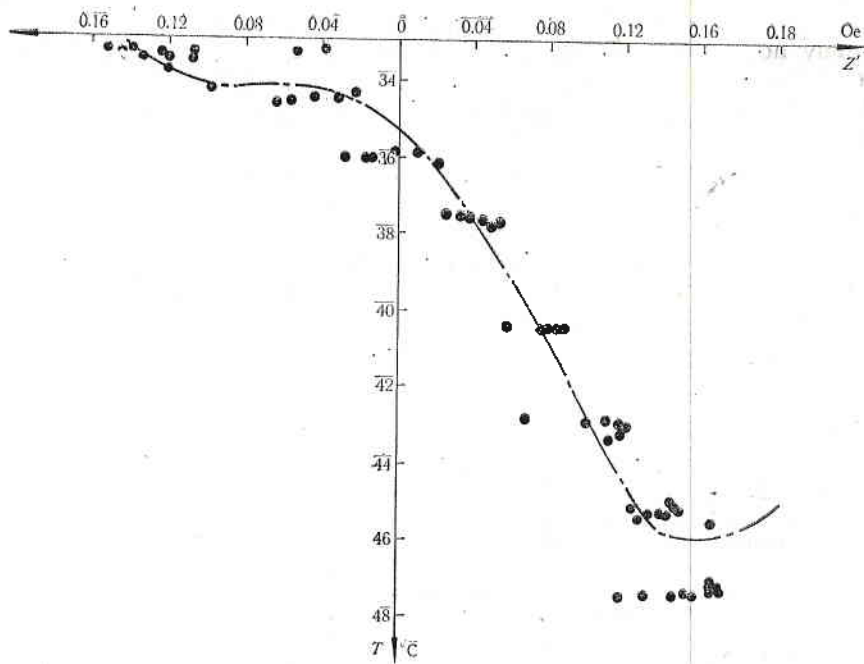


Fig. 3. Regression dependence between the vertical component of the nondipole geomagnetic field ( $h=10$  km) and the average atmospheric temperature ( $h=10$  km)

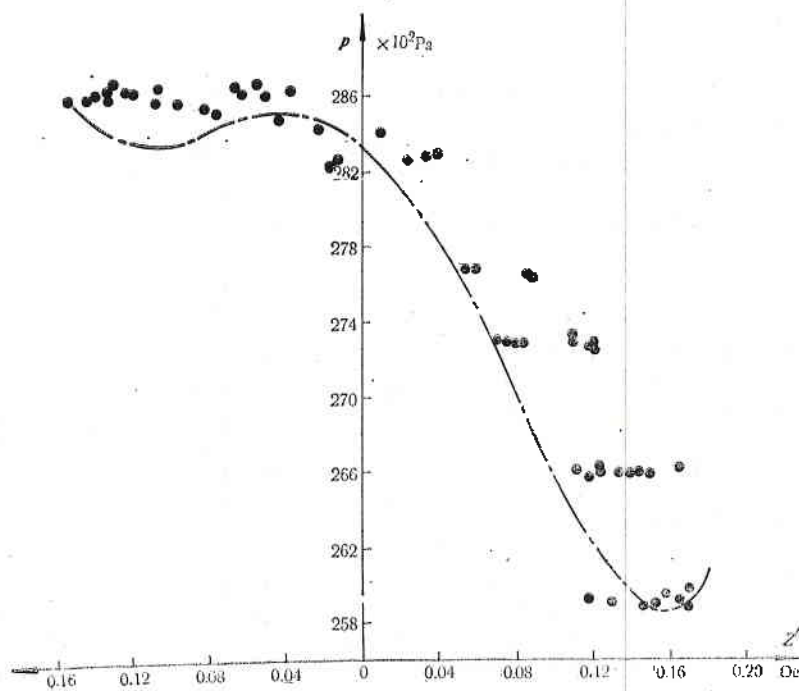


Fig. 4. Regression dependence between the vertical component of the nondipole geomagnetic field ( $h=10$ ) and the average atmospheric pressure ( $h=10$  km)

dipole geomagnetic field are incorporated, are more strongly expressed than the ones related with the components of the observed geomagnetic field. This can be interpreted in terms of the fact that the dipole field due to its smooth development of spatial variations results in "polishing" of the examined dependences and does not provide for the manifestation of peculiarities of the geomagnetic field that are directly affecting the temperature and the pressure at a given level. That is why the reduction of the dipole term provides for a possibility to study the relationship of the nondipole geomagnetic field with the mentioned meteo-fields. The field thus obtained contains all the specifics of the observed one and much more clearly represents its characteristic. The analysis performed confirms this fact as well.

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## Зависимости между аэродинамическими и геофизическими параметрами для района Южноатлантической геомагнитной аномалии

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(Резюме)

В настоящей работе рассматриваются статистические зависимости между аэродинамическими и геофизическими параметрами для района Южноатлантической геомагнитной аномалии. Исследуются 35 параметров, характеризующих поведение некоторых метеополей, поведение геомагнитных полей — наблюдаемого и недипольного, а также проявления солнечной активности. Применяется корреляционный и регрессионный анализ для обработки данных. Как следствие этого получены наилучшие регрессионные модели, описывающие существующие экспериментальные данные при помощи полиномов с второй до четвертой степени.

Установлены очень четко выраженные криволинейные зависимости между полной интенсивностью недипольного геомагнитного поля для  $h=10$  км, с одной стороны, и температурой и давлением для  $h=10$  км — с другой, а также между вертикальной составляющей недипольного геомагнитного поля и теми самыми параметрами, снова для  $h=10$  км. Все они описываются полиномами второй либо четвертой степени.